Correlation - Products

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Credit Risk Elective
Spring 2009
CDO Issuance ($millions)

Source: SIFMA
SIZE of Mortgage-backed CDO Issuance ($mm),
Source: SIFMA
## Banks “Bet” on the AAA tranches:
**Holders of Mortgage-backed Debt**

<table>
<thead>
<tr>
<th>Category</th>
<th>Loans</th>
<th>HELOC</th>
<th>Agency MBS</th>
<th>Non-Agency AAA</th>
<th>CDO Subord</th>
<th>Non CDO Subord</th>
<th>Total</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banks &amp; Thrifts</td>
<td>2,020</td>
<td>869</td>
<td>852</td>
<td><strong>383</strong></td>
<td>90</td>
<td></td>
<td>4,212</td>
<td>39%</td>
</tr>
<tr>
<td>GSEs &amp; FHLB</td>
<td>444</td>
<td></td>
<td>741</td>
<td><strong>308</strong></td>
<td></td>
<td></td>
<td>1,493</td>
<td>14%</td>
</tr>
<tr>
<td>Brokers/dealers</td>
<td></td>
<td>49</td>
<td>100</td>
<td>130</td>
<td>24</td>
<td></td>
<td>303</td>
<td>3%</td>
</tr>
<tr>
<td>Financial Guarantors</td>
<td></td>
<td>62</td>
<td></td>
<td>100</td>
<td></td>
<td></td>
<td>162</td>
<td>2%</td>
</tr>
<tr>
<td>Insurance Companies</td>
<td></td>
<td>856</td>
<td><strong>125</strong></td>
<td>65</td>
<td>24</td>
<td></td>
<td>1,070</td>
<td>10%</td>
</tr>
<tr>
<td>Overseas</td>
<td></td>
<td>689</td>
<td><strong>413</strong></td>
<td>45</td>
<td>24</td>
<td></td>
<td>1,172</td>
<td>11%</td>
</tr>
<tr>
<td>Other</td>
<td>461</td>
<td>185</td>
<td>1,175</td>
<td><strong>307</strong></td>
<td>46</td>
<td>49</td>
<td>2,268</td>
<td>21%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>2,925</td>
<td>1,116</td>
<td>4,362</td>
<td><strong>1,636</strong></td>
<td>476</td>
<td>121</td>
<td>10,680</td>
<td></td>
</tr>
<tr>
<td><strong>% of Total</strong></td>
<td>27%</td>
<td>10%</td>
<td>41%</td>
<td>15%</td>
<td>4%</td>
<td>1%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Gyrations in the price of CDO tranches
Gyrations in corporate bond spreads
Outline

• Basket credit derivatives
  ✓ FTD swaps
  ✓ CLOs, CBOs etc.

• Correlation
  ✓ why correlation is important
  ✓ impact on FTD swaps

• CDOs and CDO tranchees

• Index products
  ✓ Again, why correlation is important
  ✓ How correlations changed during the crisis of 2007-09
Basket Default Products

- **Basket default swaps**: provides protection against one or more defaults in basket of \( n \) issuers/names.
- **First-to-default swaps**: provides protection against *first default* in basket of \( n \) issuers/names.
- **\( k^{th} \)-to-default swaps**: provides protection against *\( k \)th default* in basket of \( n \) issuers/names.
- **First-\( m \)-of-\( n \) to default swaps**: provides protection against *first \( m \) defaults* in basket of \( n \) issuers/names.
Basket Products: first-to-default and first-m-of-n to default swaps

• A first \( m\)-of-\( n\) to default swap can always be decomposed into a portfolio of first-to-default swaps (but very cumbersome for large \( n\))

• **Example**: a first 2-of-3 to default swap on firms \( A, B \) and \( C \)

• This contract may be *replicated* by the following portfolio of first to default swaps:
  - **long** position in *three first-to-default* swaps on firms \((A, B), (B, C)\) and \((A, C)\);
  - **short** one *first-to-default swap* on firms \((A, B, C)\);

• The payoffs are show on the next slide
Constructing a “first 2 of 3” to default swap from a portfolio of FTD swaps

