An empirical index of Knightian uncertainty

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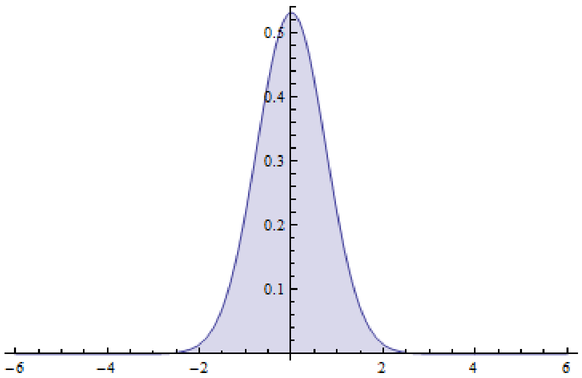
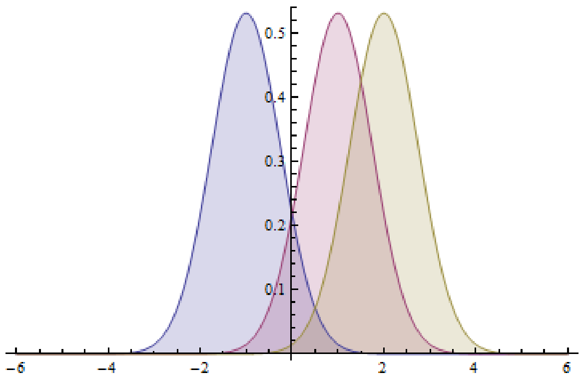
Introduction

- Risk: the state to be realized is unknown, but the probability of each state is known.
- Savage (1954): Individuals select among actions with risky outcomes by attaching a utility index and a unique probability to the outcome.
- Uncertainty: both the state to be realized and the probabilities of states are unknown.
- Ellsberg (1961):Most of the Decision makers prefer taking gambles with known probabilities over equivalent gambles with ambiguous probabilities.

Knightian uncertainty



$$E[u(\tilde{x})] = \int U(z)dF_x(z)$$



max-min expected utility (MEU) of Gilboa and Schmeidler (1989):

$$U(h) = (1-\epsilon)\int_\Omega u(h)\,dp^* + \epsilon \min_{p\in P}\int_\Omega u(h)\,dp$$

Anderson , Hansen and Sargent (2003) robustness:

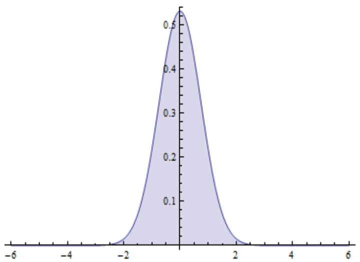
$$U^{MU}(h) = \min_{p\in\Delta(\Omega)}\left[\int_\Omega u(h)\,dp + \theta R(p \,\|\, p^*)\right]$$

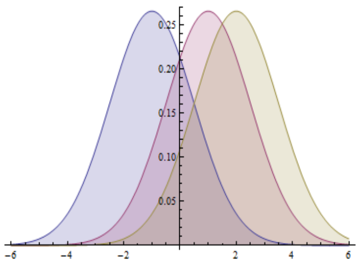
Klibanoff, Marinacci and Mukreji (2005)

$$V(f) = \int_\Delta \phi\left(\int_S u(f)\,d\pi\right)d\mu = \mathbb{E}_\mu \phi(\mathbb{E}_\pi u \circ f)$$

Motivation and contribution

- Suggest a simple methodology to rank stocks based on their realized daily distributions. (the measure should assign a higher index to the right stock)





- Examine how this index is priced in the cross-section of stock returns.

A measure of distance between distributions

- Volatility (a measure of risk), computes the degree of dispersion between *returns.*
- We want to measure the degree of dispersion between *densities*. We want to calculate an index such that: The higher the variation in probabilities, the higher the index.
- Kullback-Leibler divergence (1951):

$$J_{P(s)_t}(P(s)_{t-1}) = \sum_s P(s)_t \ln\frac{P(s)_t}{P(s)_{t-1}} + \sum_s P(s)_{t-1} \ln\frac{P(s)_{t-1}}{P(s)_t}$$

$$= \sum_s (P(s)_t - P(s)_{t-1})(\ln P(s)_t - \ln P(s)_{t-1}) \ ,$$

- $P(s)_t$ is the probability of a state of nature at date t. We need the entire distribution to calculate the above measure. Having found the daily divergences we compute the average over a month:

$$MD = \frac{1}{D}\sum_1^D J_{P(s)_t}(P(s)_{t-1})$$

Kullback-Leibler divergence, an example

Table 1
Implied distributions and ambiguity measures.

