

Strategic asset-liability allocation for foreign exchange reserves: an application to emerging markets

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In this paper we apply an asset allocation methodology based on risk factors to deal with the portfolio choice problem that Central Banks face. We start from an asset-liability management problem proposed on a set of systematic risk factors, and a portfolio of liabilities that can include explicit and contingent components. In addition, we propose a method to incorporate a capital preservation motive applying a protective put strategy. We also deal with restrictions on the investable asset universe focusing only on the systematic exposure of the portfolio. We illustrate the model for the case of Chile, a small and open economy exposed to commodities.

I. Introduction

In the context of fixed exchange rate regimes, the role of Central Banks' reserves is functioning as a buffer stock against fluctuations, (Archibald and Richmond, 1971). As most of the countries have transitioned to floating exchange rate regimes after Bretton Woods system collapsed (1972-1981), the importance of foreign exchange reserves is less clear. (Batten, 1982) summarized a popular belief among international financial economists at that time: "...since perhaps the single, most important reason for holding reserves had diminished, central banks would not be expected to hold such large stocks of foreign reserves as they had under the fixed exchange rate system". As opposed to this prediction, we can see that total Central Banks reserves are at its maximum historical level (i.e. as a percentage of global GDP or M2).

Among the arguments to justify Central Bank reserve holdings in floating exchange rate regimes, we have mercantilist motives, the goal of keeping the exchange rate undervalued to promote exports, as well as, precautionary motives, where reserves work as self-insurance against sudden-stop risk. (Dooley et al., 2003) analyze extensively the role of foreign exchange reserves and exchange rate interventions in order to maintain a currency undervaluation as part of an export-led growth strategy similar to what South East Asian countries have followed. (Rajan and Subramanian, 2011) analyze exchange rate interventions to prevent

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Dutch disease effects, an economic scenario where a sharp inflow of foreign currency (i.e. the discovery of North Sea natural gas by the Netherlands in the 60s) drives down competitiveness in other sectors. (Daude et al., 2016) focus on Central Bank interventions, especially in the spot market, as a mechanism to prevent excessive swings of the exchange rate, showing that interventions are seen as an effective “corrective” tool. From a different perspective, (Rodrik, 2006) states that raising foreign reserves is the one advice that developing countries have clearly taken to heart after the Asian Crisis, as a way to hedge changes in foreign currency liquidity. In addition, (Flood et al., 2001) show that Central Bank demand for foreign reserves takes into account the opportunity costs of holding reserves, as well as, adjustment costs of using them. (Aizenman and Lee, 2007) illustrate that sudden stops may force costly liquidation of long-term projects, and show how reserves can reduce liquidation costs and raise welfare. (Aizenman and Marion, 2004) justify that the public finance role for the Central Bank in managing the reserves is characterized by the original sin¹. (Bianchi et al., 2012) propose a model that explains how reserves can reduce output costs associated with the rollover risk of foreign borrowing. (Jeanne, 2007), and (Jeanne and Ranciere, 2011) provide a utility-based estimation and calibration of the optimal level of reserves, determined by the trade-off between the consumption smoothing benefits of reserves in the event of a sudden stop and their opportunity costs. Finally, more recent papers have intended to jointly analyze the mercantilist and precautionary motives. (Benigno and Fornaro, 2012) model how the government uses foreign exchange reserves to internalize growth externalities present in the tradable sector, as well as, to provide liquidity to the corporate sector during periods of financial stress faced by the tradables sector.

As the international finance literature tends to acknowledge that reserves may provide welfare benefits mainly from three channels: exchange rate misalignments corrections, liquidity provision during sudden stops or reductions in the probability of negative economic shocks at country level. It’s not as clear how important are the social costs of holding reserves. (Rodrik, 2006) shows that the cost is around 1 percentage point of GDP annually for developing nations. Similarly, (Calvo et al., 1991), and (Filardo and Grenville, 2012), show that sterilized interventions are costly. Therefore, from the cost perspective, we can recognize two concepts: i) The yield give-up that comes from the issuances of bills that pay a higher interest rate (in domestic currency), while reserves are invested in low yield assets (in hard currency); ii) A carry trade effect, that comes from the currency appreciation of high interest rate countries. As a result, determining the optimal level of reserves implies balancing the benefits and social costs. Quantitative rule-of-thumbs that are used to directly/indirectly determine this optimal level are: i) Total reserves should be equal to three months of imports; ii) Guidotti—Greenspan rule that states that country’s reserves should be equal

¹The original sin term was proposed by (Eichengreen et al., 2007) to make characterize countries that are not able to borrow abroad in their domestic currency.

to the short-term external debt; iii) Using a model of contingent liability management, (Caballero and Panageas, 2008) show that gains from precautionary savings can easily exceed the equivalent of cutting a country’s external liabilities by 10% of GDP; iv) The (IMF, 2011) proposed weights on different variables that would determine the optimal level of reserves. The variables considered were: M2, which captures possible outflows during a bank-run crisis; Short-term debt (STD), which includes possible outflows in the contingency of a balance sheet crisis; Other portfolio liabilities (OPL), which captures outflows that occur by the liquidation of foreign short-term portfolio investments in period of market stress; Exports (X), which measure possible outflows during a current account crisis.²

While the literature have tended to focus in the trade-offs of Central Banks reserves, only a few studies have tried to analyze the asset allocation problem of Central Banks reserves. A first motivating quote is based on (Eichengreen, 2005), that states “It may pay to hold reserves in the most liquid market, which tends to be the market in which everyone else holds reserves, but market liquidity is not all that matters. It may worth tolerating a bit less market liquidity in return for the benefits of greater diversification”. Other quantitative models such as (Papaioannou et al., 2006) analyze this problem using a mean-variance optimization framework with liquidity costs to estimate optimal portfolio weights among the main international currencies. (Zhang et al., 2013) approach the portfolio choice problem, considering a conditional value-at-risk minimization, and disappointment avoidance utility maximization. (Aizenman and Glick, 2009) study the asset allocation problem of a Central Bank that invests its reserves in order to minimize the probability of a sudden stop. (García-Pulgarín et al., 2015) solve the asset allocation problem of a Central Bank, separating the Central Bank objective in two sub-problems. First, a Safety Tranch, comprised of liquid, almost default-free and low volatile assets. Second, a Wealth Tranche, that aims to maximize the return with a broader range in the asset space and a longer investment horizon. Finally, the work that motivates this paper is (Villalon and Goldberger, 2012), in so far as I know, the first paper that applies a factor asset liability management to analyze the problem of foreign exchange reserves.

The main contribution of this paper is to apply a normative model to analyze the asset-liability allocation problem that most of the Central Banks face managing their reserves. Using a multifactor approach, related to the arbitrage pricing theory proposed by (Ross, 1976), and the empirical asset pricing literature initiated by (Fama and French, 1992). Our main argument is that Central Banks should shift from dollar allocations to risk allocations, using (Ilmanen and Kizer, 2012) words. While others papers such as, (Dynkin and Hyman, 2004) and (Ang et al., 2009), have made an explicit argument about the importance of assessing systematic and security-specific risk by a Central Bank, as well as, for active factor

²(García-Pulgarín et al., 2015) calculate the adequate level of international reserves in Colombia based on a linear combination of short-term external debt (13.3%), other portfolio liabilities (2.9%); M2 (8.7% and imports (19.2%)

investing in the case of the Norwegian Sovereign Wealth Fund. This paper follows (Villalon and Goldberger, 2012), that extends a portfolio choice model to encompass Central Banks’ multi-objectives (i.e. ‘yield give up’ minimization, liquidity risk hedging, capital preservation and reputational risk), given exogenous constraints on investable assets. Secondly, we propose an empirical methodology to measure the systematic risk exposure of investable assets and liabilities of Central Banks (explicit and contingents). Finally, we implement our methodology to the Chilean case, a small open economy with commodity exposure, documenting potential problems in the implementation, as well as, a quantitative evaluation of the trade-offs of including derivatives in the investable asset spectrum.

The rest of the paper is organized as follows. Section 2 describes the methodology and the model, and explains how assets and liabilities can be analyzed using factor models. Secondly, an optimal factor based portfolio is constructed, following an asset-liability management approach. Thirdly, a portfolio replication has to be made, incorporating only the investable assets. Finally, in order to incorporate a capital preservation objective, we apply a protective put strategy.

II. Methodology

A. Factor Model

We start from a set of n -asset classes (fixed income, equities and currencies) that are traded in the market. In our framework, we will assume that assets returns follow an approximate factor structure, consistent with (Chamberlain and Rothschild, 1982) and (Ingersoll, 1984), the generalization of the classical arbitrage pricing theory developed by (Ross, 1976). This assumption will be important for our empirical application, since idiosyncratic components of returns don’t need to be uncorrelated. In an approximate factor model, asset classes returns are given by:

$$(1) \quad r_t^n = c^n + B^n \tilde{f}_t + \epsilon_t^n$$

Here c^n denotes a n -vector of constants, \tilde{f} the k -vector of systematic risk factors, B^n the $n \times k$ -matrix of factor betas, and $\tilde{\epsilon}_t^n$ the n -vector of idiosyncratic returns. As we mentioned above, one important characteristic of an approximate factor model is that the covariance matrix of idiosyncratic returns doesn’t need to be diagonal.

The literature recognizes three types of factor models: i) Macroeconomic factors, based on observable economic and financial time series; ii) Fundamental factors, created from observable asset characteristics; iii) Statistical factors, that are unobservable and are extracted directly from asset returns. While the empirical asset pricing literature has dedicated significant effort searching for factors, see for example (Cochrane, 2011) and (Harvey et al., 2016). There is no one accepted factor model able to explain the cross-sectional variation of a large

variety of different asset classes. As a consequence, we propose different approaches to address that problem. Firstly, we can consider the global version of two traditional factor models, that are consider baseline model in the asset pricing literature. The Fama-French 3 Factor Model, (Fama and French, 2012), and the global Fama-French 5 factor model, (Fama and French, 2017). Secondly, we use a macroeconomic founded model that is an adaptation of the traditional (Chen et al., 1986). Thirdly, we analyze other factors that are used by practitioners or that could have an economic importance in term of explaining the variation of the market value of Central Banks' liabilities. Specifically, we propose a factor model based on the three-factor specification that maximizes the adjusted R-square of the time series regression of liabilities.

B. Asset Liability Management

Following Bodie and Brice (2014) we formulate the asset liability optimization model as follows:

$$(2) \quad \underset{w_f^*}{\text{maximize}} \quad E[r_f - r_l] + \frac{1}{2}(1 - \rho)\text{Var}[r_f - r_l]$$

Here the return of equity (r_e) is obtained as the difference between the factor based portfolio return (r_f) and the return of the liabilities (r_l), where $\rho > 0$ is the risk aversion coefficient. Two assumptions are implicit in this specification. First, Central Banks are mean-variance optimizers, and consequently will not hedge unanticipated shocks to time varying investment opportunities. Empirically, this assumption can be defended since, for a wide variety of preferences, hedging demands for risky assets are typically small, even nonexistent as Ait—(Ait-Sahalia and Brandt, 2008) and (Brandt, 2009) have shown. Second, we assume that Central Bank optimal portfolio choice is not influenced by Central Banks solvency or leverage.³

In this case, we can solve the unconstrained optimal factor based portfolio analytically:

$$(3) \quad w_f^* = \frac{\mu_f}{(\rho - 1)\Omega_f} + \frac{\Omega_{f_l}}{\Omega_f}$$

Here μ is a vector with the expected risk premiums, Ω_f is the variance-covariance matrix of the risk factors, and Ω is a vector that contains the covariance between the factors and the liabilities.

³(Hall and Reis, 2015) analyze financial stability of central banks. The literature tends to acknowledge that capitalization of Central Banks could affect monetary policy decisions or independence from central governments.

C. Central Bank Liabilities

The liabilities that Central Banks hedge are intimately related with the institutional framework of the country. For example, the IMF (2014) states that, in order to identify Central Banks' liabilities is important to consider the amount of debt issued, sovereign credit risk, interest rate duration and exchange rate risk, as well as, contingent liabilities which dynamics (and size) are usually unobservable. Specifically, we focus our attention to explicit debt, debt issued by the Central Bank, and contingent liabilities related to the provision of foreign exchange liquidity and financial sector solvency. While the size and the evaluation of the cost of debt can be measured from Central Bank's traded bonds or indirectly by the bonds issued by the Central Government, the market value of contingent liabilities have to be estimated. Two approaches are proposed to measure the market value of liabilities. Firstly, the implicit responsibility of financial sector's debt that a Central Bank undertake is estimated following the methodology of (Ronn and Verma, 1986). The main idea is that banks' assets follow a stochastic process with a volatility that can be estimated from banks' stock volatility. The liability for the Central Bank would be equal to a put option on banks' assets with a strike price equal to banks' debt. In a Black-Scholes framework, the equity value of a bank will be equal to a call option on assets with strike price equal to the market value of debt. In this context, (Ronn and Verma, 1986) derive a closed form solution for the equity value of a bank. As the market value of assets is unobservable, the volatility of the market value of assets' is estimated from the following set of equations.

$$\begin{aligned}
 E &= VN(x) - \rho BN(x - \sigma_V \sqrt{T}) \\
 x &= \frac{\ln\left(\frac{V}{\rho B}\right) + \sigma_V^2 T/2}{\sigma_V \sqrt{T}} \\
 \sigma_V &= \frac{\sigma_E E}{VN(x)}
 \end{aligned}
 \tag{4}$$

where ρ is a measure of the relevant threshold of debt that will capture bank's default⁴; σ_V is the volatility of banks' assets; T is the option maturity; V and E are the value of assets and equity respectively; $N(\cdot)$ is the cumulative normal distribution function.

Finally, abstracting from the dividend payments made by banks, the cost of guaranteeing \$1 for the Central Bank will be given by:

$$y = \frac{\ln(B/V) - \sigma_V^2 T/2}{\sigma_V \sqrt{T}}
 \tag{5}$$

⁴(Ronn and Verma, 1986) calibrate this number in 0.97.

On the other hand, the contingent liability related with the provision of foreign exchange liquidity is measured as a payer swaption on the spread of foreign currency borrowing. The main idea is that Central Banks provide liquidity at the 'long-run' spread, something that will be valuable in moments of transitory high short term cost of borrowing. Modelling the swap rate as the spread on foreign currency using a Vasicek model, we could estimate the value of payer swaption with a fixed swap rate following (Hiibnerl, 1997). The closed form solution of a payer swaption under a Vasicek model will be given by:

$$\begin{aligned}
\Pi &= P(r, t, T)N(-d_2) - \exp\left(r_x(s - t)\right)P(r, t, s)N(-d_1) \\
\sigma_P &= v(t, T)\frac{1 - \exp(-a(s - t))}{a} \\
d_1 &= \ln\left(\frac{P(r, t, s)\exp(r_x(s - t))}{P(r, t, T)}\right) + \sigma_P/2 \\
d_2 &= d_1 - \sigma_P \\
v^2(t, T) &= \sigma_r^2(1 - \exp(-2a(T - t)))/(2a) \\
P(r, t, T) &= A(t, T)\exp(-B(t, T)r) \\
B(t, T) &= \frac{1 - \exp(-a(T - t))}{a} \\
(6) \quad A(t, T) &= \exp\left(\frac{(B(t, T) - T + t)(a^2\bar{r} - \sigma_r^2/2)}{a^2} - \frac{\sigma^2 B(t, T)^2}{4a}\right)
\end{aligned}$$

Alternatively, we can take an empirical approach to estimate the risk exposure to Central Banks' contingent liabilities. Firstly, the financial bailout cost can be estimated indirectly from the absolute value of idiosyncratic returns of the financial sector with respect to the local stock market, that are below the 5th percentile. Secondly, the contingent liability related to the provision of liquidity in foreign currency, could be estimated as the absolute value of returns below the 5th percentile of a total return index of bonds issued in foreign currency. It's worth noting that, is assumed that financial solvency's liability returns are measured in local currency, while liquidity's liability is in foreign currency.

D. Portfolio Replication

The optimal exposure to risk factors presented above, balances the return maximization (or yield give-up minimization) and liability hedging objectives of a Central Bank with a coefficient of risk aversion ρ . Nevertheless, a Central Bank usually is restricted to a subset of asset classes that are defined exogenously (e.g. constitutional amendment). Given a m-subset of investable assets, the Central Bank will replicate the optimal factor based portfolio minimizing the weighted

difference exposure to systematic risks, as follow:

$$(7) \quad \underset{w_a^*}{\text{minimize}} \quad [w_f^* - w_a^* B^m] W [w_f^* - w_a^* B^m]^T \quad \text{subject to} \quad w_a^* \geq 0, w_a^* \leq 1$$

Where w_a^* is the portfolio that replicates the systematic exposure of the optimal factor based allocation (w_f^*); B^m is a matrix of k-factor betas for the m-subset of investable assets; W is a weighting matrix with $\sum_{i=1}^k d_{ii} = 1$ and $0 \leq d_{ii} \leq 1$.

E. Capital Preservation

In addition to the return maximization and liability hedging objectives, (Berkeelaar et al., 2010) and others argue that Central Banks have capital preservation and short-term liquidity needs. In order to incorporate this third objective, we propose a protective put approach.

The protective put strategy is implemented assuming a Black and Scholes framework, such that a Central Bank would be able to replicate a put option with a strike price equal to the current level of reserves (δ). In this setting a put option can be replicated shorting δ_P units of the underlying asset, in this case the foreign exchange reserves with value S_t , given a time horizon τ). At the same time that we invest ϕ_P at the risk-free rate. The dynamic replication can be formulated analytically as follows:

$$(8) \quad \begin{aligned} P_t^* &= \varphi_P + S_t \Delta_P \\ &= PV(\delta)N(-d_2) - S_t N(-d_1) \end{aligned}$$

Where $N(\cdot)$ is the cumulative standard normal distribution; $PV(\delta)$ is equal to the present value at the risk-free rate of the minimum safety level (δ), taking an horizon τ ; d_1 and d_2 are the well-known expressions of the Black-Sholes formula.⁵

Finally, the reserves at time t will be allocated between the risk-free asset and the portfolio that replicates the systematic exposure of the optimal factor based allocation, as follows:

$$(9) \quad PV(\delta)N(-d_2) + S_t(1 - N(-d_1))$$

In order to implement the protective put strategy, two important parameters are needed. The volatility of the portfolio that replicates the systematic exposure of the optimal factor based allocation and the put option maturity. Firstly, the

⁵ As is well known from the Black-Scholes formula, the expressions d_1 and d_2 are given by:

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{\tau}} \left(\ln \frac{S_t}{\delta} \right) + \left(r + \frac{1}{2}\sigma^2 \right) \tau \\ d_2 &= d_1 - \sigma\sqrt{\tau} \end{aligned}$$

Here σ is the volatility of the portfolio that replicates the systematic exposure of the optimal factor based allocation.

volatility of the reserves' portfolio is estimated as the sample standard deviation of historical returns of the proposed asset allocation. Secondly, we assume a 1 year maturity as the relevant horizon for the Central Bank.

Finally, we propose a straightforward comparison of three competitive objectives that Central Banks desire to attain: i) Yield-give up minimization; ii) Tracking error with respect to the positive returns of the liabilities portfolio; iii) Minimization of the maximum drawdown of the reserves' portfolio. Acknowledging, that the importance of each of this objectives can be also endogenous to the relevant institutional factors of each country, we propose a simple starting point based on a z-score ranking similar to one used by (Chincarini, 2006) in stock rankings. The main idea is that for each of the replicating portfolios, given by the liabilities proxy and the factor model, we calculate the z-score for the specific metric, and we calculate a total score for each of the portfolios.