Payoff on Contracts*

<table>
<thead>
<tr>
<th>Contract</th>
<th>Holding</th>
<th>None</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>AB</th>
<th>AC</th>
<th>BC</th>
<th>ABC</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTD on AB</td>
<td>+1</td>
<td>0</td>
<td>-L</td>
<td>-L</td>
<td>0</td>
<td>-L</td>
<td>-L</td>
<td>-L</td>
<td>-L</td>
</tr>
<tr>
<td>FTD on AC</td>
<td>+1</td>
<td>0</td>
<td>-L</td>
<td>0</td>
<td>-L</td>
<td>-L</td>
<td>-L</td>
<td>-L</td>
<td>-L</td>
</tr>
<tr>
<td>FTD on BC</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>-L</td>
<td>-L</td>
<td>-L</td>
<td>-L</td>
<td>-L</td>
<td>-L</td>
</tr>
<tr>
<td>FTD on ABC</td>
<td>-1</td>
<td>0</td>
<td>+L</td>
<td>+L</td>
<td>+L</td>
<td>+L</td>
<td>+L</td>
<td>+L</td>
<td>+L</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>0</td>
<td>-L</td>
<td>-L</td>
<td>-L</td>
<td>-2L</td>
<td>-2L</td>
<td>-2L</td>
<td>-2L</td>
</tr>
<tr>
<td>First 2 to Def on ABC</td>
<td></td>
<td>0</td>
<td>-L</td>
<td>-L</td>
<td>-L</td>
<td>-2L</td>
<td>-2L</td>
<td>-2L</td>
<td>-2L</td>
</tr>
</tbody>
</table>

*Note:* “L” represents the loss given default
Effect of Correlation on FTD Swaps

• With *low correlation* a diversified portfolio of credits will have a loss distribution that is *centred* around the *expected value*
  ✓ *small chance* of large portfolio *losses*
  ✓ *small chance* of low portfolio *losses*

• As correlation increases the probability of both small and large portfolio losses increases

• With *perfect correlation* either
  ✓ 0% credits default
  or
  ✓ 100% credits default
Effect of Correlation – Two Assets

• Suppose assets A & B both have default probabilities of 0.5
Limits on FTD Pricing

• Zero correlation:
  ✓ the default intensity for the first-to-default is simply the sum of the individual credit intensities.
  ✓ FTD spread in this case is close to the sum of the spreads

• 100% correlation (not often observed)
  ✓ FTD spread is close to the maximum spread in the basket.
The Distribution of Portfolio Losses

• *Default* is a *binomial* event: it happens or it doesn’t

• With a fixed recovery rate the distribution of portfolio losses is the distribution of the *number* of *defaults*

• But *difficult* to include default *correlation* directly into standard binomial framework
Loan Loss Distribution with $p = 1\%$ and $\rho = 12\%$ and $0.6\%$
The Basel II Capital Structure Rules and Generalisations

• Under the new *Basel II* rules, *bank capital* for banks using the so-called “internal ratings based (IRB) approach” is calculated using a version of this model (derived originally by Vasicek)
  ✓ But model assumed *time-invariant* correlations!

• The approach can be generalised to accommodate multiple factors (e.g., industry sectors, geographical characteristics etc.)
  ✓ this provides a potentially *better characterisation* of *asset correlation*
  ✓ but at the expense of having *no simple formula* for the distribution of loan losses
Warren Buffet on Derivatives – March 2003

• I view derivatives as *time bombs*, both for the parties that deal in them and the economic system.

• I believe, however, that the macro picture is dangerous and getting more so. *Large amounts of risk, particularly credit risk, have become concentrated in the hands of relatively few derivatives dealers*, who in addition trade extensively with one another. The troubles of one could quickly infect the others.

• In my view, *derivatives are financial weapons of mass destruction*, carrying dangers that, while now latent, are potentially lethal.

Berkshire Hathaway annual report for 2002
Examples of Other Basket Products

• **Index** Products
  ✓ CDX, iTraxx, etc.

• **Tranched** index products
  ✓ structure
  ✓ motivation
  ✓ valuation and hedging
Credit Index Products
Index Products

• **CDS index** .. just a *portfolio* of single-name CDS
  ✓ *protection seller* provides protection (and receives premium) on *portfolio of names* (rather than on just one).