	Stock 1				Stock 2				Stock 3			
	G-B	G-S	L-S	L-B	G-B	G-S	L-S	L-B	G-B	G-S	L-S	L-B
Day 1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Day 2	0.26	0.26	0.24	0.24	0.30	0.30	0.20	0.20	0.30	0.20	0.20	0.30
Day 3	0.255	0.255	0.245	0.245	0.275	0.275	0.225	0.225	0.20	0.30	0.30	0.20
	0.001				0.025				0.10			

Deriving the index

- Option prices represent investor expectations about future events. We use the option skew to calculate daily risk-neutral implied density.

- For this draft, we have used Bloomberg, Bloomberg data is limited to seven daily fixed-maturity IVs and it covers 2005-2012.

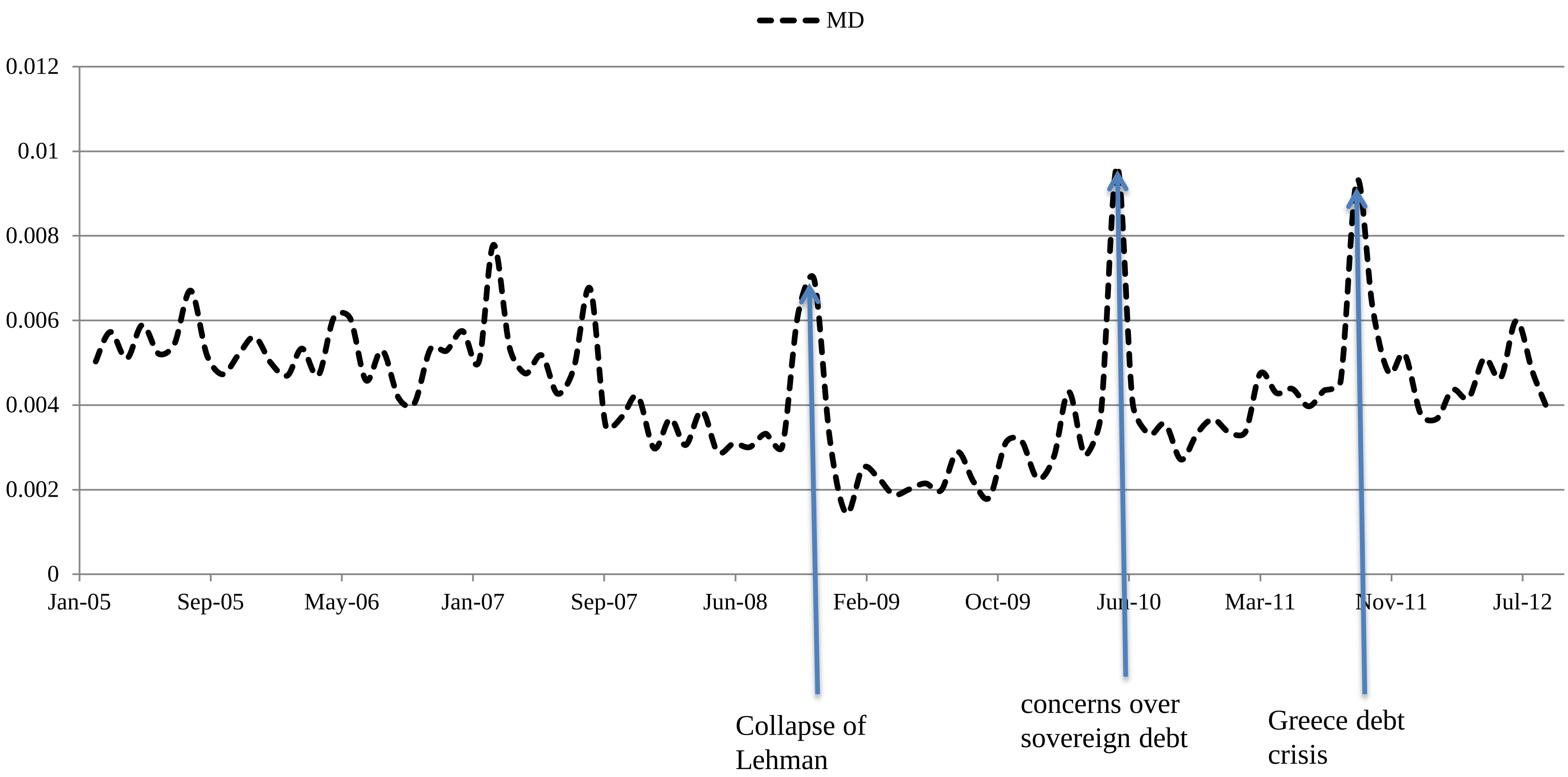
- We estimate the density using a monotonous transformation of IVs to probabilities, for example:

$$P(0.975 < x < 1) = P(1 < x < 1.025) = \Phi\left(\frac{1.025-\mu}{\sigma_1}\right) - \Phi\left(\frac{1-\mu}{\sigma_1}\right)$$

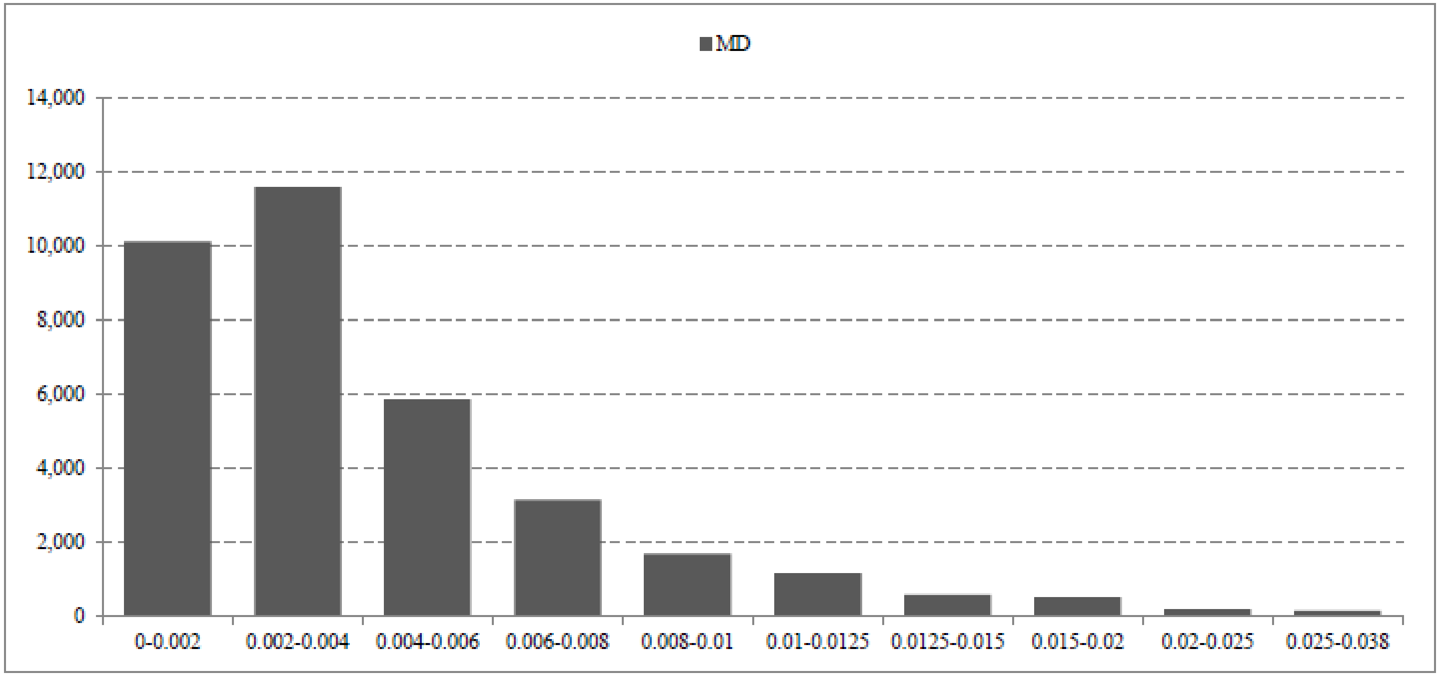
Where $\Phi(\bullet)$ is the cumulative distribution function

Then $P(1.025 < x < 1.05)$ is calculated using IV of option with 102.5% moneyness level, $P(1.05 < x < 1.1)$ uses 105% and so on.