III. An Illustration: the case of Chile

A. Context

Chile is a small (0.38% of World GDP at PPP in 2015), open economy (60% openness index in 2015) that is mainly exposed to commodity prices (metals and mining represent 57% of the total exports in 2014⁶). The Central Bank of Chile is an autonomous entity granted by Chile's National Constitution. According to the Basic Constitutional Act of the Central Bank of Chile, its main objectives are to safeguard the stability of the currency and the normal functioning of internal and external payments. Since 1999, the foreign Exchange policy is led by a floating exchange rate, although the Central Bank maintained the right to intervene in the foreign exchange markets.

Historically, the foreign exchange reserves were mainly used by the Chilean Central Bank to maintain the external value of their currencies at fixed rate. Nevertheless, currently the role of reserves are more related to self-insurance and foreign liquidity, De Gregorio (2011).

Consistently with IMF's guidelines for foreign exchange reserve management, the Chilean Central Bank manages reserves with the following strategy: i) A liquidity tranche (24% of the total reserves); ii) A medium-term tranche (61% of the total reserves); iii) A diversification tranche (25%). These three sub-portfolios, plus cash maintained by the Chilean Treasury in the Central Bank, and other assets (special drawing rights and gold) compose the total foreign exchange reserves. In term of currency composition, the main exposures are: US Dollar (65%), Euro (17%), Australian Dollar (5%), and Canadian Dollar (5%). The average duration is 25 months.

In a way to summarized historical events that have been relevant for foreign exchange reserves management in emerging economies in general, and for the

⁶The Atlas of Economic Complexity (2014).

Chilean case in particular, we described briefly four events: the 1982 Chilean Banking Crisis, the Asian Crisis, the interventions post-floating exchange rate regime, and the 2008 Financial Crisis.

1982 FINANCIAL CRISIS

(Harberger, 1985) documents that foreign exchange reserves were roughly US\$ 1 billion in 1978. At the end of 1979 they have grown up to US\$ 2.3 billion, reaching US\$ 4 billion during the period 1980-1981. In mid-1981 when Chile entered its worst economic crisis since the 1930s, the peg was at the fixed rate of 1 USD = 39 CLP. In June 1982, a 18% devaluation was announced and a further monthly devaluation of 0.8% was pre-announced. As of May 1983 the foreign exchange reserves had fallen -44% in YoY basis. Given the significant exposure to foreign exchange risk of the private sector, the currency depreciation lead to a collapse of the financial system. The government acted trough the Central Bank and other agencies, managing the liquidation process of financial institutions, purchasing non-performing loans, creating a program of subsidized foreign currency for debtors, mediating debt restructuring, and finally creating a recapitalization program called 'Popular Capitalism' that offered loans to acquire stocks in industrial and financial institutions. (Restrepo et al., 2009) estimate that the total cost of the 1982 Financial Crisis for the Chilean Central Bank was roughly 40 percentage points of the GDP.

ASIAN CRISIS

(Cowan and De Gregorio, 2007) document the Chilean experience before and after the Asian Crisis. During the 1990s, the Chilean floating currency was pegged to a band. During a period of significant capital inflows – given by the good economic expectations and the high interest rate gap between local and international yields – foreign exchange rates grew consistently (+38% between Nov-97/Jan-96). In 1997, when the Tom Yam Kung crisis started in Thailand, a significant outflow from emerging markets occurred. The reaction of the Chilean Central bank was defending the Chilean Peso, lifting the monetary policy rate from 9% to 19%, and selling US\$ 4 billion in reserves. In September 1999, after liquidating US\$ 4 billion (roughly 25%) the Central Bank moved to a freely floating exchange rate.

EXCHANGE RATE INTERVENTIONS POST-FLOATING REGIME

Since the adoption of the fully floating regime in 1999, the Central Bank has intervened the exchange market in four occasions. As (Claro and Soto, 2013) document, in 2001 and 2002 interventions were mainly explained by the financial turmoil in Argentina and the political election in Brazil. In 2001, the intervention program consisted of spot sales of US dollars, and the program was implemented through the issuance of dollar-denominated debt. Later, in 2008

after a significant appreciation of the Chilean peso, and considering that reserves indicators (reserves to imports; reserves to M2; reserve to GDP) were relatively low, (De Gregorio, 2011). The Central Bank decided to increase the amount of reserves in 5 percentage points of the GDP (US\$ 8 billion). In September 2008, the Central Bank abandoned the plan following Lehman Brothers' collapse, after accumulating only US\$ 5.75 billion (around 75% of the original reserve acquisition plan). Finally, the last intervention occurred in January 2011, the Central Bank decided to intervene, increasing its foreign exchange reserves from 13 to 17% of the GDP. A side effect of these last two sterilized interventions has been an increasing currency mismatch between assets (reserves in hard currencies) and liabilities (currently issued only local currency).

2008 FINANCIAL CRISIS

In September 2008, motivated by the foreign currency liquidity situation experienced after the Lehman Brothers collapse. The Chilean Treasury decided to bid US\$ 700 million in US dollar deposits at 3.39%, and also inject US\$ 1.05 billion in the local financial system from the Sovereign Wealth Funds. In addition, the Central Bank decided to offer liquidity in local and foreign currency through swaps and repos with local banks. As (García Cicco and Kawamura, 2014) document, the collateral assets accepted by the Central Bank, in a first phase (starting in October, 2008) were only banks deposits, while in a second phase (from January, 2009) the list was further expanded, including government bonds.

In conclusion, as the (of Chile, 2012) states, the main function of the foreign exchange reserves is to ensure access to liquidity in foreign currency in order to intervene in the foreign exchange market or to provide temporary liquidity in foreign currency in specific exceptional circumstances. In absolute terms, Chile currently has US\$ 39.7 billion, while in relative terms reserves are about 16% of the GDP and 27.6% of the M2. In Figure 1 we can see the three most common benchmarks used to evaluate the size of foreign exchange reserves. As we can see, Chile has a level of reserves above the different rules, three months of imports is equal to 7.01% of GDP, the Guidotti rule implies a 5.66% of GDP and IMF (2011) would be equivalent to 7.4% of GDP⁷.

B. Systematic Factors

As a starting point, we describe a set of factor models that mimic different global systematic risks that explain the variation of investable assets, as well as, the relevant liabilities for the Central Bank. The global version of the three factor model, (Fama and French, 2012), is a natural benchmark used in the literature. The Fama-French 3 factor model (FF3) has three factors: the market risk premium (MRP), Small minus Big (SMB), and High minus Low (HML). In

⁷The IMF (2011) rule is implemented as the sum of 40% of the foreign short term debt, 5% of M3 and 5% of total exports.

the interpretation of (Vassalou, 2000), the market risk premium would be mainly related to surprises in GDP growth, while the SMB and the HML factors are mainly related to systematic default risk. The global version of the five factor model, (Fama and French, 2017), adds a robust versus weak profitability (RMW), and a low versus high investment (CMA) factor.⁸ In a production asset pricing model, (Hou et al., 2014) show how this two new factors are a consequence of firms rational investment policies. (Chen et al., 1986) propose a macroeconomic founded factor model that takes the market risk premium (MRP), inflation surprises, industrial production surprises, the term premium (TP) and the expected inflation.⁹

Finally, we propose a set of other factors that can be potentially important. A Economic factor, that is constructed as the excess of return of equities, commodities and real estate over the short term interest rate¹⁰; a Credit factor that is constructed as the excess of return of high yield bonds over investment grade corporate bonds¹¹; a EM equities factor that is constructed as the excess of return of emerging market stocks over the stocks of developed markets¹²; a liquidity factor that is constructed from the differential return of global small and large stocks¹³; a Real Rates factor that is measured by the return of inflation linked bonds¹⁴; an Inflation factor that is constructed by the differential return between nominal and inflation-linked bonds¹⁵; a Carry Trade factor, (Burnside et al., 2011), that is measured by the return of investing in currencies of countries with high interest rates versus low interest rate currencies¹⁶; a Commodity factor that is constructed as the excess of return of commodities over the short-term interest rate¹⁷; a Emerging Market Currency factor that is constructed as the return of a basket of relevant emerging market currencies against the US dollar¹⁸.

C. Liabilities

From the Chilean Central Bank perspective we assume that the relevant liabilities are the explicit debt (nominal and inflation linked) outstanding in local

⁸The global version of the Fama-French factors are available in Kenneth French's website.

⁹In our implementation of the model we measure inflation surprises and industrial production surprises using the global version of Citi's inflation and economic surprise indexes that are available in Bloomberg. The term premium (TP) is measured by the excess of return XX index and XX index. The expected inflation is measured by the difference in the average yield of a nominal global bond index (BofA Merrill Lynch Global Government) and the inflation-linked global bond index (BofA Merrill Lynch Global Inflation-Linked Government).

¹⁰Equally weighted return of MSCI ACWI Index, Bloomberg Commodity Index and SP Global REIT Index USD over the BofA Merrill Lynch US 3-Month Treasury Bill Index

¹¹Return of the BofA Merrill Lynch US High Yield Index over the BofA Merrill Lynch 1-10 Year AAA-A US Corporate Government Index.

¹²MSCI Emerging Markets Index over the MSCI World Index.

¹³MSCI World Small Cap Index over the MSCI World Large Cap Index

¹⁴Barclays World Inflation Linked Bonds TR

¹⁵BofA Merrill Lynch-Global Government over the Barclays World Inflation Linked Bonds TR

¹⁶Deutsche Bank Currency Carry USD Excess Return Index

¹⁷Bloomberg Commodity Index over the BofA Merrill Lynch US 3-Month Treasury Bill Index

¹⁸Equally weighted return between CNY/USD, BRL/USD, RUB/USD, INR/USD and ZAR/USD, such that a positive return is equal to an appreciation of EM currencies.

currency, as well as, contingent liabilities related to financial sector solvency and foreign liquidity. The risk exposure to explicit liabilities is measured from a total return index of bonds issued by the Central Government and the Central Bank in local currency provided by Riskamerica, a private company that provides fair value pricing for the Chilean fixed income market, Figure 2.

On the other hand, we estimate the risk exposure to contingent liabilities applying the methodologies described above. Firstly, we estimate the value of the liability associated to financial sector solvency following the option pricing approach. As we have described above, relevant parameters that are needed are: i) The volatility of equity of banks, that is obtained from a GARCH (1,1) estimation on monthly returns of the MSCI Chile Banks index (1989-2016); ii) The assets and liabilities of banks, obtained annually from the Superintendency of Banks and Financial Institutions. In the Figure 3 we present our estimated cost of the liability as a percentage of \$1 of the underlying asset. The variations in the market value of the liability related to the provision of a bailout, can be calculated as the monthly percentage of estimated value. Alternatively, we can estimate the returns related to the contingent liability of providing a financial bailout, from the absolute value of the residuals that are below its 5th percentile, obtained from the regression of the MSCI Chile Banks index on the MSCI Chile index. As we can see, this will imply that the estimated returns will follow a jump style process. In Figure 3, we document: (a) the estimated market value of the contingent liability, following the option pricing methodology; (b) the returns of the contingent liability estimated as the changes in the estimated market value; (c) the returns of the contingent liability estimated from the idiosyncratic returns methodology.

Secondly, we estimate the value of the liability associated to liquidity in foreign currency. As we have described above, relevant parameters that are needed are: i) The long-run interest rate spread in foreign currency, that is obtained from a Hodrick-Prescott filter estimation on the spread between the borrowing rate in foreign currency paid by the Chilean financial sector over the Libor-3m (1992-2016); ii) The volatility of the interest rate spread that is obtained as the sample standard deviation of the spread; iii) The external short term debt of Chile. In Figure X we present our estimated cost of the liability as a percentage of the GDP. The variations in the market value of the liability related to the provision of foreign, can be calculated as the monthly percentage of estimated value. Alternatively, we can estimate the returns related to the contingent liability of providing foreign liquidity, from the absolute value of the returns that are below its 5th percentile of the JPM EMBI Global Chile – Total Return Index. In Figure 4, we document: (a) the estimated market value of the contingent liability, following the swaption pricing methodology; (b) the returns of the contingent liability estimated as the changes in the estimated market value; (c) the returns of the contingent liability estimated from the idiosyncratic returns methodology.

Finally, we construct four liabilities portfolios based on the different methodologies used to estimate each component of the contingent liabilities, and also

the relative size of the different components that are assumed to construct the portfolio. Firstly, we can estimate the relative size of each components, as: i) The explicit Central Bank debt (7% of GDP); ii) The contingent liability associated to a financial bailout (0.5% of GDP), calculated as the average cost presented in Figure 3 multiplied by an average Credit to GDP (IMF) of 62%; iii) The contingent liability associated to foreign liquidity (7% of GDP), calculated as the average external debt to GDP (IMF). Alternatively, we consider a 10% weight for the explicit debt, and equal weights for the two contingent liabilities analyzed (45%).

D. Factor Models

We start from a large set of asset classes that include government and corporate fixed income, investment grade and high yield bonds, developed and emerging market bonds and equities, and derivatives. In Table 1 we estimate time series regressions for the Global Fama-French 3 Factor Model. As we can see, from these regressions we can learn from the exposure of different assets to the relevant systematic risks. For example, we can see that the MSCI Chile has a positive exposure to the three factors, and the three factors can explain 26.7% of its variance. While the Put Option SP index has a negative exposure to the Market Risk Premium, and positive exposure to the other two factors, and the fraction of the variance that is explained by the model is 34%.

Similarly, in Table 2 we document the time series regressions for the Global Fama-French 5 Factors. In Table 3 the Chen, Roll, and Ross Model regressions are documented

Finally, in Table 4 to Table 7 we calculate the same time series regressions for the macro-factors proposed above. Each of the Macro Models documented below is decided based on the maximization of the adjusted- R^2 of the liabilities portfolio. In other words, for each of the time series of liabilities returns estimated we find the combination of at most three factors that better explain the variation of the liabilities portfolio.

As we can see, certain models are better explaining the variation of equities or fixed income assets. Having different factor models will allow us to learn how important is to be able to explain the variation of traded assets versus the non-tradable liabilities.

E. Optimal Factor Allocation

Given the factor models, and the returns of the liabilities portfolios described above. In Figure 5 to 11 we document the optimal factor allocation for each factor-liabilities combination. As we can see, from the analytic solution of the optimal portfolio, the allocation depends on the expected risk premium, the correlation among factors, and the correlation of each factor with the liabilities portfolio. For example, in Figure 5 we show that at low levels of risk aversion, significantly

high long positions in the HML factor are calculated. Conversely, in Figure 6 we show that the optimal factor allocation would be a long position in the RMW factor and a short position in the HML factor. As is well known from the mean-variance optimization literature, changes in parameters have large impacts in the optimal allocation. Consequently, the decision with respect to the relevant factor model, and the risk aversion parameter, are a key steps of the asset allocation process.

F. Portfolio Replication

As the optimal factor allocation are possible not investable directly. However, currently there are ETFs that intend to replicate the most common risk factors such as iShares Russell Microcap Index IWC (Size), Guggenheim SP 500 Pure Value RPV (Value) or QuantShares U.S. Market Neutral Momentum Fund (Momentum). We apply the replication method proposed in Section II.D, based on the asset classes' exposure to the systematic risks that are estimated by each factor model, presented in Figure 12 to Figure 17, including equities and short sales constraints. As we can see, in Figure 12 we present the asset allocation for the Fama-French 3 Factor Model by each of the proposed proxies of Central Bank's liabilities, considering only fixed income assets. As we can see, at lower level of risk aversion the optimal reserves portfolio would be 100% in High Yield Bonds, while at higher level of risk aversions the reserves' portfolio is more diversified, specially to government bonds of developed markets and Chinese money market instruments. Similar results are presented in Figure 13 for the Fama-French 5 Factor Model. In Figure 14, we show the fixed income replicating portfolio for the Macro Model I. As we can see, the portfolios are more exposed to US Treasuries, and the diversification is shifted to Korean money market instruments, High Yield bonds and emerging market bonds in hard currency. In Figure 15, the replicating portfolio for the Macro Model II is mainly exposed to Chinese money market instruments. In the case of the Macro Model III, Figure 16, the largest weight in the portfolios are emerging market bonds in local currency. In Figure 17, the allocation for the Macro Model IV are mainly exposed to global inflation-linked bonds and Australian government bonds. Alternatively, we can replicate the optimal factor allocation for a broader set of asset classes. In Figure 18, we present the results for the Fama-French 3 Factor Model. As we can see, the most important asset classes are Copper, developed countries equities and put options on the SP 500. In Figure 19, we present the results for the Fama-French 5 Factor Model, the most important asset class is Japanese equities. For the Macro Model I, in Figure 20, the most important assets are swaptions and Global Government bonds. In Figure 21, the results for the Macro Model II show that the most important assets are emerging market equities and swaptions. The results presented in Figure 22 are the replicating portfolios for the Macro Model III, the most important assets are swaptions and put options on the SP 500. In Figure 23, the Macro Model IV replicating portfolios are mainly exposed to the

global inflation-linked bonds.

G. Evaluation the Replicating Portfolios

As we have seen, the decision with respect to the relevant factor model, the measurement of the liabilities, or the risk aversion of the Central Banker influence the asset allocation significantly. In order to compare the different replicating portfolios obtained by factor model and liabilities proxy, we apply the protective put strategy proposed in the methodological section to the portfolios that are consistent with an arbitrary risk aversion of 20. Therefore, for each of the selected portfolios we estimate the in-sample volatility of the historical monthly returns. The main input to estimate the allocation in the risk-free asset that would be needed to replicate a protective put of an at-the-money put option on the risky portfolio. As we can see, in Figure 24 we document the allocation in the risk-free asset that is obtained by the proposed protective put strategy. The fixed income strategies tend to be less volatile, and consequently they require a lower investment in the risk-free asset.

Considering the different replicating portfolios weighted by their required allocation in the risk-free asset, we provide a quantitative comparison for the models that we have proposed, as well, different proxies of the liabilities that are relevant for the Central of Chile. In Figure 25, we show the relevant metrics for the fixed income spectrum, yield give-up, tracking error of liabilities, and maximum draw-down. Similarly, in Figure 25 we document the same metrics for the replicating portfolios constructed for the broader spectrum of assets.

Given the metrics, in Figure 27 we can calculate the z-score for each metric, and calculate the total score by factor model and proposed liabilities proxy. Given the magnitude of the metrics, has to be interpreted as lower is better. Interestingly, we can see that the total scores tend to favor factor models that include macroeconomic factors, that are usually not supported by the data in the empirical asset pricing literature. For example, if we consider only fixed income strategies, and the Liabilities 1 and 4, the replicating portfolio with the best score is the Macro Model II; in the case of Liabilities 2 the best replicating portfolio is the Macro Model IV; for Liabilities 3 is Macro Model II. On the other hand, if we consider the entire spectrum of investable asset classes, we can see that the best replicating portfolios are the Macro Model IV, for Liabilities 1, 2 and 4, the Chen, Roll and Ross for Liabilities 3. In Figure 28, we document the composition of the risky portion of the reserves portfolio that would be selected given by the z-score rule, and a fixed income spectrum. As is worth notice, Asia Pacific government bonds are include in the allocation, as well as, European bonds or US Treasuries in the case of Liabilities 3. Conversely, in Figure 29, we show the portfolios that would be selected if there is no restriction to the fixed income spectrum. Interestingly, the portfolio allocation change significantly. While a global diversified index of inflation-linked bonds is important three of the cases, emerging market bonds and equities, commodities such as gold, and derivatives have important weights in the

allocation.