MD over time


MD
0.012
0.01
0.008
0.006
0.004
0.002
0
Jan-05
Sep-05
May-06
Jan-07
Sep-07
Jun-08
Feb-09
Oct-09
Jun-10
Mar-11
Nov-11
Jul-12
Collapse of Lehman
concerns over sovereign debt
Greece debt crisis

Histogram of the MD index (all stocks)



Summary statistics and cross-sectional (Spearman) correlations

	Ret*	MD*	Vol*	Beta	Ln(S)	B/M	S.T.R.*	Mom*	Kurt	Skew	Turn
Excess Returns	1										
MD	-0.06	1									
Volatility	-0.01	-0.21	1								
Beta	0.01	-0.28	0.38	1							
ln(Size)	-0.04	0.14	-0.21	-0.20	1						
Book to Market	0.01	0.01	0.09	0.15	-0.07	1					
Short-term reversal	-0.02	-0.07	-0.16	0.01	0.03	-0.09	1				
Momentum	-0.01	0.02	-0.14	0.01	0.10	-0.24	0.21	1			
Kurtosis	-0.02	0.10	0.03	-0.16	-0.09	-0.12	-0.04	-0.10	1		
Skewness	0.00	-0.09	0.03	0.05	-0.04	-0.09	0.06	0.10	0.10	1	
Turnover	0.01	-0.28	0.62	0.37	-0.38	0.07	-0.07	-0.09	0.03	0.00	1
Mean	**0.75**	**0.42**	**8.26**	**1.13**	**23.3**	**0.45**	**0.90**	**10.60**	**6.30**	**0.12**	**0.23**
Standard deviation	**8.33**	**0.37**	**4.70**	**0.43**	**1.00**	**0.30**	**8.19**	**32.10**	**3.85**	**0.67**	**0.15**

Sorts

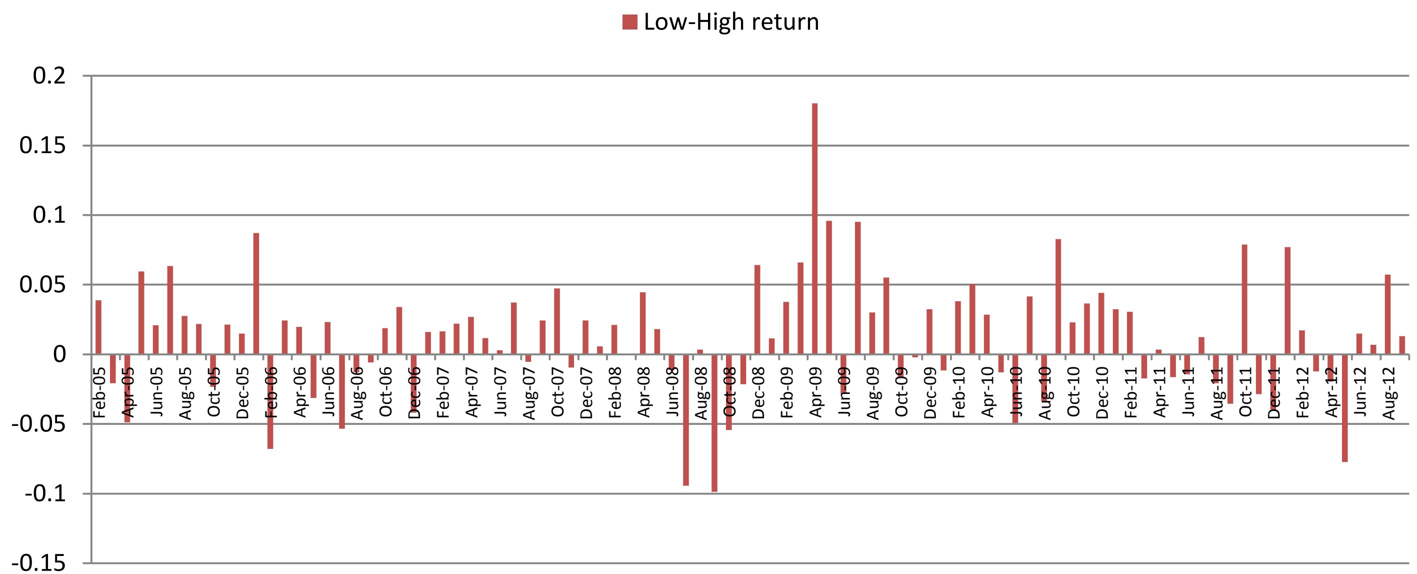
Table 2
Control variable sorts.

Our sample of stocks is first sorted into five portfolios based on MD and we calculate the average of each control variable for each quintile. Columns 1-5 report the calculated average for each variable. Next, every month we regress the difference between the averages of the asset pricing variables in the lowest and highest ambiguity quintiles on a constant (one), the coefficient of this constant is reported in the 6th column (Low−High), the corresponding *t*-statistics is reported in the last column and is Newey-West corrected. The coefficients that are significant at 10 percent level are presented in **bold** face. Variables expressed in percentage are distinguished by *.

	Portfolio						
	Low	2	3	4	High	Low−High	*t*-statistic
MD	0.14	0.23	0.33	0.48	0.94	-0.81	
Excess Returns*	1.16	0.90	0.71	0.62	0.36	**0.80**	2.30
Volatility*	9.90	8.79	8.12	7.60	7.08	**2.82**	12.70
Beta	1.34	1.22	1.12	1.03	0.92	**0.43**	16.70
ln(Size)	23.00	23.38	23.40	23.50	23.51	**-0.51**	5.85
Book to Market	0.43	0.45	0.46	0.44	0.45	-0.01	-0.63
Short-term reversal*	1.16	1.09	0.83	0.76	0.51	**0.66**	1.89
Momentum*	15.86	12.00	10.30	8.35	6.75	**9.11**	2.67
Kurtosis	6.19	6.14	6.15	6.26	6.77	**-0.58**	3.74
Skewness	0.17	0.13	0.12	0.10	0.10	**0.07**	1.91
Turnover	0.30	0.25	0.21	0.19	0.18	**0.12**	47.30

Low-High strategy has a Sharpe ratio of 0.82.

Low-High MD portfolio performance



- 48.6% higher returns than market portfolio during the crisis

- 39.3% higher returns than market portfolio before-after crisis

Mean portfolio returns

Mean Returns

Panel A. Beta, Turnover and MD

	Small Beta			Medium Beta			Large Beta		
Ambiguity quintiles	Low Turn	Average Turn	High Turn	Low Turn	Average Turn	High Turn	Low Turn	Average Turn	High Turn
Low	0.56	0.78	1.09	0.94	0.77	1.46	1.01	1.28	0.92
Medium	0.41	0.60	0.74	0.75	0.55	0.73	0.64	0.99	0.85
High	0.13	0.48	0.49	0.53	0.18	0.55	0.42	0.59	0.67
Low−High	**0.43**	0.30	0.59	**0.40**	0.18	**0.91**	**0.59**	0.69	0.25
t-statistic	2.55	0.84	1.39	1.94	0.85	3.68	1.88	1.63	0.56

Panel B. Size, Book-to-Market and MD

	Low Book-to-Market			Medium Book-to-Market			High Book-to-Market		
Ambiguity quintiles	Small Cap	Medium Cap	Large Cap	Small Cap	Medium Cap	Large Cap	Small Cap	Medium Cap	Large Cap
Low	1.97	0.86	0.61	1.70	0.87	0.48	1.06	0.82	0.65
Medium	1.24	0.26	0.82	0.96	0.60	0.50	0.89	0.80	0.46
High	0.64	0.73	0.39	0.75	0.34	0.40	0.52	0.41	0.31
Low−High	**1.33**	0.13	0.22	**0.96**	**0.53**	0.09	0.54	0.41	0.34
t-statistic	3.73	0.30	0.98	2.62	1.84	0.40	1.52	0.88	0.94