IV. Conclusion

The portfolio choice problem that Central Banks face in the investment process of their foreign exchange reserves is subject to different requirements. Historically, reserves were mainly used to intervene the foreign exchange rate market. However, in a world that is more dominated by floating exchange rate systems, Central Bank reserves have continued to be important. Independently of the rationality behind the amount of reserves that a Central Bank hold, countries desire to minimize the so-called 'yield give up'. In other words, they prefer to reduce the social cost of holding reserves, that tend to exist because reserves are invested in lower-yielding assets with respect to its cost of funding. On the other hand, Central Banks in modern economies play a role of lender of last resort. This is requirement was specially important during the Great Recession. For example, Central Banks in some cases had to provide liquidity in foreign or local currency, or in some cases even deal with solvency problem of financial institutions. All of these objectives are mediated by the institutions of each country, and Central Banks in principle would have to invest in a way that maximize its capacity to act, given the constraints that its face, such as capital preservation requirements, reputational risk, or restricted investment universe. In this paper we propose an asset allocation methodology for Central Bank reserves. We assume that Central Banks are risk averse mean-variance agents that care about a specific number of observable risk factors. As a result, we start from an institution that decide a factor allocation in an asset-liability framework. The relevant liabilities are composed by explicit debt that pays a variable cost that is observable, and two unobserved contingent liabilities that are related with a potential bailout of the banking sector and the provision of foreign liquidity. In addition, we include a capital preservation motives applying a protective put strategy. Finally, given the relevant investment universe, we search for the investable portfolio that has the desired systematic risk exposure, and balance the different objectives of the Central Bank (yield give-up minimization, hedging of liabilities, and capital preservation). We illustrate the model for the case of Chile, a small open economy that is exposed to commodities. Based on different assumptions, we measure the different components of the liabilities that the Central Bank faces, as well as, we test different factor models. Importantly, we find that factor models that are based on macroeconomic factors lead to a better balance of the different objectives. We also find that, if we focus on fixed income strategies the portfolios that result include Asia Pacific government bonds, European bonds and US Treasuries. However, if a more ample spectrum of assets is considered, a more global diversified index of inflation-linked bonds is chosen, emerging market bonds and equities, commodities such as gold, and derivatives, such as swaptions or put option are also included.

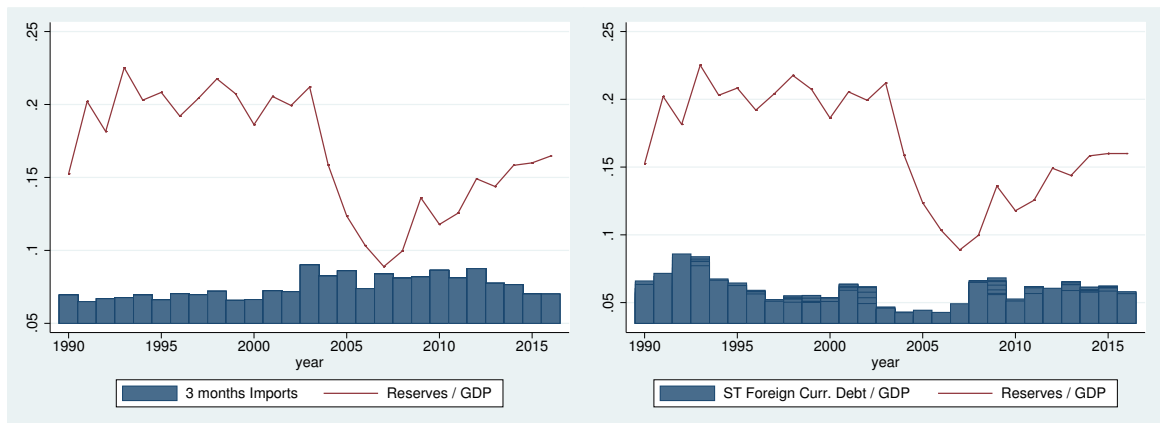
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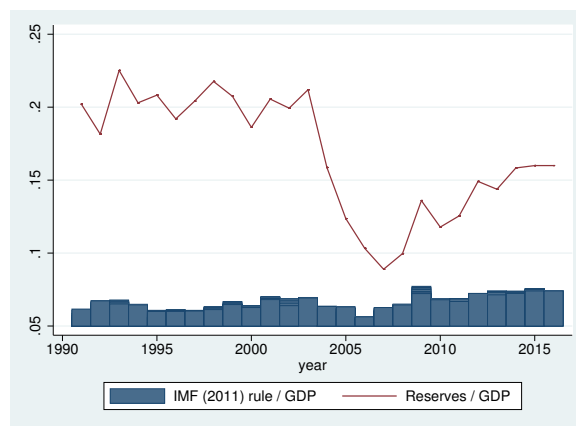
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(a) Central Bank Reserves - 3 months imports rule

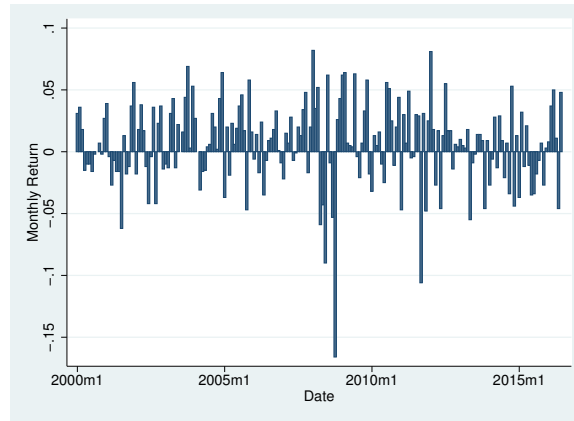
(b) Central Bank Reserves - Guidotti's rule

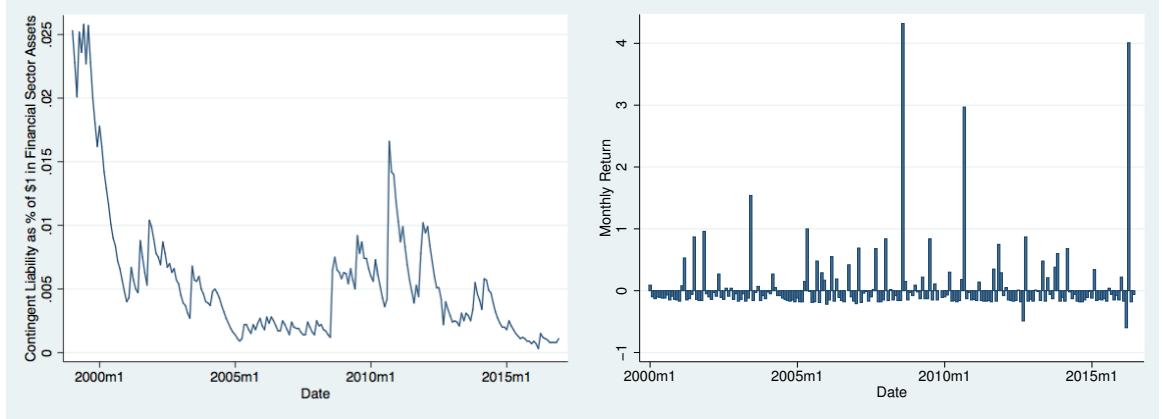


(c) Central Bank Reserves - IMF (2011)

Figure 1. : Chile Central Bank Reserves - Benchmarks as a % of GDP

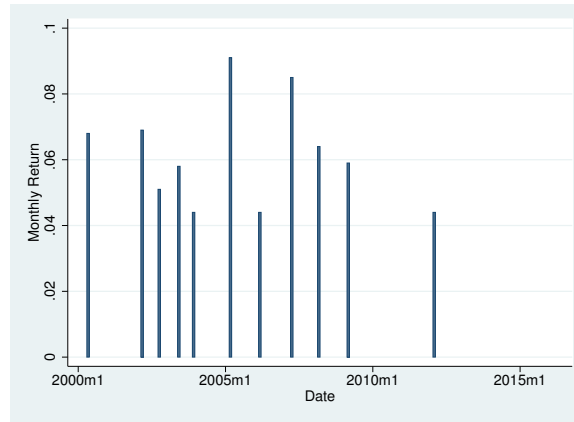
Figure 2. : Returns of Market Value of Debt





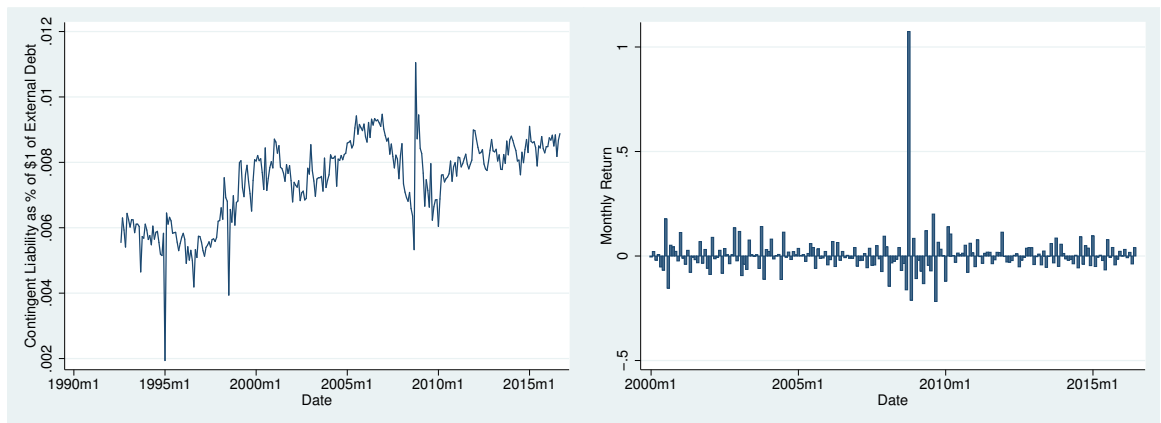
(a) Contingent Liability - Financial Bailout

(b) Return of Contingent Liability - Financial Sector Solvency (Option Pricing Methodology)



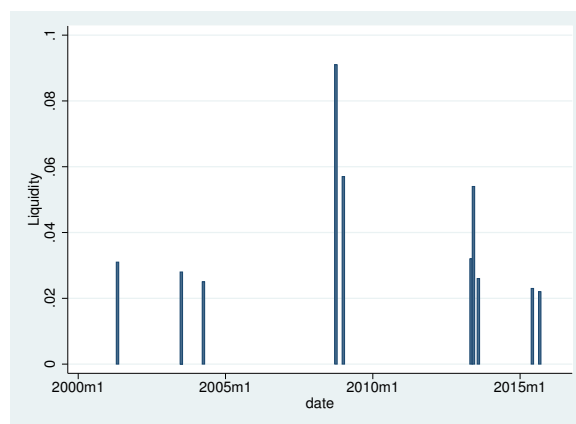
(c) Return of Contingent Liability - Financial Sector Solvency (Idiosyncratic Return Methodology)

Figure 3. : Contingent Liability - Financial Sector Solvency



(a) Contingent Liability - Foreign Liquidity

(b) Return of Contingent Liability - Foreign Liquidity (Swaption Methodology)



(c) Return of Contingent Liability - Foreign Liquidity (Return Methodology)

Figure 4. : Contingent Liability - Foreign Currency Liquidity

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
MRP	0.816*** (0.100)	-0.0487* (0.0195)	-0.0152 (0.0145)	-0.0170 (0.0129)	-0.0470* (0.0195)	-0.0474* (0.0237)	-0.0319* (0.0147)	-0.0274 (0.0458)	0.368*** (0.0460)	0.465*** (0.0503)	0.194*** (0.0340)
SMB	0.652** (0.160)	-0.0884* (0.0356)	-0.0667* (0.0259)	-0.0625 (0.0260)	-0.0414 (0.0249)	-0.114** (0.0392)	-0.0779** (0.0284)	-0.0477 (0.0289)	0.246* (0.121)	0.265*** (0.0714)	0.0688 (0.0449)
HML	0.138 (0.151)	-0.0139 (0.0336)	0.0126 (0.0236)	-0.0079 (0.0260)	-0.0190 (0.0249)	-0.0829 (0.0428)	-0.0169 (0.0249)	-0.0771 (0.0752)	0.119 (0.0836)	0.137* (0.0625)	0.122*** (0.0346)
Const.	0.00476 (0.00355)	0.00535*** (0.000752)	0.00571*** (0.000632)	0.00359*** (0.000570)	0.00443*** (0.000638)	0.00757*** (0.000970)	0.00685*** (0.000669)	0.00333*** (0.000789)	0.00291 (0.00189)	0.00300* (0.00139)	0.00342** (0.00107)
N adj. R-sq	313 0.266	313 0.036	313 0.012	313 0.012	313 0.032	313 0.030	313 0.023	312 0.001	312 0.195	223 0.374	235 0.247
	JPM GBLEM	JPM EMBI	B Barclays World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI World	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
MRP	0.0529 (0.0293)	0.448*** (0.0775)	0.177*** (0.0473)	1.063*** (0.0154)	0.978*** (0.00561)	0.976*** (0.00633)	0.995*** (0.00793)	1.196*** (0.0558)	1.089*** (0.0405)	-0.00874 (0.0173)	-0.0447** (0.0171)
SMB	0.0767 (0.0657)	0.0104 (0.0774)	0.0928 (0.0573)	0.792*** (0.0421)	-0.229*** (0.0156)	-0.191*** (0.0139)	-0.148*** (0.0156)	0.597*** (0.120)	0.181 (0.0359)	-0.0466 (0.0371)	-0.0690 (0.00959)
HML	0.120* (0.0498)	0.0239 (0.0756)	0.133** (0.0423)	0.190*** (0.0384)	-0.0655*** (0.0108)	0.0128 (0.0112)	0.0138 (0.0131)	0.0608 (0.0985)	0.0758 (0.0934)	-0.0520 (0.0348)	-0.00959 (0.0382)
Const.	0.00313* (0.00132)	0.00597** (0.00217)	0.00384** (0.00139)	0.000723 (0.000579)	0.000149 (0.000240)	0.000577* (0.000246)	0.000457 (0.000311)	0.000922 (0.00239)	-0.00263 (0.00167)	0.00626*** (0.000774)	0.00602*** (0.000774)
N adj. R-sq	210 0.023	271 0.274	235 0.143	258 0.364	266 0.392	313 0.390	313 0.385	313 0.637	313 0.720	313 0.002	313 0.022
	Bofa EMBI Plus Mexico	Nikkei	Oil	Gold	Copper	Chilean Peso	Bofa EMBI Plus Chile	Bofa Global Gov.	Put Option Sk&P	Swaption US 3m	
MRP	0.257*** (0.0459)	1.056*** (0.0624)	0.394* (0.185)	0.0864 (0.0814)	0.728*** (0.120)	0.274*** (0.0569)	0.334*** (0.0692)	0.0875*** (0.0248)	-2.886*** (0.286)	-0.407 (0.282)	
SMB	-0.0539 (0.0776)	0.220 (0.181)	1.021*** (0.220)	0.477*** (0.115)	0.340 (0.181)	0.224** (0.0785)	0.154 (0.0856)	-0.0277 (0.0528)	0.210 (0.443)	0.656 (0.461)	
HML	-0.0374 (0.0667)	-0.0341 (0.143)	0.168 (0.232)	0.00157 (0.0090)	0.460** (0.151)	0.0331 (0.0664)	-0.0169 (0.0748)	0.0729 (0.0460)	0.0937 (0.366)	0.184 (0.359)	
Const.	0.00648*** (0.00179)	-0.00333 (0.00248)	0.00482 (0.00555)	0.00479 (0.00272)	0.000171 (0.00395)	-0.00345* (0.00165)	0.00178 (0.00205)	0.00475*** (0.00106)	0.0317** (0.0105)	0.00734 (0.0122)	
N adj. R-sq	271 0.134	313 0.524	313 0.067	313 0.044	313 0.180	313 0.182	229 0.239	313 0.037	313 0.344	236 0.003	

Table 1—: Global Fama-French 3 Factors

Standard errors in parentheses $*p < 0.05$ $**p < 0.01$ $***p < 0.001$

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
MRP	0.748*** (0.113)	-0.00702 (0.0214)	-0.00546 (0.0173)	-0.0121 (0.0209)	-0.0282 (0.0219)	-0.00467 (0.0274)	-0.0136 (0.0160)	-0.0020 (0.0579)	0.351*** (0.0562)	0.444*** (0.0500)	0.253*** (0.0370)
SMB	0.408* (0.165)	-0.0348 (0.0379)	-0.0539 (0.0290)	-0.0548 (0.0295)	-0.0174 (0.0290)	-0.0544 (0.0419)	-0.0605 (0.0315)	-0.0008 (0.0238)	0.265 (0.138)	0.282*** (0.0773)	0.162** (0.0569)
HML	0.301 (0.236)	-0.102* (0.0517)	-0.000239 (0.0401)	-0.0126 (0.0444)	-0.0627 (0.0434)	-0.151* (0.0655)	-0.0674 (0.0460)	-0.0180 (0.0525)	0.262 (0.139)	0.229** (0.0858)	0.0296 (0.0790)
RMW	-0.00470 (0.261)	0.165* (0.0646)	0.0442 (0.0471)	0.0225 (0.0513)	0.0095 (0.0513)	0.200** (0.0608)	0.0414 (0.0430)	0.0599 (0.0873)	0.161 (0.129)	0.131 (0.117)	0.291** (0.0686)
CMA	-0.588 (0.305)	0.154* (0.0653)	0.0349 (0.0513)	0.0195 (0.0582)	0.0763 (0.0555)	0.125 (0.0553)	0.108 (0.0588)	-0.102 (0.194)	-0.360* (0.164)	-0.279* (0.102)	0.0658 (0.102)
Const.	0.00546 (0.00383)	0.00417*** (0.000813)	0.00550*** (0.000681)	0.00340*** (0.000565)	0.00405*** (0.000678)	0.00640*** (0.00103)	0.00655*** (0.000905)	0.00321*** (0.000905)	0.00267 (0.00208)	0.00291 (0.00150)	0.00196 (0.00112)
N adj. R-sq	313 0.272	313 0.068	313 0.009	313 0.005	313 0.037	313 0.048	313 0.030	312 -0.000	312 0.211	223 0.591	235 0.287
	JPM GBL-EM	JPM EMBI	Barclays World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI World	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
MRP	0.116** (0.0389)	0.415*** (0.0676)	0.234*** (0.0503)	1.075*** (0.0172)	0.969*** (0.00763)	0.960*** (0.00807)	0.986*** (0.0101)	1.113*** (0.0508)	1.159*** (0.0565)	0.0170 (0.0203)	-0.0264 (0.0104)
SMB	0.136 (0.0746)	-0.00230 (0.0808)	0.180** (0.0676)	0.792*** (0.0402)	-0.242*** (0.0143)	-0.199*** (0.0118)	-0.151*** (0.0147)	0.571*** (0.116)	0.250* (0.115)	-0.0163 (0.0380)	-0.0521 (0.0381)
HML	-0.0453 (0.0858)	0.137 (0.172)	0.0602 (0.0814)	0.00168 (0.0537)	-0.0160 (0.0164)	0.0753*** (0.0135)	0.0957*** (0.0212)	0.458** (0.138)	-0.214 (0.146)	-0.122* (0.0521)	-0.0701 (0.0363)
RMW	0.183 (0.108)	-0.0126 (0.152)	0.345** (0.119)	-0.132* (0.0557)	-0.0191 (0.0224)	0.0188 (0.0227)	0.0494 (0.0275)	0.166 (0.176)	0.0735 (0.163)	0.0753 (0.0571)	0.0283 (0.0322)
CMA	0.222 (0.116)	-0.200 (0.227)	0.0207 (0.115)	0.117 (0.0631)	-0.0124 (0.0198)	-0.0573* (0.0267)	-0.111*** (0.0279)	-0.920*** (0.206)	0.449* (0.177)	0.124 (0.0667)	0.124 (0.0735)
Const.	0.00188 (0.00136)	0.00634** (0.00215)	0.00226 (0.00153)	0.000744 (0.000615)	0.000370 (0.000275)	0.00068* (0.000260)	0.000479 (0.000339)	0.00133 (0.00245)	-0.00368* (0.00179)	0.00578*** (0.000839)	0.00660*** (0.000794)
N adj. R-sq	210 0.046	271 0.272	235 0.181	258 0.965	266 0.992	313 0.991	313 0.987	313 0.667	313 0.728	313 0.010	313 0.029
	Bofa EMBI Plus Mexico	Nikkei	Oil	Gold	Copper	Chilean Peso	Bofa EMBI Plus Chile	Bofa Global Gov.	Put Option SK&P	Swaption US 3m	
MRP	0.263*** (0.0485)	1.177*** (0.0861)	0.277 (0.234)	0.105 (0.131)	0.577*** (0.134)	0.273*** (0.0674)	0.383*** (0.0839)	0.152*** (0.0296)	-2.516*** (0.351)	-1.062** (0.364)	
SMB	-0.0319 (0.0845)	0.306 (0.181)	0.873** (0.275)	0.493*** (0.131)	0.206 (0.212)	0.211* (0.0870)	0.163 (0.0984)	0.0469 (0.0568)	0.286 (0.467)	-0.0426 (0.533)	
HML	0.00274 (0.124)	-0.655** (0.215)	0.444 (0.397)	-0.165 (0.183)	0.964*** (0.229)	-0.0390 (0.108)	-0.214 (0.117)	-0.106 (0.0715)	-0.130 (0.655)	1.375 (0.748)	
RMW	0.124 (0.139)	-0.109 (0.240)	-0.252 (0.320)	-0.0629 (0.206)	-0.252 (0.316)	-0.0983 (0.144)	-0.0336 (0.159)	0.198* (0.0910)	0.210 (0.742)	-2.481* (1.158)	
CMA	-0.0802 (0.168)	1.062*** (0.266)	-0.751 (0.513)	0.166 (0.237)	-0.957** (0.301)	0.0795 (0.135)	0.312* (0.156)	0.292** (0.0901)	0.299 (0.556)	-1.756 (1.015)	
Const.	0.00609** (0.00191)	-0.00455 (0.00245)	0.00660 (0.00668)	0.00459 (0.00284)	0.00248 (0.00419)	-0.00326 (0.00183)	0.00128 (0.00235)	0.00350*** (0.00104)	0.0303** (0.0014)	0.0210 (0.0146)	
N adj. R-sq	271 0.131	313 0.565	313 0.665	313 0.940	313 0.159	313 0.178	229 0.246	313 0.677	313 0.341	236 0.636	