Panel C. Skewness, Kurtosis and MD

	Small Kurtosis			Medium Kurtosis			Large Kurtosis		
Ambiguity quintiles	Small Skewness	Medium Skewness	Large Skewness	Small Skewness	Medium Skewness	Large Skewness	Small Skewness	Medium Skewness	Large Skewness
Low	1.66	1.20	1.07	1.04	0.76	0.80	0.73	0.63	1.22
Medium	0.80	0.90	0.57	0.71	0.70	0.77	1.01	0.41	0.68
High	0.65	0.69	0.56	0.19	0.40	0.23	0.24	0.60	0.69
Low−High	**1.01**	0.51	0.51	**0.85**	0.37	**0.57**	0.50	0.03	0.52
t-statistic	2.43	1.30	1.60	2.87	1.02	1.70	1.41	0.08	1.40

Factor alphas

Table 4

Tests of CAPM, three and four factor models for equally weighted portfolios sorted on MD.

This table displays alphas and factor loadings for five portfolios sorted by MD. The portfolios are formed as in Table 4. We run CAPM, Fama-French three factor model and Fama-French-Carhart four factor model by estimating: $r_{it} - r_{ft} = \alpha + \beta(r_{Mt} - r_{ft}) + sSMB_t + hHML_t + mUMD_t$ for every month from February 2005 to September 2012. r_{it} is the return on the equally weighted portfolio, r_{Mt} is the return on the value weighted market portfolio (S&P500). SMB, HML and UMD are downloaded from Kenneth French's website. Newey-West t-statistics (12 month lag) are reported in parenthesis. We report the Low minus High alpha for the three models and the corresponding t-statistics in Panel B. The coefficients that are significant at 10 percent level are presented in **bold** face.

Panel A. Factor loadings

Quintile	alpha (%)	$R_M - R_f$	SMB	HML	UMD
Low	**0.66**	**1.13**			
	(1.98)	(14.90)			
	0.58	**1.08**	**0.53**	**-0.30**	
	(2.17)	(26.19)	(4.89)	(-2.11)	
	0.58	**1.08**	**0.53**	**-0.30**	0.00
	(2.17)	(25.49)	(4.87)	(-2.03)	(-0.04)
2	**0.42**	**1.06**			
	(1.75)	(24.86)			
	0.38	**1.01**	**0.37**	-0.08	
	(1.90)	(26.22)	(5.50)	(-1.25)	
	0.38	**0.99**	**0.37**	**-0.11**	-0.05
	(1.96)	(22.65)	(5.22)	(-1.76)	(-1.94)
3	0.21	**1.01**			
	(1.25)	(24.29)			
	0.17	**0.94**	**0.37**	-0.04	
	(1.35)	(31.81)	(6.71)	(-0.96)	
	0.18	**0.93**	**0.37**	**-0.07**	**-0.05**
	(1.36)	(25.55)	(7.38)	(-1.76)	(-3.47)
4	0.20	**0.89**			
	(1.42)	(17.62)			
	0.18	**0.85**	**0.22**	-0.01	
	(1.56)	(18.30)	(4.32)	(-0.47)	
	0.18	**0.85**	**0.22**	-0.01	0.00
	(1.55)	(17.52)	(4.34)	(-0.46)	(-0.06)
5	0.05	**0.70**			
	(0.36)	(16.73)			
	0.04	**0.67**	**0.14**	0.04	
	(0.32)	(13.46)	(2.16)	(0.53)	
	0.05	**0.69**	**0.14**	0.07	0.06
	(0.42)	(14.10)	(1.99)	(1.09)	(1.54)

Panel B. Difference in alphas

Model	Low minus High Alpha (%)	*t*-statistics
CAPM	**0.62**	1.71
3F	**0.54**	1.83
4F	**0.54**	1.84

FM regressions of MD on firm characteristics

Table 5

Fama MacBeth predictive regressions of ambiguity indices on firm characteristics.