Table 2—: Global Fama-French 5 Factors

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
Inf. Surp.	-0.190 (0.143)	-0.0270* (0.0132)	0.0139 (0.0277)	0.0124 (0.0662)	0.00507 (0.0254)	-0.0004 (0.0075)	-0.0403** (0.0184)	-0.0650*** (0.00310)	-0.00306 (0.00306)	0.0283 (0.0461)	0.00688 (0.0343)
Econ. Surp.	0.0145 (0.0496)	0.00130 (0.00315)	-0.00696 (0.00859)	-0.00326 (0.0522)	-0.0116 (0.00813)	0.00785 (0.00926)	-0.00645 (0.00996)	-0.00310 (0.00375)	-0.0413 (0.0336)	-0.00389 (0.0121)	0.00497 (0.0421)
MRP	0.026*** (0.0109)	0.00266 (0.00109)	0.0118 (0.0202)	-0.001 (0.0103)	-0.0198 (0.0247)	0.000673 (0.0199)	-0.0186 (0.0218)	0.0271* (0.0131)	0.552*** (0.0785)	0.540*** (0.0662)	0.323*** (0.0316)
TP	-0.144 (0.201)	0.384*** (0.0175)	0.236*** (0.0241)	0.082*** (0.0143)	0.236*** (0.0332)	0.445*** (0.0332)	0.160*** (0.0530)	-0.0103 (0.0125)	0.138 (0.107)	0.0040 (0.107)	0.388*** (0.0404)
Expt. Inf.	-2.090 (1.287)	-0.0226 (0.110)	-0.017 (0.185)	0.0482 (0.140)	-0.0054 (0.234)	-0.293 (0.234)	-0.348 (0.286)	0.304** (0.101)	0.417 (1.106)	-0.309 (0.395)	-0.310 (0.273)
Const.	0.0311 (0.0186)	0.00207 (0.00147)	0.00589* (0.00238)	0.00690 (0.00187)	0.00292 (0.00348)	0.00604 (0.00309)	0.00964** (0.00293)	0.000501 (0.00131)	-0.00586 (0.0149)	0.00710 (0.0115)	0.00425 (0.00882)
N adj. R-sq	163 0.403	163 0.873	163 0.408	163 0.383	163 0.474	163 0.467	163 0.381	162 0.158	162 0.441	163 0.648	163 0.610
	JPM GBL-EM	JPM EMBI	Barclays World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI World	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
Inf. Surp.	-0.022 (0.0460)	-0.0506 (0.0403)	-0.0285 (0.0497)	-0.0385 (0.0333)	-0.0186 (0.0129)	-0.0168 (0.00946)	-0.0207 (0.0110)	-0.0639 (0.0891)	0.0204 (0.0654)	-0.0279 (0.0178)	-0.0475* (0.0197)
Econ. Surp.	0.0102 (0.0119)	-0.0277 (0.0157)	0.00498 (0.0161)	0.00835 (0.0115)	0.00151 (0.00455)	0.00290 (0.00404)	0.00235 (0.00404)	-0.00890 (0.0284)	0.00605 (0.0187)	-0.00470 (0.00608)	-0.00966 (0.00773)
MRP	0.151*** (0.0320)	0.437*** (0.0554)	0.342*** (0.0428)	1.127*** (0.0289)	0.958*** (0.0119)	0.978*** (0.00815)	1.010*** (0.00863)	1.305*** (0.0590)	1.013*** (0.0440)	0.018 (0.011)	-0.0439* (0.0169)
TP	0.388*** (0.0418)	0.359*** (0.0823)	0.415*** (0.0626)	-0.0376 (0.0663)	0.00420 (0.0130)	0.00636 (0.00916)	0.0110 (0.0119)	0.0586 (0.0876)	0.105 (0.0569)	0.296*** (0.0192)	0.214*** (0.0234)
Expt. Inf.	-0.557 (0.286)	-0.252 (0.329)	0.403 (0.458)	-0.532 (0.322)	0.311** (0.112)	0.218** (0.0807)	0.125 (0.0819)	-0.737 (0.027)	-0.639 (0.493)	-0.143 (0.159)	-0.0711 (0.164)
Const.	0.00818* (0.0037)	0.00669 (0.00794)	-0.00456 (0.00632)	0.0816 (0.0132)	-0.00238*** (0.00150)	-0.00888*** (0.00106)	-0.00285** (0.00102)	0.00897 (0.00839)	0.00562 (0.00682)	0.00465* (0.00216)	0.00371* (0.00231)
N adj. R-sq	163 0.416	163 0.605	163 0.517	163 0.905	163 0.987	163 0.993	163 0.991	163 0.758	163 0.800	163 0.719	163 0.549
	Bofa EMBI Plus Mexico	Nikkei	Oil	Gold	Copper	Chilean Peso	Bofa EMBI Plus Chile	Bofa Global Gov.	Put Option S&P	Swaption US 3m	
Inf. Surp.	-0.0733 (0.0373)	0.0401 (0.0833)	-0.00515 (0.224)	-0.169 (0.157)	-0.123 (0.192)	-0.0670 (0.0982)	-0.0743 (0.0997)	-0.0467 (0.0460)	1.055* (0.522)	0.669 (0.483)	
Econ. Surp.	-0.0169 (0.0173)	0.00487 (0.0285)	0.0901 (0.0719)	0.0672 (0.0427)	0.118 (0.0678)	-0.00387 (0.0286)	-0.0109 (0.0283)	0.00760 (0.0117)	0.238 (0.180)	0.0885 (0.140)	
MRP	0.318*** (0.0361)	0.844*** (0.0657)	0.695*** (0.200)	0.296* (0.127)	0.852*** (0.199)	0.429*** (0.0928)	0.439*** (0.0947)	0.177*** (0.0315)	-3.869*** (0.174)	-1.429** (0.488)	
TP	0.516*** (0.0581)	0.133 (0.0812)	-0.490* (0.198)	0.433* (0.187)	-0.412 (0.236)	0.0926 (0.125)	0.0927 (0.127)	0.382*** (0.0414)	0.0562 (0.395)	-3.529*** (0.843)	
Expt. Inf.	-0.234 (0.513)	-0.698 (0.822)	3.182 (1.994)	-1.006 (1.327)	-0.719 (1.978)	-0.391 (0.877)	-0.518 (0.905)	-0.525 (0.286)	5.864 (2.972)	1.074 (5.384)	
Const.	0.00533 (0.00631)	0.00735 (0.0113)	-0.0361 (0.0277)	0.0192 (0.0185)	0.0156 (0.0280)	0.00570 (0.0236)	0.00809 (0.0130)	0.00769* (0.00838)	-0.0413 (0.0400)	0.103 (0.0745)	
N adj. R-sq	163 0.620	163 0.573	163 0.215	163 0.075	163 0.285	163 0.241	163 0.236	163 0.421	163 0.453	163 0.282	

Table 3—: Chen, Roll, and Ross Model

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
Economic	0.002*** (0.176)	-0.022 (0.0373)	0.0247 (0.0229)	-0.00339 (0.0199)	-0.00778 (0.0240)	0.0176 (0.0435)	0.0135 (0.0261)	-0.0109 (0.0143)	0.447*** (0.085)	0.377*** (0.0657)	0.174*** (0.0462)
Inflation	-0.0379 (0.372)	0.0714 (0.0893)	0.0627 (0.0581)	0.0615 (0.0394)	0.0178 (0.0274)	0.0635 (0.0467)	0.146* (0.0701)	-0.0094 (0.0248)	-0.178 (0.244)	-0.597*** (0.157)	-0.0455 (0.0636)
EM Curr.	1.041*** (0.251)	-0.0469 (0.0310)	-0.0657** (0.0237)	-0.00902 (0.0259)	-0.0724** (0.0271)	-0.120** (0.0446)	-0.0612* (0.0293)	0.039* (0.0177)	0.295* (0.115)	0.251*** (0.0684)	0.162* (0.0636)
Const.	0.3010** (0.00340)	0.00426*** (0.000856)	0.00431*** (0.000738)	0.00225*** (0.000486)	0.00319*** (0.000770)	0.00512*** (0.00106)	0.00544*** (0.000640)	0.0036*** (0.000359)	0.00440 (0.00296)	0.00605*** (0.00127)	0.00535*** (0.00102)
N adj. R-sq	234 0.397	234 0.023	234 0.017	234 0.005	234 0.029	234 0.031	234 0.064	234 0.031	234 0.256	222 0.580	234 0.363
	JPM GBL-EM	JPM EMBI	Borchays World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI World	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
Economic	0.0321 (0.0573)	0.339*** (0.0910)	0.0985 (0.0543)	0.883*** (0.0738)	0.829*** (0.0692)	0.840*** (0.0715)	0.811*** (0.0715)	0.909*** (0.132)	0.747*** (0.0985)	0.0109 (0.0322)	0.00670 (0.0277)
Inflation	0.276*** (0.129)	-0.0117 (0.227)	-0.590*** (0.122)	-0.290 (0.162)	-0.233 (0.149)	-0.246 (0.145)	-0.266 (0.149)	-0.353 (0.277)	-0.149 (0.249)	0.0693 (0.0676)	0.229*** (0.0632)
EM Curr.	0.280*** (0.0637)	0.459* (0.190)	0.149* (0.0635)	0.502*** (0.116)	0.342** (0.113)	0.353** (0.108)	0.433*** (0.117)	1.030*** (0.240)	0.690*** (0.130)	-0.0569 (0.0388)	-0.0730 (0.0423)
Const.	0.0622*** (0.00125)	0.00993*** (0.00149)	0.00521*** (0.00119)	0.00791*** (0.00226)	0.00458* (0.00196)	0.00480* (0.00191)	0.00516** (0.00193)	0.00224** (0.00300)	0.00447 (0.00296)	0.00444*** (0.00705)	0.00520*** (0.00096)
N adj. R-sq	269 0.149	234 0.382	234 0.300	234 0.603	234 0.604	234 0.626	234 0.642	234 0.586	234 0.499	234 0.616	234 0.116
	Bofa EMBI Plus Mexico	Nikkei	Oil	Gold	Copper	Chilean Peso	Bofa EMBI Plus Chile	Bofa Global Gov.	Put Option SK&P	Swaption US 3m	
Economic	0.249*** (0.0575)	0.682*** (0.0957)	0.0290 (0.275)	0.0138 (0.140)	0.514** (0.177)	0.146 (0.0936)	0.153 (0.0987)	0.0085 (0.0543)	-2.858*** (0.466)	-0.503 (0.456)	
Inflation	0.0937 (0.133)	-0.0654 (0.314)	-1.448* (0.572)	0.156 (0.422)	-0.518 (0.474)	-0.0455 (0.261)	-0.0744 (0.287)	0.410*** (0.122)	0.157 (0.107)	-0.113 (0.153)	
EM Curr.	0.263* (0.104)	0.360* (0.147)	0.847* (0.416)	0.493** (0.168)	0.832*** (0.215)	0.517*** (0.114)	0.471*** (0.119)	0.140* (0.0635)	-1.799** (0.577)	-1.156* (0.534)	
Const.	0.00012*** (0.00141)	0.00240 (0.00342)	0.0105 (0.00696)	0.00950** (0.00342)	0.00087* (0.00468)	0.00122 (0.00179)	0.00556** (0.00191)	0.00521*** (0.00119)	0.0134 (0.019)	-0.000379 (0.0120)	
N adj. R-sq	234 0.253	234 0.265	234 0.122	234 0.061	234 0.235	234 0.305	228 0.272	234 0.113	234 0.334	234 0.640	

Table 4—: Macro Model I

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
Economic	0.150*** (0.125)	0.103*** (0.0270)	0.0601** (0.0207)	0.0271 (0.0204)	0.0314 (0.0235)	0.102* (0.0399)	0.0387 (0.0318)	-0.0309 (0.0453)	0.432*** (0.0796)	0.108*** (0.0238)	0.228*** (0.0392)
Credit	0.046*** (0.171)	-0.203*** (0.0415)	-0.117*** (0.0330)	-0.0597* (0.0247)	-0.113** (0.0423)	-0.253*** (0.0332)	-0.101* (0.0457)	-0.00718 (0.0159)	0.124 (0.109)	0.863*** (0.0389)	0.0155 (0.0690)
EM	0.069*** (0.0778)	-0.00279 (0.0147)	0.00507 (0.0150)	0.00758 (0.0170)	0.0123 (0.0128)	0.0312 (0.0220)	-0.0140 (0.0156)	0.0416 (0.0357)	0.12** (0.0419)	0.0319 (0.0170)	0.0393 (0.0280)
Const.	0.00507* (0.00288)	0.00673*** (0.000665)	0.00590*** (0.000600)	0.00353*** (0.000571)	0.00444*** (0.000620)	0.00738*** (0.000920)	0.00694*** (0.000638)	0.00299* (0.00116)	0.0412* (0.00182)	0.00404*** (0.000641)	0.00411*** (0.00106)
N adj. R-sq	317 0.448	317 0.173	317 0.037	317 0.004	317 0.037	317 0.076	317 0.032	317 -0.001	317 0.221	222 0.395	234 0.217
	JPM GBLEM	JPM EMBI	Borclays World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI World	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
Economic	0.194*** (0.0483)	0.381*** (0.0544)	0.238*** (0.0525)	0.091*** (0.0858)	0.737*** (0.0796)	0.886*** (0.0681)	0.891*** (0.0689)	0.886*** (0.0681)	0.92*** (0.107)	0.0988*** (0.0252)	0.0728* (0.0287)
Credit	-0.256*** (0.0619)	0.131 (0.101)	-0.0216 (0.0857)	0.753*** (0.110)	0.542*** (0.101)	0.446*** (0.0898)	0.448*** (0.0900)	0.446*** (0.0898)	0.258 (0.142)	-0.176*** (0.0373)	-0.185*** (0.0367)
EM	0.0693 (0.0388)	0.392*** (0.0717)	0.0338 (0.0374)	0.173*** (0.0480)	-0.00104 (0.0498)	-0.0483 (0.0412)	0.0110 (0.0421)	0.952*** (0.0412)	0.143 (0.0819)	0.0139 (0.0181)	-0.0101 (0.0192)
Const.	0.00385** (0.00126)	0.00679*** (0.00177)	0.00488*** (0.00134)	0.00412* (0.00181)	0.00246 (0.00165)	0.00306* (0.00152)	0.00291 (0.00154)	0.00306* (0.00152)	0.000240 (0.00254)	0.00635*** (0.000744)	0.00713*** (0.000711)
N adj. R-sq	209 0.085	270 0.397	234 0.137	257 0.670	265 0.626	317 0.633	317 0.641	317 0.845	317 0.414	317 0.658	317 0.681
	Bofa EMBI Plus Mexico	Nikkei	Oil	Gold	Copper	Chilean Peso	Bofa EMBI Plus Chile	Bofa Global Gov.	Put Option SK&P	Swaption US 3m	
Economic	0.308*** (0.0508)	0.576*** (0.127)	0.0334 (0.217)	0.214 (0.116)	0.475** (0.149)	0.164* (0.0689)	0.152 (0.0781)	0.249*** (0.0398)	-2.814*** (0.465)	-1.664** (0.542)	
Credit	-0.0884 (0.0636)	0.197 (0.182)	0.487 (0.346)	-0.202 (0.189)	0.539** (0.202)	0.256* (0.101)	0.240* (0.119)	-0.258*** (0.0526)	-1.367* (0.597)	1.669* (0.646)	
EM	0.246*** (0.0577)	-0.114 (0.0974)	0.285* (0.128)	0.274*** (0.0581)	0.297** (0.093)	0.166*** (0.0341)	0.229*** (0.0432)	-0.0125 (0.0242)	-0.195 (0.264)	-0.396 (0.275)	
Const.	0.00763*** (0.00157)	-0.000275 (0.00331)	0.00503 (0.00538)	0.00470 (0.00259)	0.00316 (0.00380)	-0.00317* (0.00146)	0.00243 (0.00185)	0.00591*** (0.000995)	0.0235* (0.0105)	0.00470 (0.0116)	
N adj. R-sq	270 0.229	317 0.241	317 0.038	317 0.068	317 0.171	317 0.242	228 0.268	317 0.117	317 0.277	235 0.655	

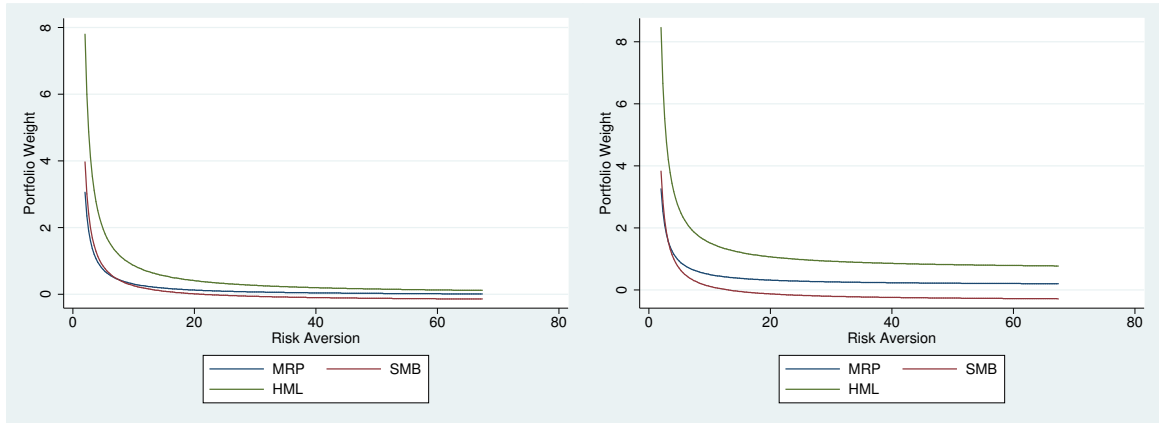
Table 5—: Macro Model II

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
EM	0.872*** (0.0851)	-0.0326 (0.0177)	-0.00889 (0.0157)	0.00382 (0.0178)	-0.00487 (0.0143)	0.00285 (0.0242)	-0.0334 (0.0173)	-0.0324 (0.0374)	0.183*** (0.0528)	0.111 (0.0579)	-0.00215 (0.0373)
EM Curr.	0.292 (0.244)	-0.0107 (0.0244)	-0.0234 (0.0207)	-0.00186 (0.0326)	-0.0362 (0.0199)	-0.0487 (0.0382)	-0.0168 (0.0265)	0.264 (0.219)	0.245** (0.0841)	0.486*** (0.135)	0.277** (0.0843)
Const.	0.00836* (0.00335)	0.00437*** (0.000762)	0.00484*** (0.000669)	0.0027*** (0.000553)	0.00327*** (0.000648)	0.00538** (0.000959)	0.00553*** (0.000619)	0.0057*** (0.00122)	0.00661** (0.00223)	0.00816*** (0.00145)	0.00619*** (0.000965)
N adj. R-sq	275 0.366	275 0.007	275 0.001	275 -0.007	275 0.007	275 0.002	275 0.016	275 0.159	275 0.105	222 0.322	234 0.177
	JPM GBI-EM	JPM EMBI	Barchys World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI World	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
EM	-0.0441 (0.0424)	0.351*** (0.0787)	-0.00987 (0.0428)	0.211** (0.0811)	0.0487 (0.0775)	0.139* (0.0652)	0.206** (0.0651)	1.130*** (0.0632)	0.431*** (0.0895)	0.00627 (0.0197)	-0.0316 (0.0207)
EM Curr.	0.252*** (0.0579)	0.334* (0.139)	0.291** (0.0877)	0.916*** (0.206)	0.814*** (0.200)	0.459*** (0.162)	0.457** (0.166)	0.450** (0.162)	0.383 (0.197)	-0.0170 (0.0347)	-0.0178 (0.0329)
Const.	0.00494*** (0.00127)	0.0115*** (0.00158)	0.00669*** (0.00122)	0.0118*** (0.00248)	0.00653*** (0.00219)	0.00910*** (0.00236)	0.0007*** (0.00237)	0.00910*** (0.00236)	0.00526 (0.00277)	0.00501*** (0.000714)	0.00537*** (0.000711)
N adj. R-sq	209 0.069	270 0.336	234 0.127	257 0.339	265 0.280	275 0.164	275 0.197	275 0.657	275 0.232	275 -0.006	275 0.009
	Bofa EMBI Plus Mexico	Nikkel	Oil	Gold	Copper	Chilean Peso	Bofa EMBI Plus Chile	Bofa Global Gov.	Put Option S&P	Swaption US 3m	
EM	0.209*** (0.0587)	0.162 (0.106)	0.428** (0.146)	0.250*** (0.0688)	0.445*** (0.112)	0.185*** (0.0393)	0.174** (0.0563)	-0.0172 (0.0277)	-0.570 (0.340)	0.113 (0.304)	
EM Curr.	0.240** (0.0806)	0.266 (0.238)	0.373 (0.258)	0.217* (0.0886)	0.477* (0.198)	0.283*** (0.0800)	0.448*** (0.126)	0.0844* (0.0398)	-1.945** (0.653)	-1.533* (0.611)	
Const.	0.00689*** (0.00161)	0.00328 (0.00354)	0.0113* (0.00550)	0.00736** (0.00280)	0.0107* (0.00433)	0.00134 (0.00154)	0.00552*** (0.00167)	0.00536*** (0.00111)	0.00324 (0.019)	-0.00258 (0.013)	
N adj. R-sq	270 0.205	275 0.042	275 0.067	275 0.091	275 0.129	275 0.22	228 0.250	275 0.013	275 0.115	235 0.039	

Table 6—: Macro Model III

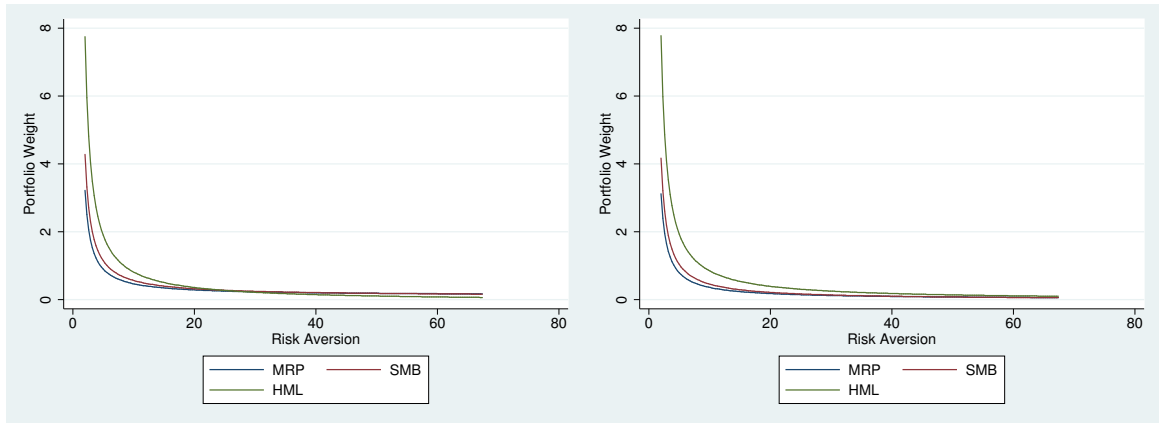
	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
Real Rates	1.228*** (0.275)	0.0581* (0.0358)	0.0194 (0.0249)	-0.0171 (0.0392)	0.000734 (0.0385)	-0.00339 (0.0390)	0.0413* (0.0176)	0.993*** (0.148)	0.919*** (0.136)	0.714*** (0.0459)
TP	-0.964*** (0.161)	0.394*** (0.0187)	0.246*** (0.0237)	0.248*** (0.0270)	0.443*** (0.0248)	0.204*** (0.0367)	-0.0272 (0.0140)	-0.382*** (0.106)	-0.429*** (0.0320)	0.00474 (0.0026)
Const.	0.00360 (0.00419)	0.00221*** (0.000340)	0.00332*** (0.000577)	0.00234*** (0.000613)	0.00354*** (0.000675)	0.00466*** (0.000570)	0.00334*** (0.000351)	0.000217 (0.00248)	0.00304 (0.00169)	0.00102 (0.000608)
N adj. R-sq	234 0.179	234 0.868	234 0.433	234 0.400	234 0.617	234 0.333	234 0.021	234 0.209	222 0.303	234 0.740
	JPM GBLEM	JPM EMBI	Barclays World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
Real Rates	0.573*** (0.0959)	0.575*** (0.147)	1 (.)	1.272*** (0.198)	1.120*** (0.155)	1.192*** (0.167)	1.624*** (0.254)	1.346*** (0.168)	0.0312 (0.0304)	-0.0620 (0.0383)
TP	0.189*** (0.0323)	-0.0392 (0.0779)	0 (.)	-0.864*** (0.116)	-0.745*** (0.103)	-0.779*** (0.104)	-1.047*** (0.144)	-0.731*** (0.114)	0.307*** (0.0199)	0.282*** (0.0262)
Const.	0.00253 (0.00101)	0.00504* (0.00204)	0 (.)	0.00409 (0.00308)	0.00171 (0.00264)	0.00175 (0.00268)	0.00175 (0.00409)	-0.00123 (0.00306)	0.00303*** (0.000444)	0.00433*** (0.000592)
N adj. R-sq	209 0.552	234 0.117	234 1.000	234 0.288	234 0.291	234 0.310	234 0.251	234 0.267	234 0.659	234 0.439
	Bofa EMBI Plus Mexico	Nikkei	Oil	Gold	Copper	Chilean Peso	Bofa Global Gov.	Put Option S&P	Swaption US 3m	
Real Rates	0.404*** (0.0853)	1.045*** (0.187)	1.875*** (0.345)	0.966*** (0.136)	1.780*** (0.351)	0.741*** (0.136)	0.606*** (0.0828)	-2.517*** (0.308)	-2.923** (0.079)	
TP	0.254*** (0.0646)	-0.500*** (0.137)	-1.240*** (0.203)	0.0197 (0.170)	-1.343*** (0.165)	-0.324*** (0.0890)	0.161*** (0.0326)	2.242*** (0.531)	-2.206*** (0.491)	
Const.	0.00454*** (0.00140)	-0.00168 (0.00396)	0.00382 (0.00586)	0.00167 (0.00293)	0.00387 (0.00482)	-0.00338 (0.00201)	0.000312 (0.000919)	0.0220 (0.0148)	0.0312* (0.0129)	
N adj. R-sq	234 0.265	234 0.128	234 0.176	234 0.171	234 0.280	234 0.212	234 0.648	234 0.076	234 0.317	

Table 7—: Macro Model IV



(a) Liabilities I

(b) Liabilities II



(c) Liabilities III

(d) Liabilities IV

Figure 5. : Optimal Factor Allocation / Fama-French 3 Factor Model by Liabilities Portfolio

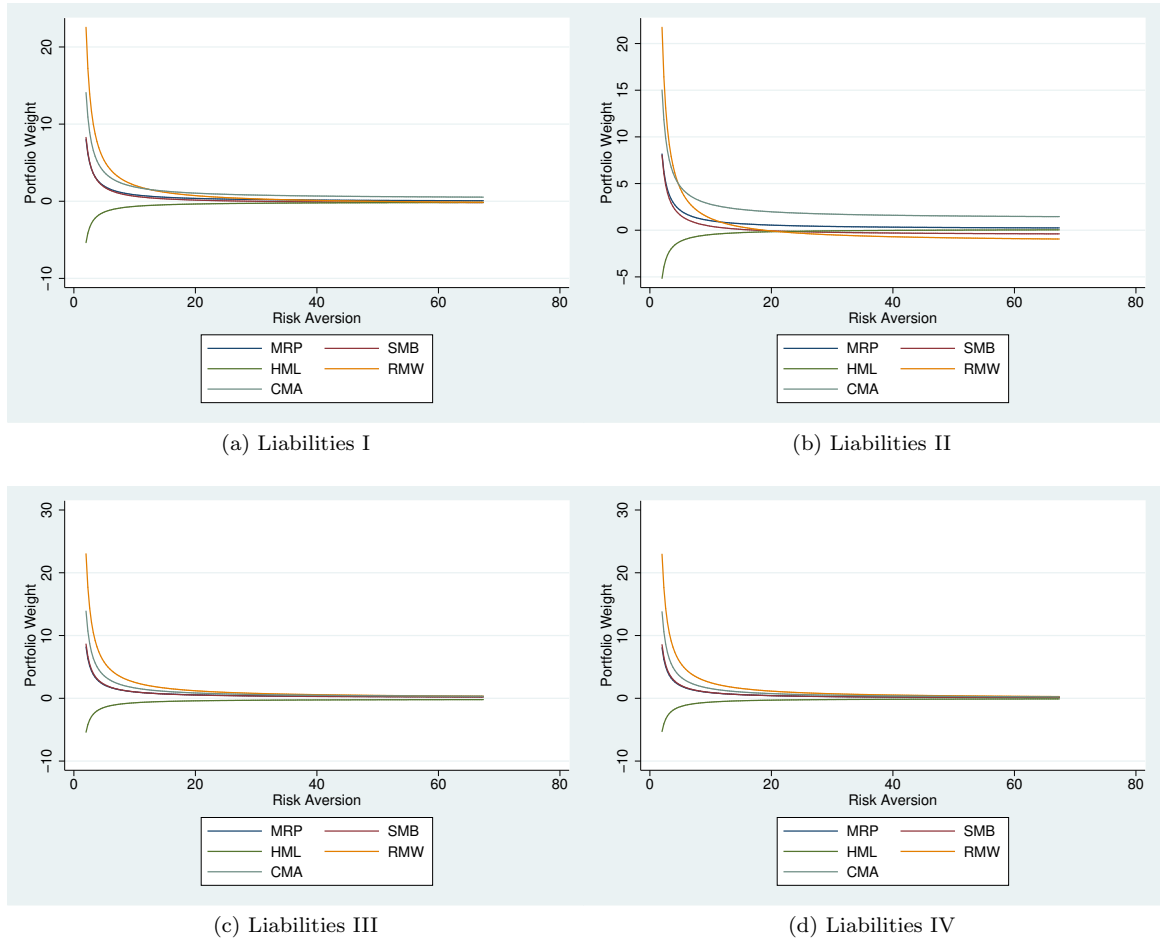


Figure 6. : Optimal Factor Allocation / Fama-French 5 Factor Model by Liabilities Portfolio

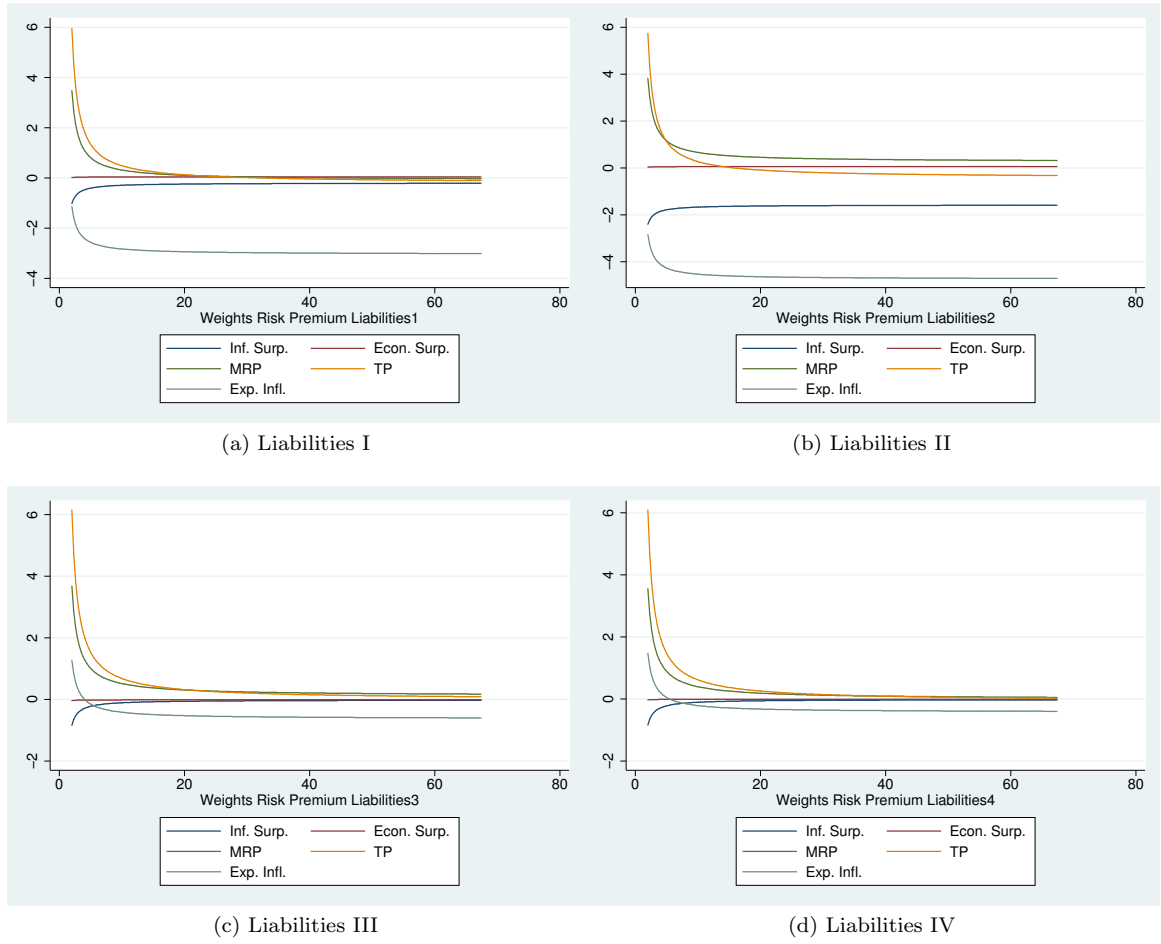


Figure 7. : Optimal Factor Allocation / Chen, Roll Ross Factor Model by Liabilities Portfolio