We run Fama and Macbeth (1973) monthly cross-sectional regressions from February 2005 to September 2012. The dependent variable is the distance between two implied distributions on two consecutive trading days (MD), averaged over a month(*t*). Beta is measured by regressing weekly stocks returns on market return (S&P 500) from *t-24* to *t-1*. Ln(size) is the natural logarithm of market capitalization on the last day of month *t*. Book-to-Market is the ratio of most recent book value to market capitalization on the last day of month *t*. Short-term reversal is last month's return. Momentum is the cumulative return from *t-12* to *t-1*. Kurtosis is the fourth-order centralized moment of daily returns from *t-11* to *t*. Skewness is third-order centralized moment of daily returns from *t-11* to *t*. Turnover is the ratio of total number of shares traded over the last month to total number of shares outstanding. In All of the independent variables are lagged one period, and the dependent variable is MD. Newey-West t-statistics (12 month lag) are reported in parenthesis. The coefficients that are significant at 10 percent level are presented in **bold** face. Constant terms are omitted.

(*N* =31,480)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(11)	(12)	(13)
Volatility	**-0.038**											**-0.022**
	(-6.42)											(-6.57)
Beta		**-0.003**								**-0.003**	**-0.003**	**-0.001**
		(-7.83)								(-9.34)	(-9.52)	(-8.31)
Ln(size)			**<0.001**							-0.000	-0.000	**<-0.001**
			(1.83)							(-0.41)	(-0.53)	(-1.73)
Book-to-Market				**0.121**						**0.164**	**0.147**	**0.121**
				(2.11)						(4.71)	(4.18)	(5.84)
Short term reversal					**-0.003**							**-0.001**
					(-2.82)							(-2.53)
Momentum						**-0.001**					**-0.001**	**-0.001**
						(-2.45)					(-2.62)	(-1.73)
Kurtosis							**<0.001**					**<0.001**
							(2.87)					(3.52)
Skewness								**-0.001**				**-0.001**
								(-2.70)				(-1.70)
Turnover									**-0.008**			**-0.003**
									(-6.03)			(-5.91)
R-squared	0.091	0.074	0.023	0.008	0.017	0.014	0.009	0.007	0.067	0.107	0.114	0.172

FM regressions of return on MD and controls

Panel A. MD (*N* =31,480)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
MD	**-1.016**	**-0.875**	**-0.829**	**-0.901**	**-0.989**	**-0.944**	**-0.873**	**-1.001**	**-0.993**	**-0.880**	**-0.744**	**-0.655**	**-0.551**
	(-2.01)	(-2.02)	(-2.23)	(-1.85)	(-2.04)	(-1.89)	(-1.79)	(-2.11)	(-2.00)	(-1.98)	(-2.00)	(-1.72)	(-1.69)
Volatility		0.041											0.026
		(1.60)											(1.05)
Beta			0.001								0.000	-0.001	-0.001
			(0.41)								(0.15)	(-0.29)	(-0.60)
Ln(size)				**-0.002**							**-0.002**	**-0.002**	**-0.002**
				(-3.10)							(-3.84)	(-3.89)	(-3.27)
Book-to-Market					-0.209						-0.244	-0.110	-0.092
					(-0.48)						(-0.60)	(-0.34)	(-0.30)
Short term reversal						-0.008							-0.022
						(-0.54)							(-1.38)
Momentum							0.003					0.004	0.003
							(0.54)					(0.70)	(0.65)
Kurtosis								0.000					0.000
								(0.03)					(0.63)
Skewness									0.000				-0.000
									(0.19)				(-0.31)
Turnover										**0.012**			0.001
										(1.98)			(0.21)
R-squared	0.019	0.047	0.063	0.029	0.038	0.042	0.046	0.026	0.024	0.043	0.085	0.105	0.143

MD is strongly associated with future returns.