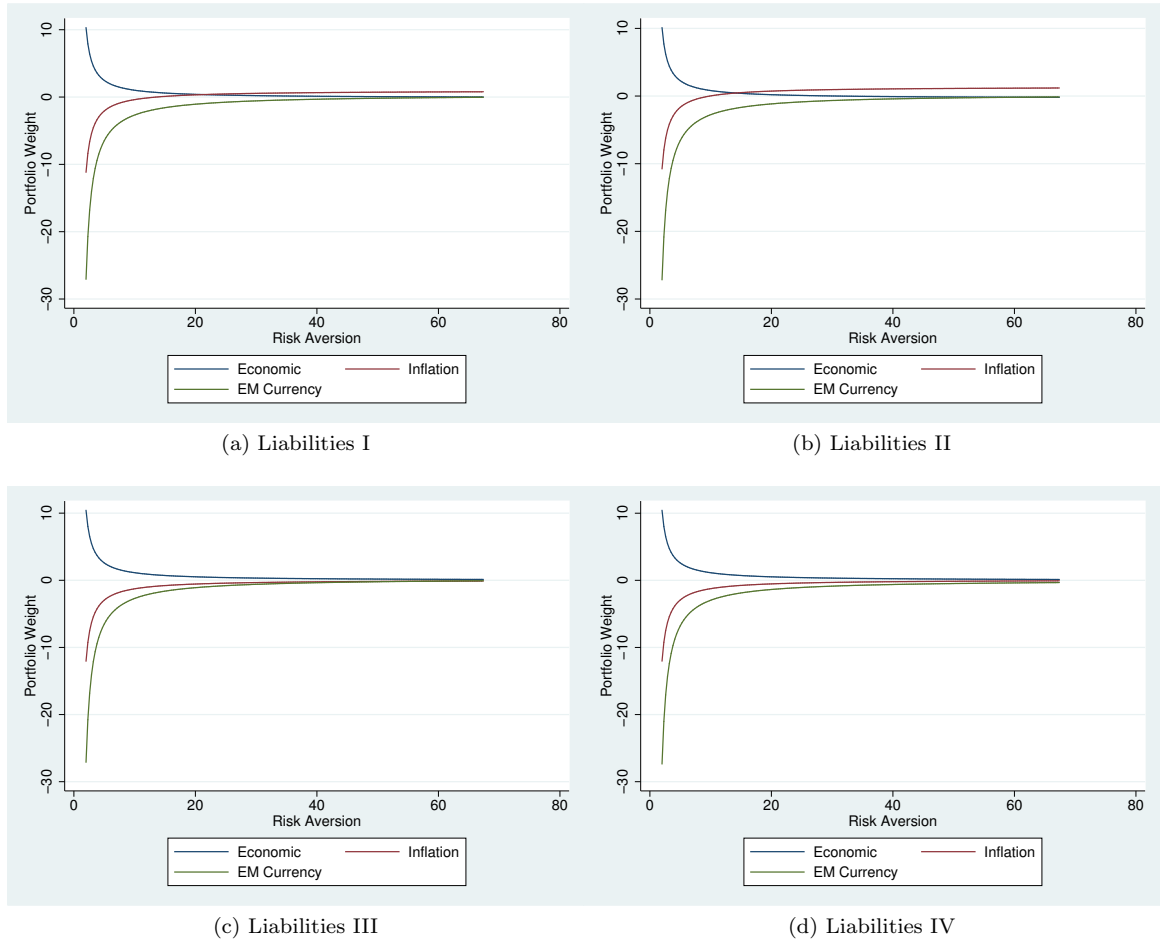
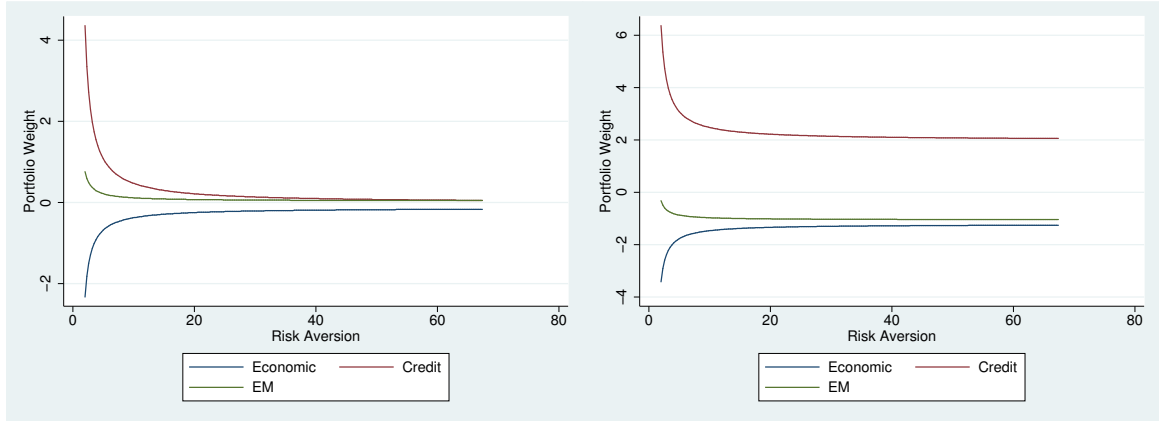
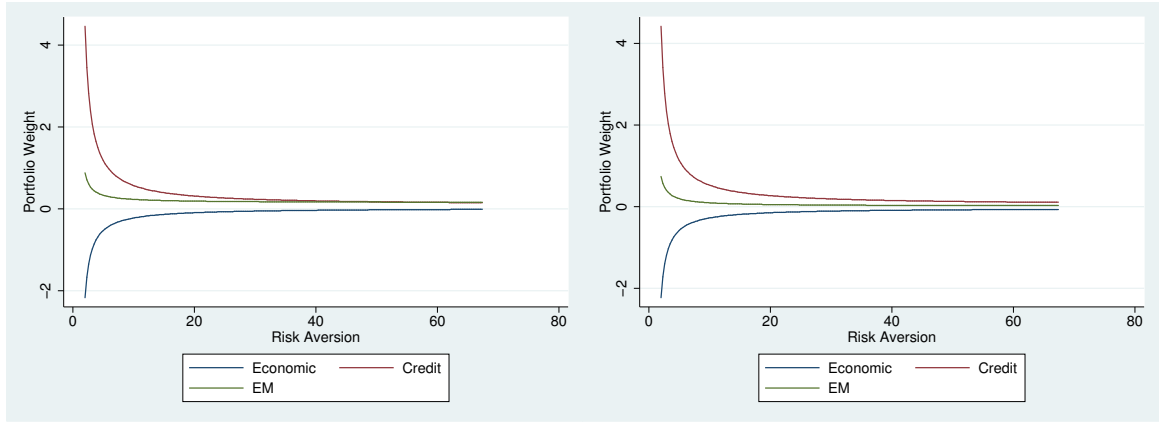


Figure 8. : Optimal Factor Allocation / Macro Model I by Liabilities Portfolio



(a) Liabilities I

(b) Liabilities II



(c) Liabilities III

(d) Liabilities IV

Figure 9. : Optimal Factor Allocation / Macro Model II by Liabilities Portfolio

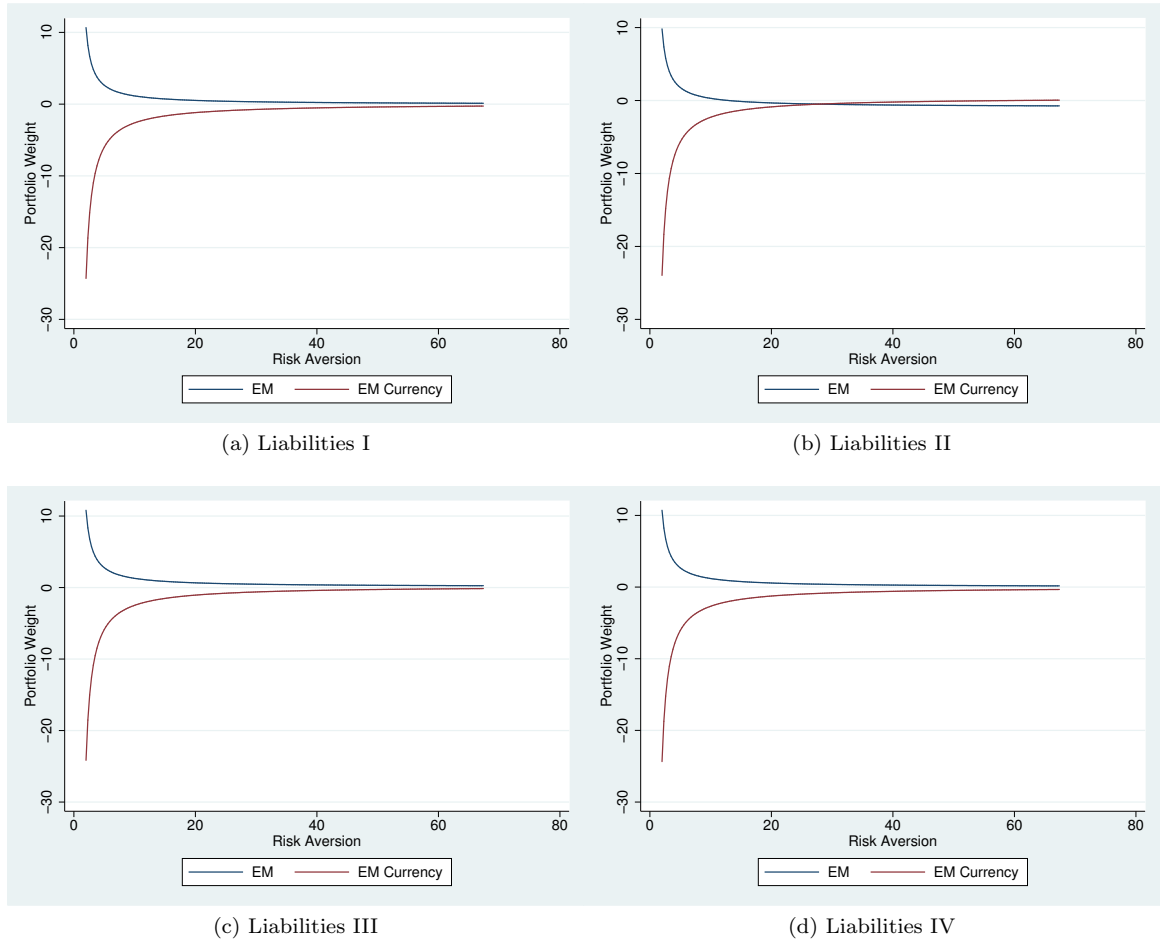


Figure 10. : Optimal Factor Allocation / Macro Model III by Liabilities Portfolio

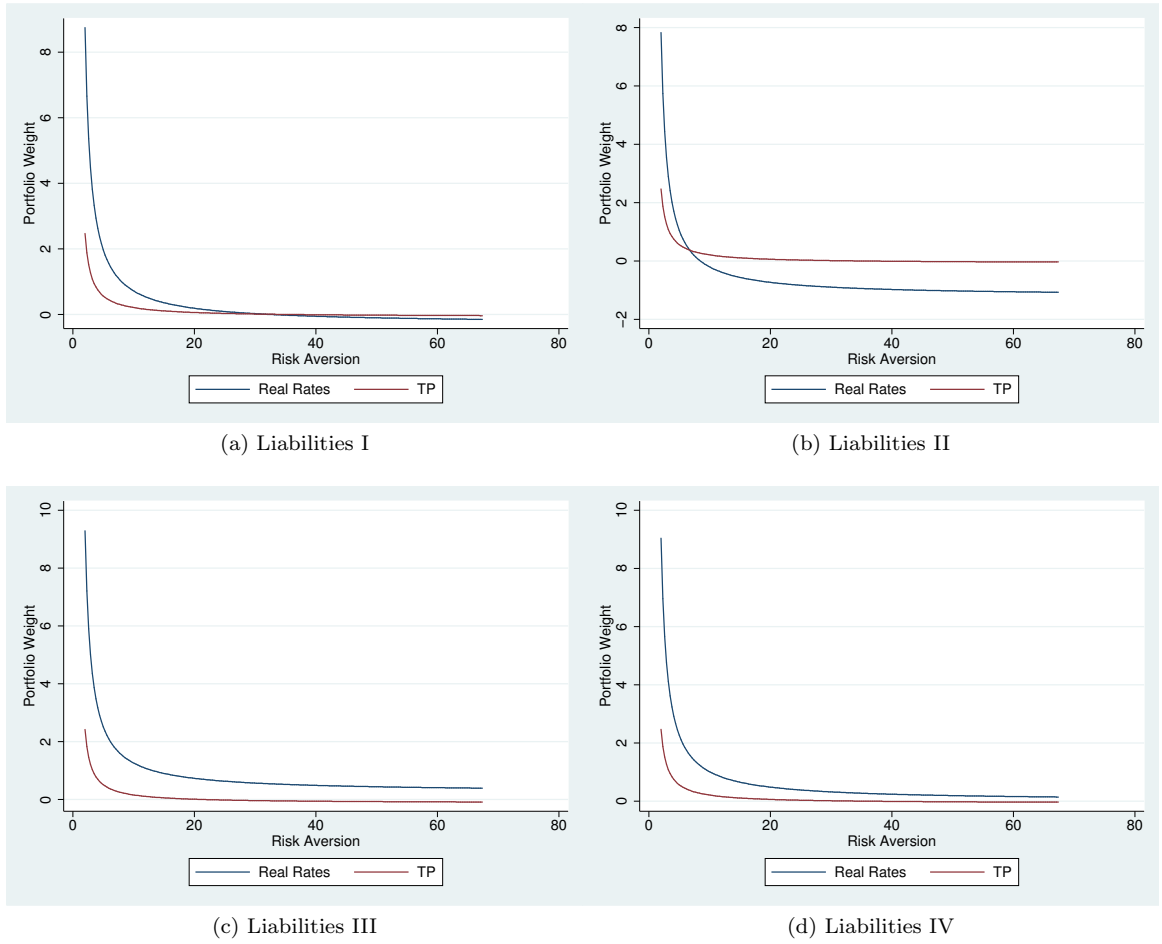
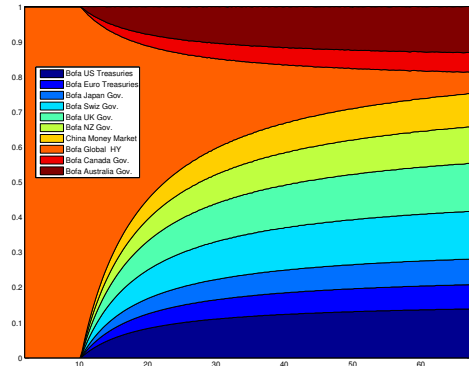
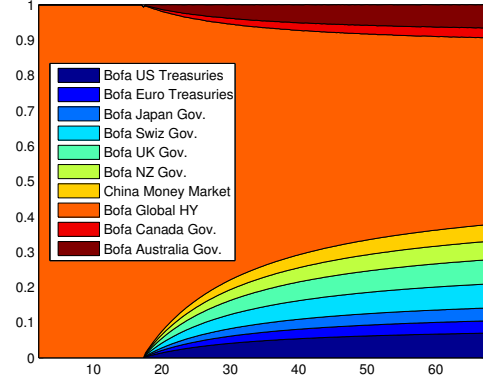


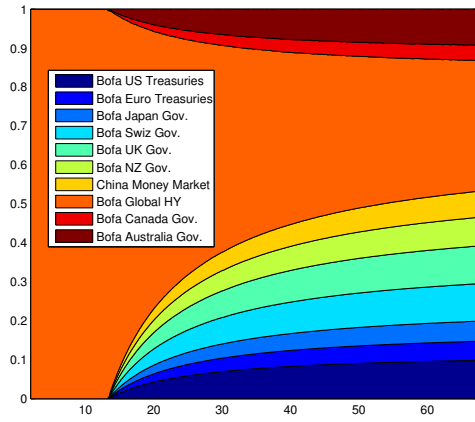
Figure 11. : Optimal Factor Allocation / Macro Model IV by Liabilities Portfolio



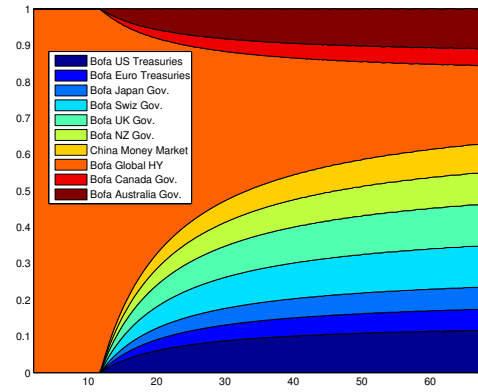
(a) Liabilities I



(b) Liabilities II

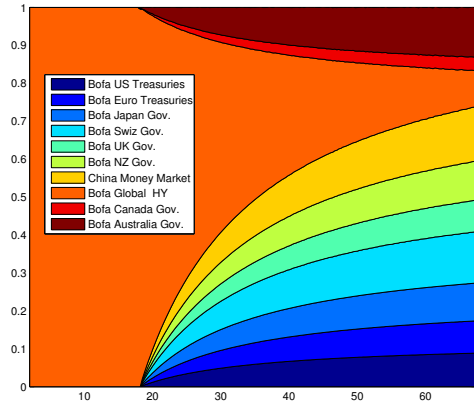


(c) Liabilities III

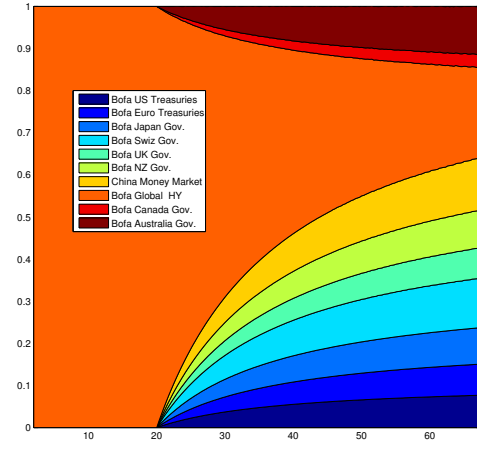


(d) Liabilities IV

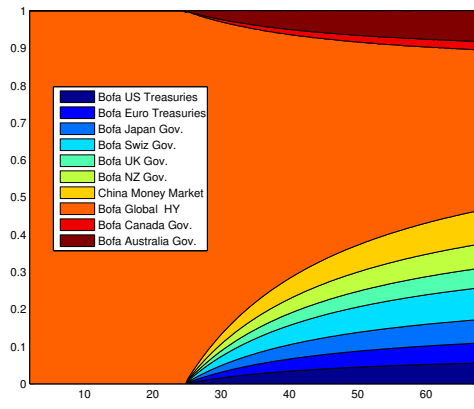
Figure 12. : Replicating Factor Portfolio / Fama-French 3 Factor Model / Fixed Income Set



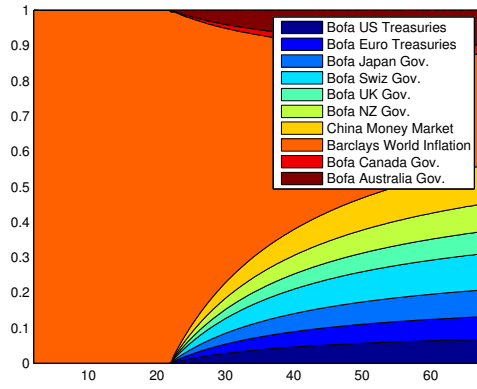
(a) Liabilities I



(b) Liabilities II

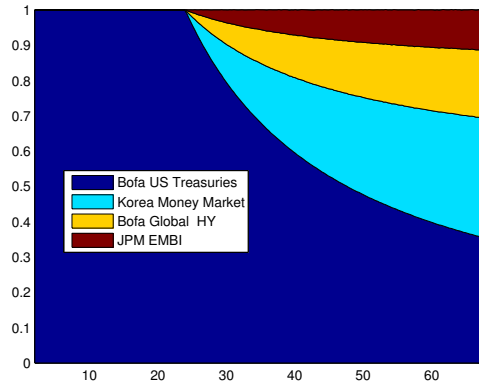


(c) Liabilities III

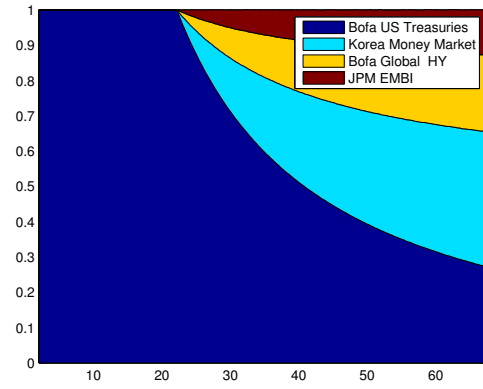


(d) Liabilities IV

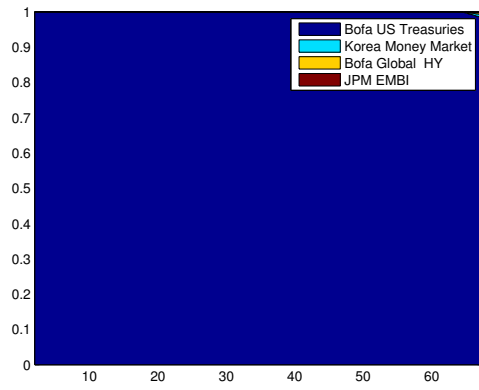
Figure 13. : Replicating Factor Portfolio / Fama-French 5 Factor Model / Fixed Income Set



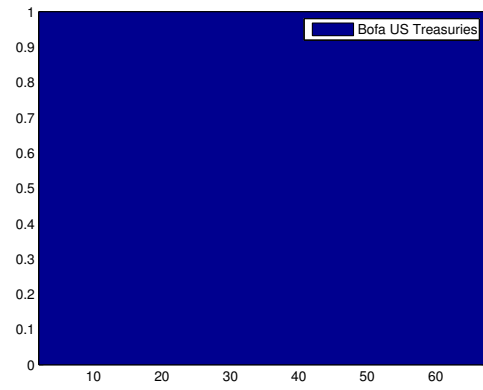
(a) Liabilities I



(b) Liabilities II

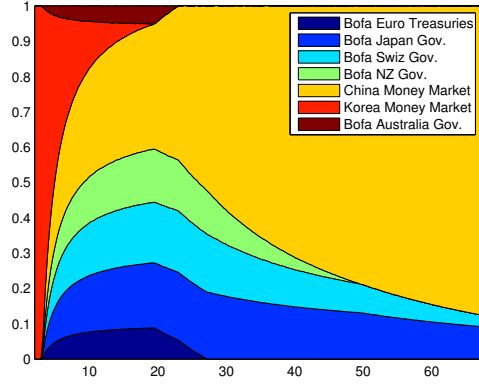


(c) Liabilities III

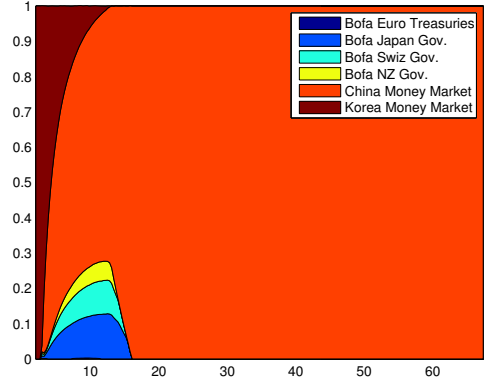


(d) Liabilities IV

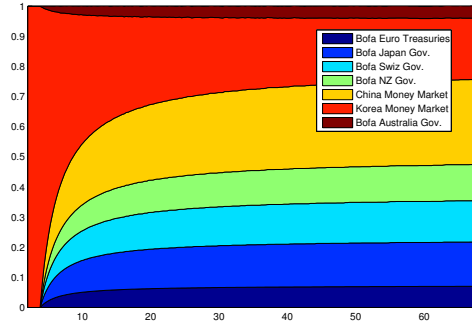
Figure 14. : Replicating Factor Portfolio / Macro Model I / Fixed Income Set



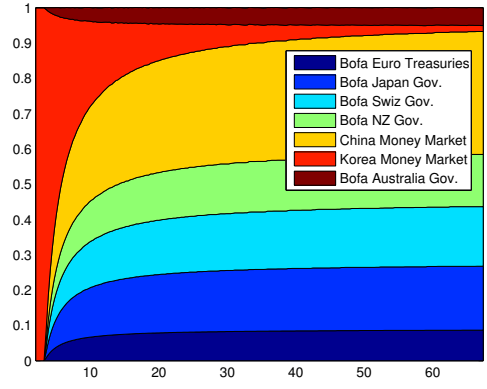
(a) Liabilities I



(b) Liabilities II

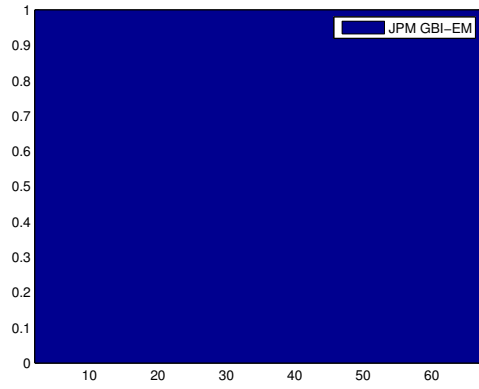


(c) Liabilities III

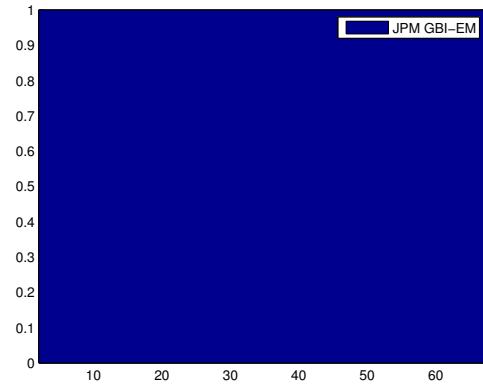


(d) Liabilities IV

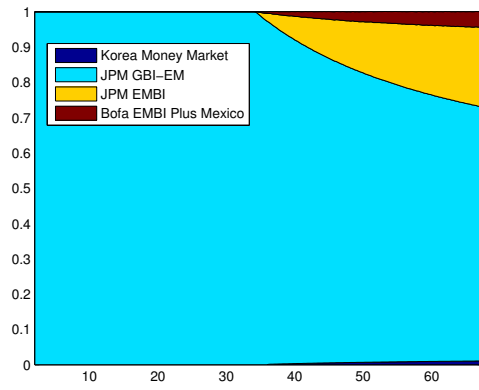
Figure 15. : Replicating Factor Portfolio / Macro Model II / Fixed Income Set



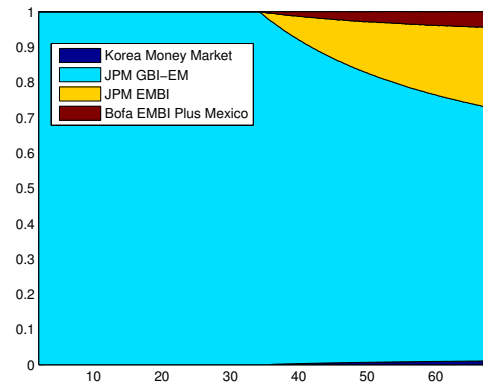
(a) Liabilities I



(b) Liabilities II

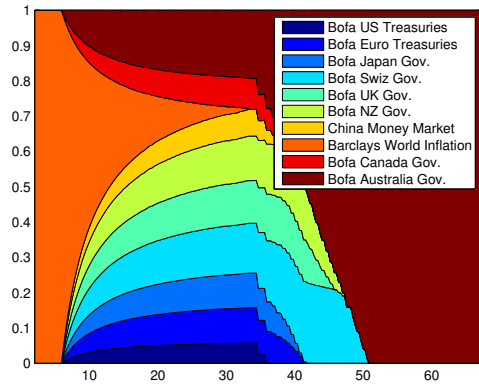


(c) Liabilities III

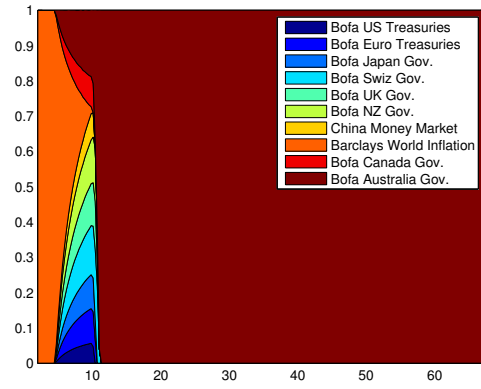


(d) Liabilities IV

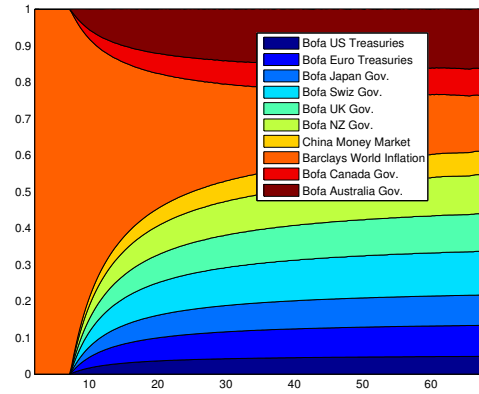
Figure 16. : Replicating Factor Portfolio / Macro Model III / Fixed Income Set



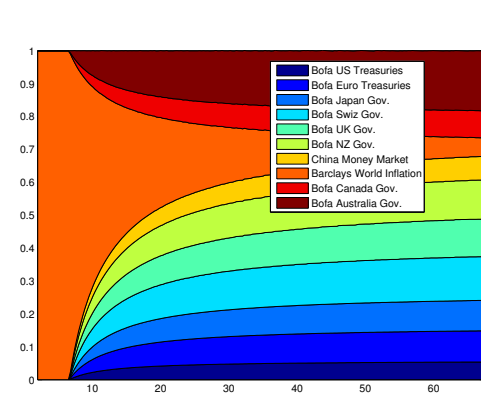
(a) Liabilities I



(b) Liabilities II

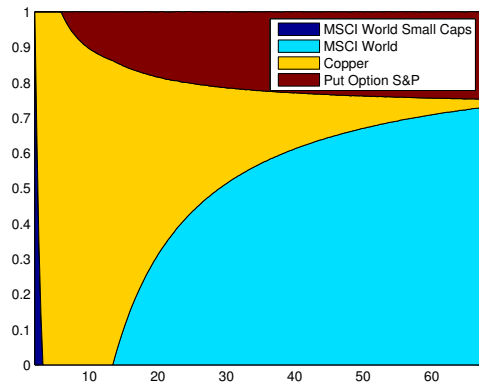


(c) Liabilities III

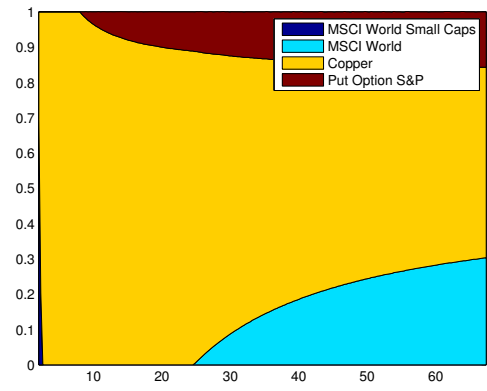


(d) Liabilities IV

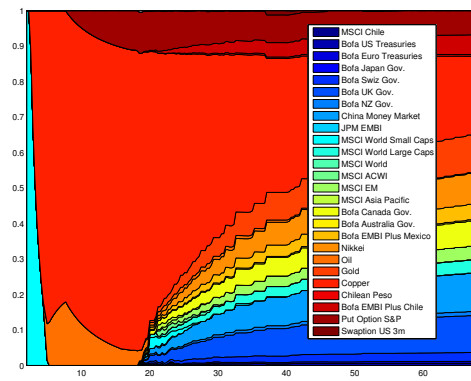
Figure 17. : Replicating Factor Portfolio / Macro Model IV / Fixed Income Set



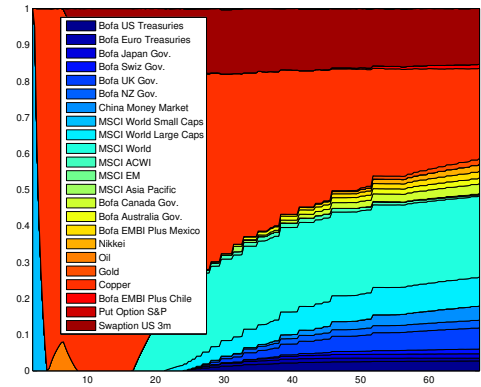
(a) Liabilities I



(b) Liabilities II

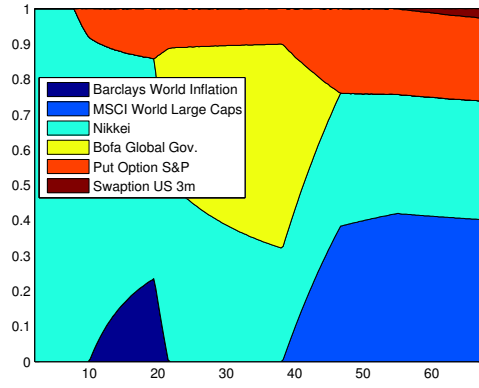


(c) Liabilities III

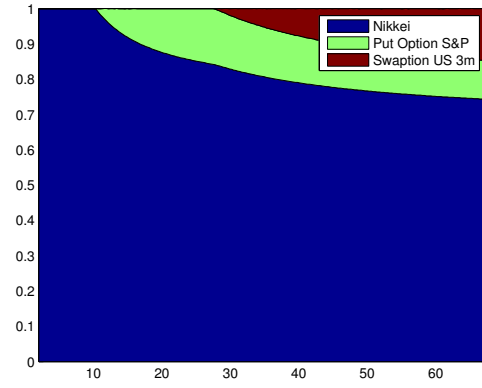


(d) Liabilities IV

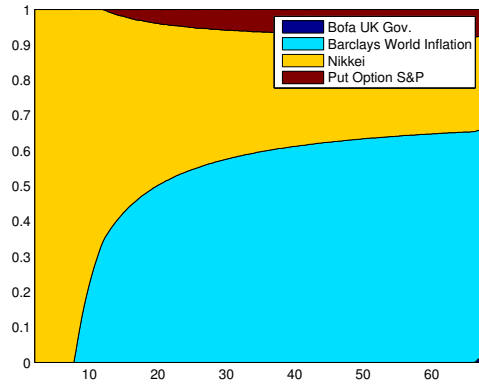
Figure 18. : Replicating Factor Portfolio / Fama-French 3 Factor Model / All Assets



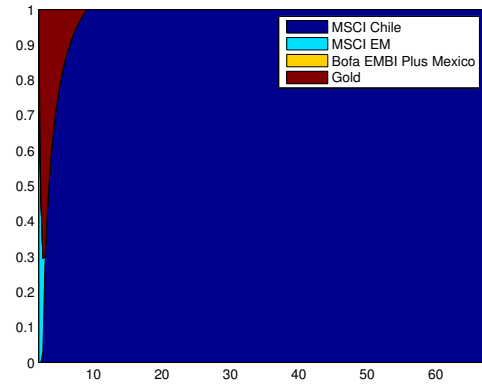
(a) Liabilities I



(b) Liabilities II

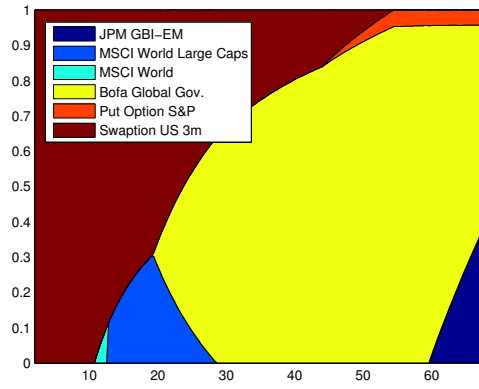


(c) Liabilities III

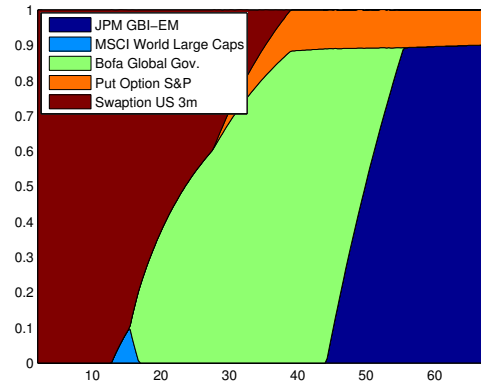


(d) Liabilities IV

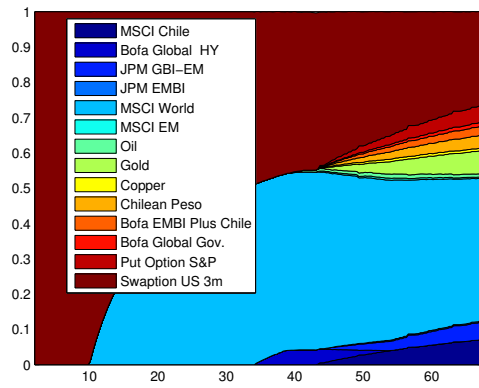
Figure 19. : Replicating Factor Portfolio / Fama-French 5 Factor Model / All Assets



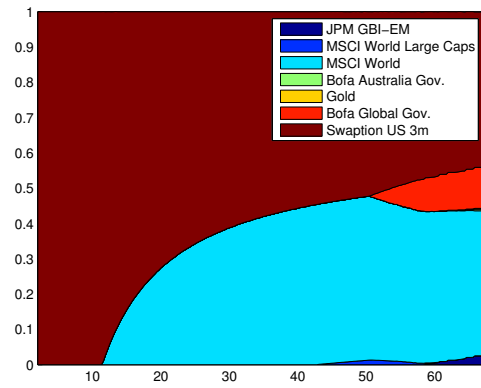
(a) Liabilities I



(b) Liabilities II

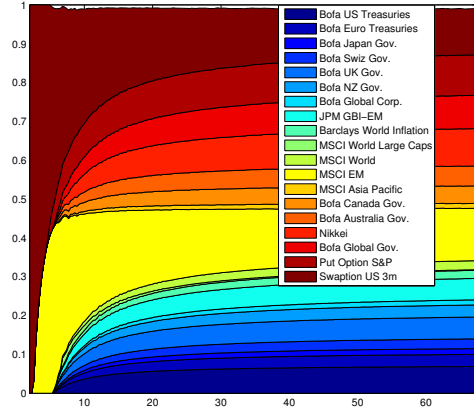


(c) Liabilities III

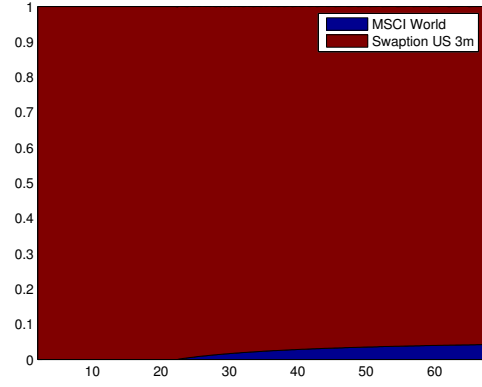


(d) Liabilities IV

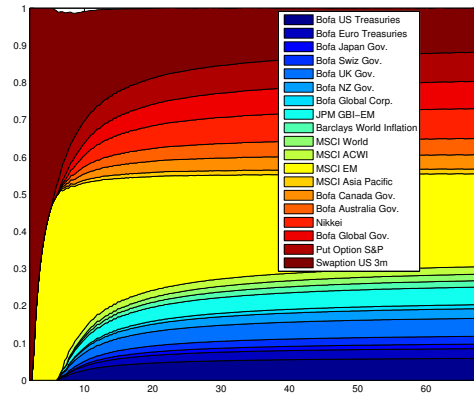
Figure 20. : Replicating Factor Portfolio / Macro Model I / All Assets



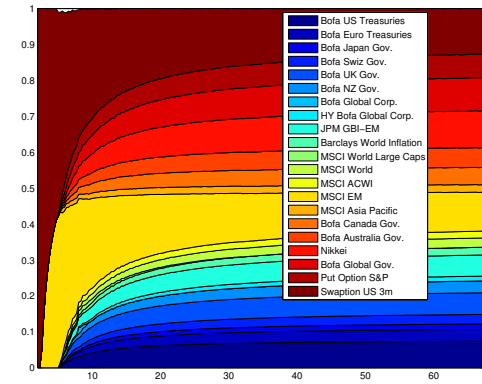
(a) Liabilities I



(b) Liabilities II

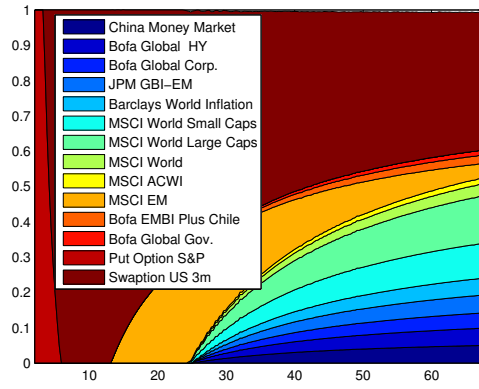


(c) Liabilities III

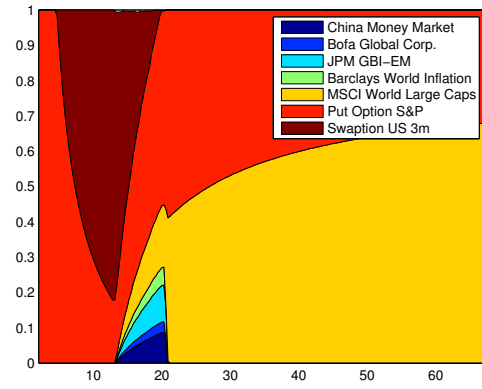


(d) Liabilities IV

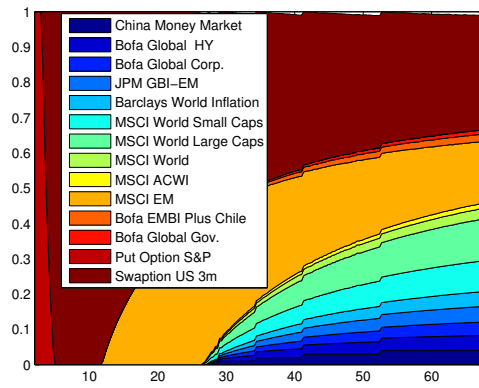
Figure 21. : Replicating Factor Portfolio / Macro Model II / All Assets