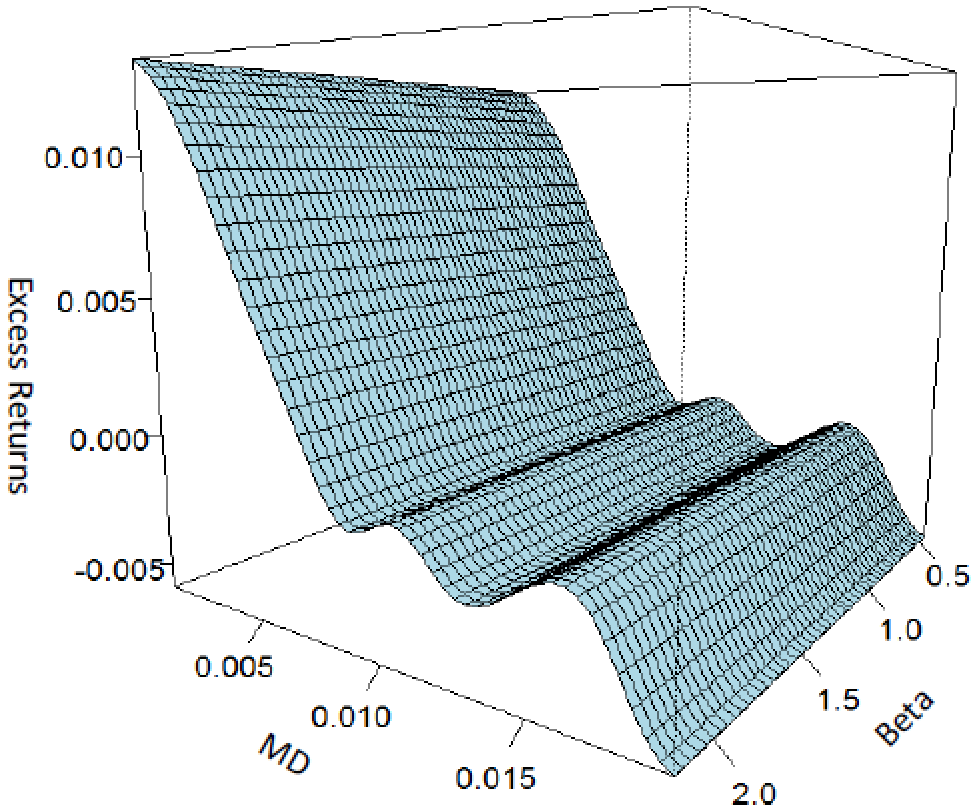
Sub-period analysis

Table 7
Sub-period Analysis.

Each month we sort our sample of stocks into five portfolios based on their level of ambiguity and calculate the return of each equally weighted portfolio over the following month. Average return of each portfolio and the difference between returns of the low minus high ambiguity portfolio is reported for each sub-period. All values are calculated using the procedure in Table 2, which is calculating returns for equally weighted portfolios and regressing those return on a constant (one). All returns are in percentages and differences that are significant at 10 percent level are presented in **bold** face.

	Mean Excess Return						
Period	Low	2	3	4	High	Low − High	*t*-stat
2005-2007	1.30	1.01	0.74	0.68	0.53	**0.77**	1.66
2008-2009	0.68	0.25	0.13	-0.03	-0.23	0.91	0.89
2010-2012	1.36	1.22	1.04	0.98	0.61	**0.75**	1.81

Robustness



Risk, MD and return surface – estimated using 25 portfolios based on Beta and MD

Possible explanations

- Bier and Connell (1994) : **Optimistic** people show greater ambiguity seeking
- In Pulford (2009): Highly **optimistic** people feel that "**luck is on their side**".
- Dimmock et al (2012): Ambiguity aversion has a negative relation with **participation in stock market**.
- Chakravarty and Roy (2009) : People are ambiguity neutral over gains and **ambiguity seekers over losses**.
- Other studies that found ambiguity seeking attitude: Einhorn and Hogarth (1986), Di Mauro and Maffioletti (1996), Abdellaoui et al (2005), Brenner and Izhakian.

Summary

- This study proposes an easy methodology to construct an index of realized Knightian uncertainty, the index is the dispersion between daily risk-neutral densities.
- We find that the proposed index is strongly associated with lower future returns.
- Portfolio containing stocks with a low ambiguity index outperforms portfolio containing high ambiguity stocks by 8.47% annually.

Future work

- Currently we are using OptionMetrics, we are able to:

1. Estimate the density using a nonparametric approach,
2. Extend the sample period to 17 years (1996-2012).
3. Control for more pricing factors

Thank You!