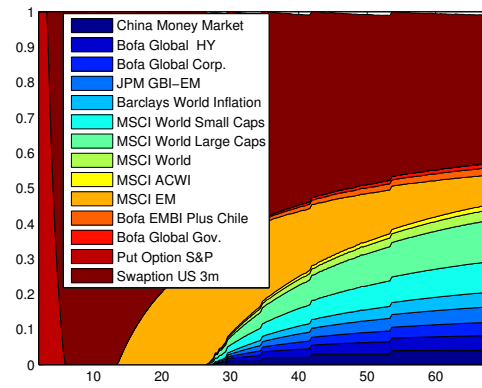
(a) Liabilities I



(b) Liabilities II

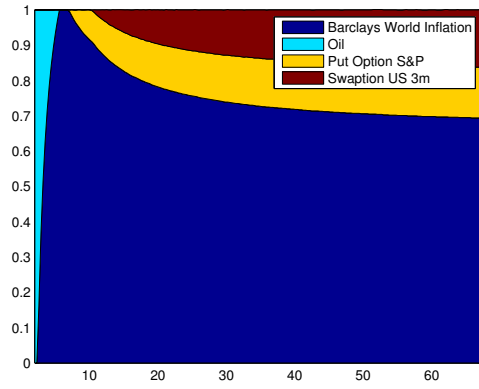


(c) Liabilities III

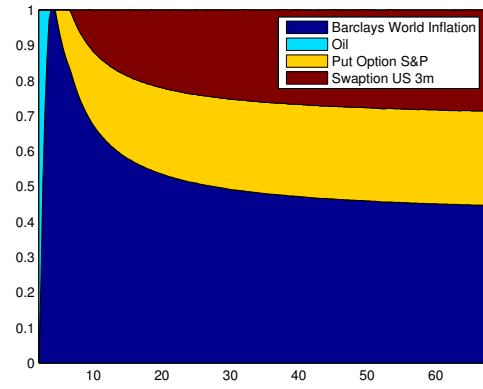


(d) Liabilities IV

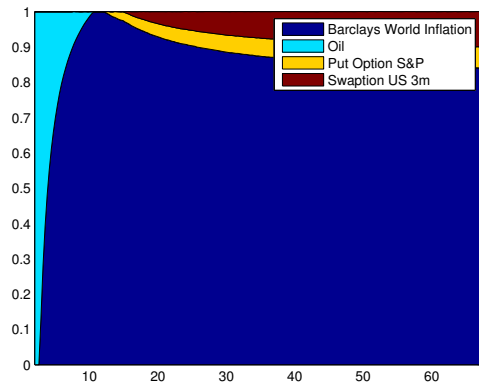
Figure 22. : Replicating Factor Portfolio / Macro Model III / All Assets



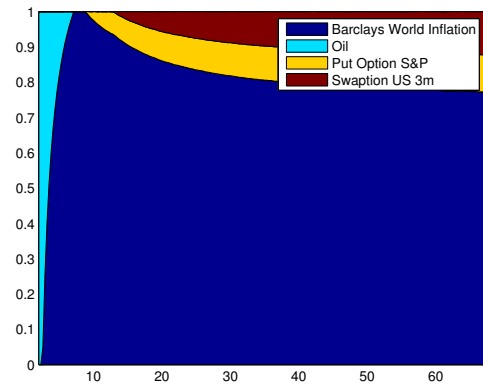
(a) Liabilities I



(b) Liabilities II

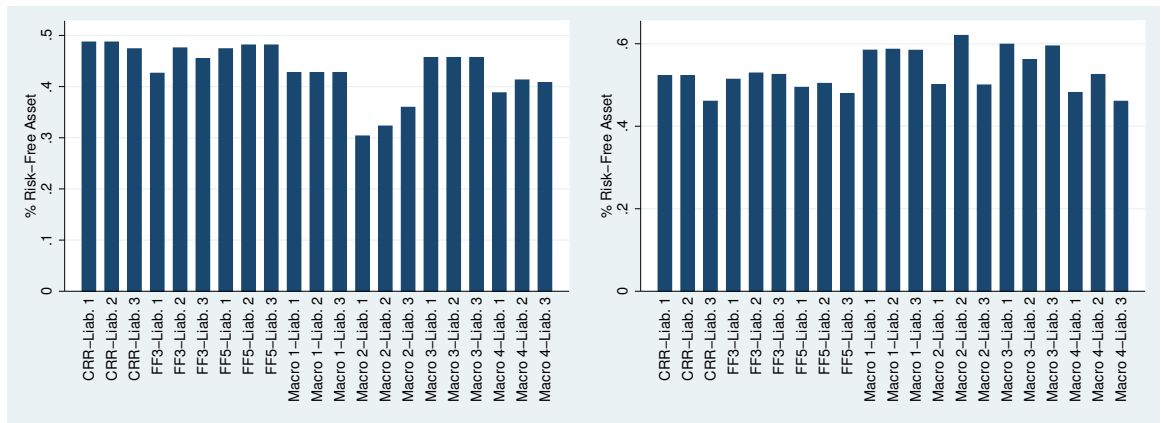


(c) Liabilities III



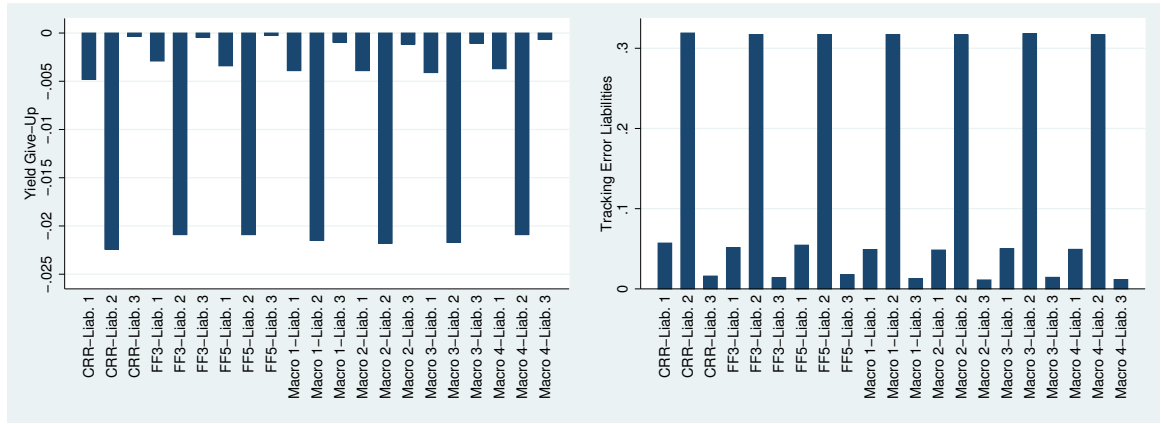
(d) Liabilities IV

Figure 23. : Replicating Factor Portfolio / Macro Model IV / All Assets



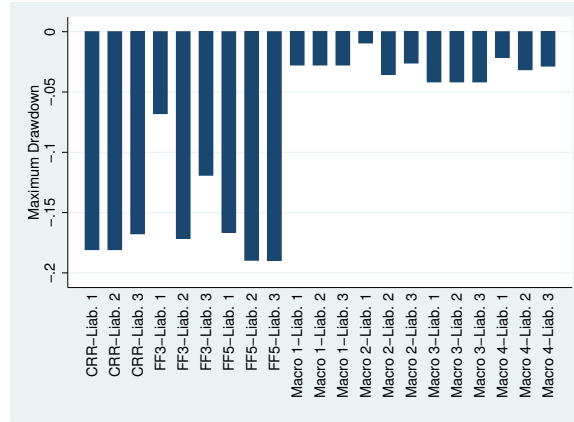
(a) % Invested in the Risk-Free Asset - Fixed Income Strategies (b) % Invested in the Risk-Free Asset - All Assets

Figure 24. : Protective Put - % Invested in the Risk-Free Asset



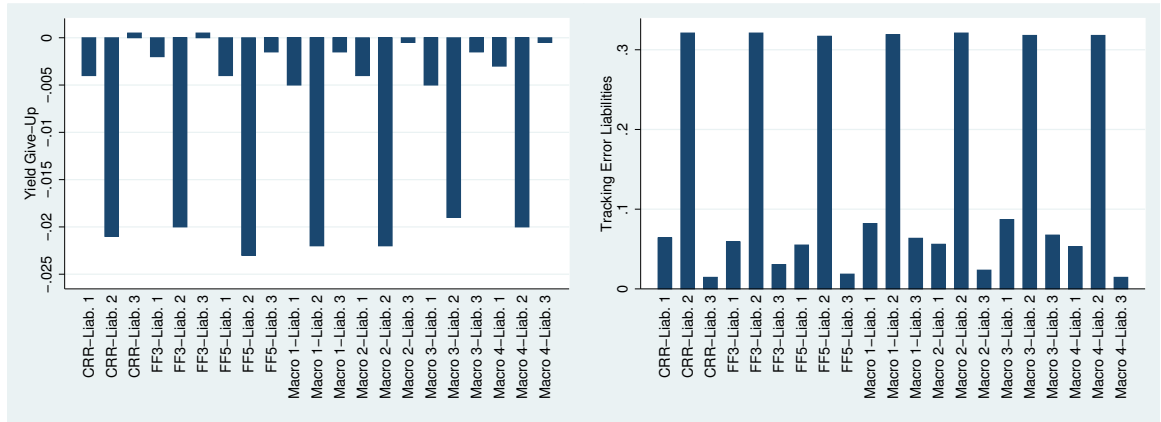
(a) Yield Give-Up

(b) Tracking Error of Liabilities



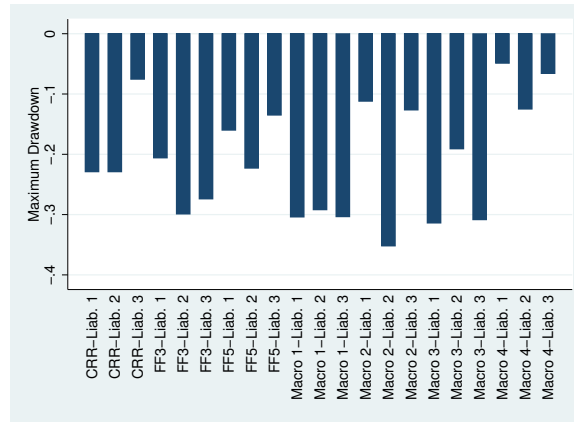
(c) Maximum Drawdown

Figure 25. : Replicating Portfolio Metrics - Fixed Income Strategies



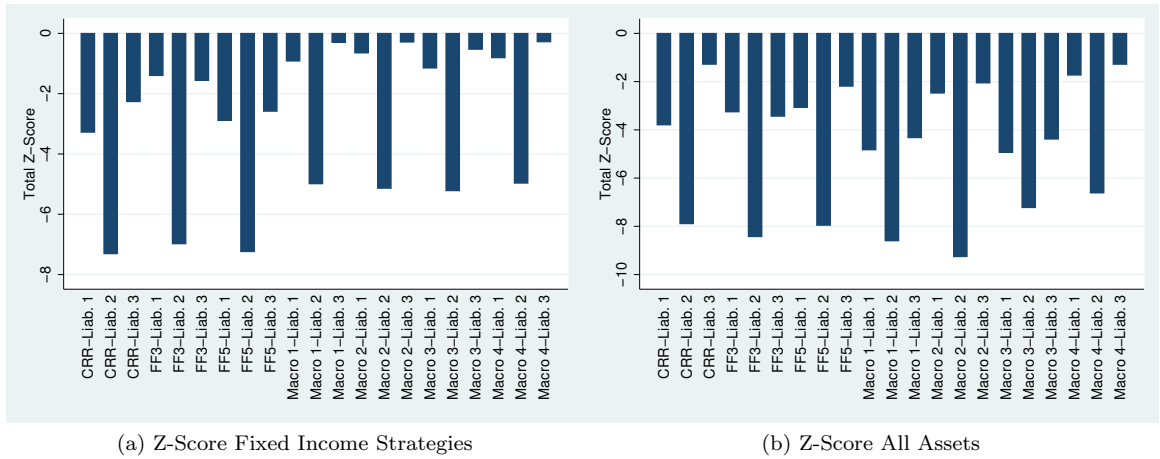
(a) Yield Give-Up

(b) Tracking Error of Liabilities



(c) Maximum Drawdown

Figure 26. : Replicating Portfolio Metrics - All Assets



(a) Z-Score Fixed Income Strategies

(b) Z-Score All Assets

Figure 27. : Total Z-Score by Asset Spectrum

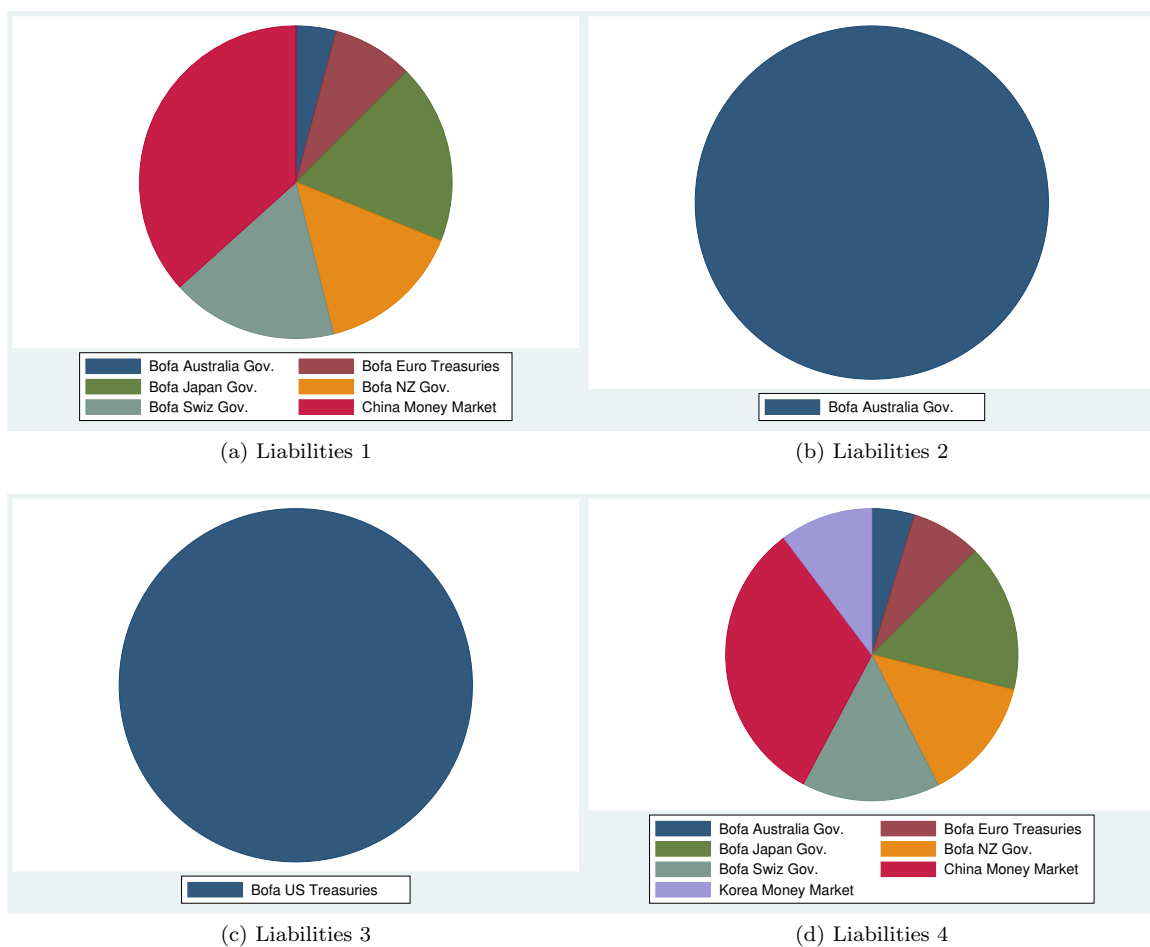


Figure 28. : Best Replicating Portfolio by Z-Score Rule - Fixed Income Strategies

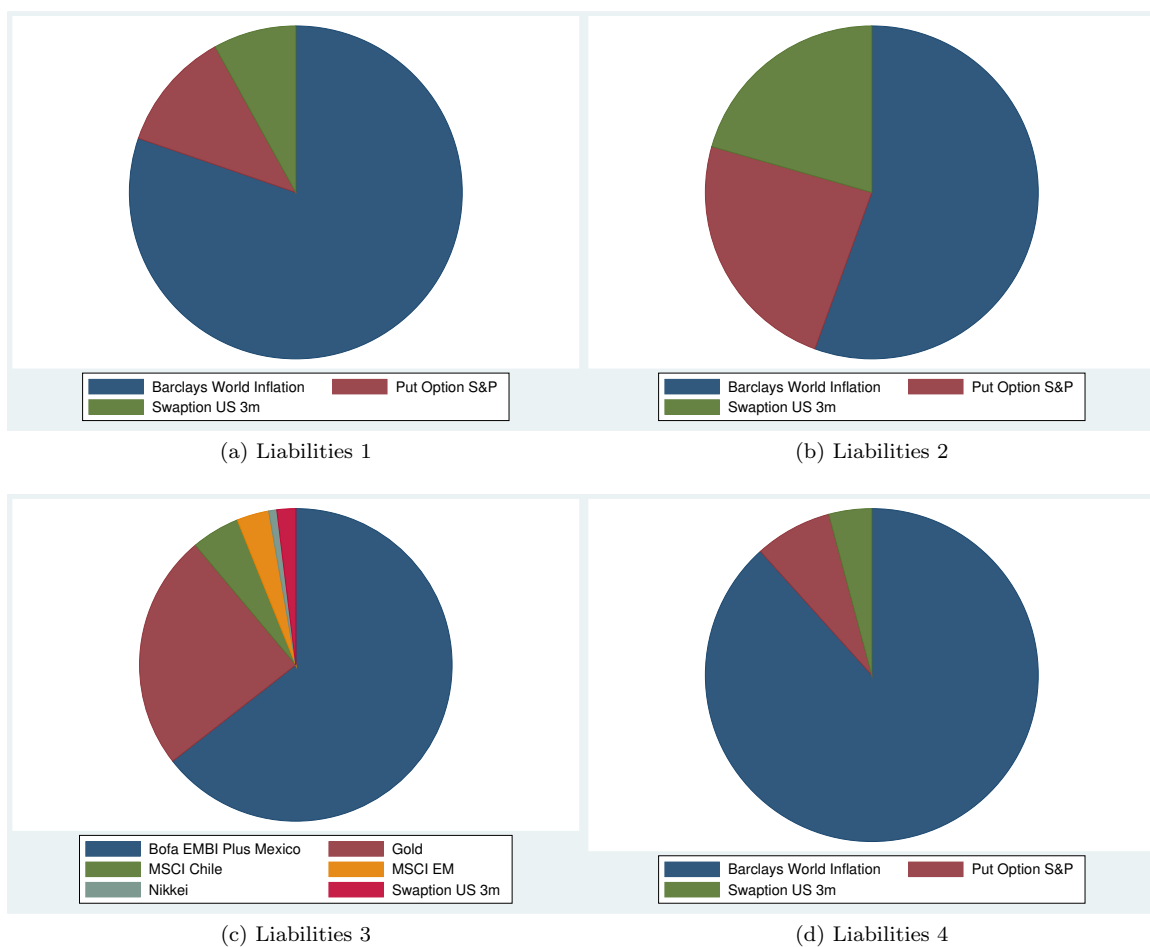


Figure 29. : Best Replicating Portfolio by Z-Score Rule - All Assets