• **Wide range** of indices
  ✓ CDX
  ✓ iTraxx
  ✓ …. and many sub-indices
# CDS indices

By region

<table>
<thead>
<tr>
<th>Master</th>
<th>North America</th>
<th>Europe</th>
<th>Japan</th>
<th>Asia excl Japan</th>
<th>Australia</th>
<th>Emerging markets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CDX.NA.IG (125)</td>
<td>iTraxx Europe (125)</td>
<td>iTraxx CJ</td>
<td>iTraxx Asia</td>
<td>iTraxx</td>
<td>CDX.EM</td>
</tr>
<tr>
<td></td>
<td>CDX.NA.HY (100)</td>
<td>iTraxx Corporate (52)</td>
<td>(50)^2</td>
<td>Asia</td>
<td>Australia (25)</td>
<td>(14)^3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>iTraxx Crossover (30)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sub-indices</td>
<td>Financials (24)</td>
<td>Financials (15)</td>
<td>Financials</td>
<td>Korea (8)</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>Consumer (34)</td>
<td>Autos (10)</td>
<td>(10)</td>
<td>Greater</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Energy (15)</td>
<td>Consumer cyclicals (15)</td>
<td>Capital</td>
<td>China (9)^6</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Industrials (30)</td>
<td>Consumer non-</td>
<td>goods (10)</td>
<td></td>
<td>None</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TMT (22)</td>
<td>cyclicals (15)</td>
<td>Tech (10)</td>
<td></td>
<td>None</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HiVol (30)</td>
<td>Energy (20)</td>
<td>HiVol (10)</td>
<td></td>
<td>None</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B (44)</td>
<td>Industrials (20)</td>
<td></td>
<td></td>
<td>None</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB (43)</td>
<td>TMT (20)</td>
<td></td>
<td></td>
<td>None</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HB (30)</td>
<td>HiVol (30)</td>
<td></td>
<td></td>
<td>None</td>
<td></td>
</tr>
</tbody>
</table>

1 Earlier generations of DJ Trac-x and iBoxx indices are still traded. This table summarises the composition of the most recently issued series, DJ CDX and DJ iTraxx, which are a by-product of the merger between the DJ Trac-x and iBoxx families. The number of reference entities in each index is given in parentheses.  
2 Maximum of 10 names in a given sector.  
3 Includes only sovereigns: Brazil, Bulgaria, Colombia, Korea, Malaysia, Mexico, Panama, Peru, the Philippines, Romania, Russia, South Africa, Turkey and Venezuela.  
4 Includes the largest, most liquid non-financial names from the iBoxx EUR Corporate bond index.  
5 Most liquid non-financial names rated BBB/Baa3 or lower and on negative outlook.  
6 Includes China, Hong Kong SAR and Taiwan (China), with at least two names from each.  
7 Includes India, Malaysia, the Philippines, Singapore and Thailand.
Sectors and ratings distributions

<table>
<thead>
<tr>
<th>Sectors</th>
<th>North America¹</th>
<th>Europe²</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FIN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IND</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TMT</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ratings³</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
</tr>
<tr>
<td>AA</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>BBB</td>
</tr>
</tbody>
</table>

Note: CNS = consumer, EN = energy, FIN = financials, IND = industrials, TMT = technology, media and telecommunications. For Europe, CNS includes consumer cyclicals and consumer non-cyclicals, and IND includes industrials and autos.

¹ DJ CDX.NA.IG.3. ² DJ iTraxx Europe Series 2. ³ Average rating from Moody’s, Standard & Poor’s and Fitch (where available).

Sources: Bloomberg; BIS calculations.

Graph 1
Dow Jones CDX.NG.IG Structure

Single Name Credit Default Swaps

Buyer

Notional: USD 10MM

Notional x [ ] bp p.a.

Credit Risk of XYZ

Seller

Delivery of 10MM Principal XYZ
Senior Unsecured Obligation

Occurrence of a Credit Event (Physical Settlement)

$10MM Cash

Index Products
Dow Jones CDX.NA.IG

125 equally weighted names

Source: Morgan Stanley
Dow Jones CDX.NG.IG: Portfolio Characteristics

• **Composition** of the IG Index
  ✓ 125 most liquid investment grade credits domiciled in North America;
  ✓ determined by Dow Jones
  ✓ initially strictly investment grade credits; no high yield or distressed credits are included

• **Equal weightings** for all credits

• **Diverse pool** of credits across industry sectors: represents a broad exposure to the investment grade corporate market

• **Reference Entities cannot be added** and will only be **removed** upon the triggering of a **credit event**

• A majority vote is required by members to determine which reference entities will be removed from the Index
Why are the Index Products Important?

• As with *single name CDS*, provides ability for investors to:
  ✓ take *long* and *short* positions in credit
  ✓ take credit exposure either *leveraged* or *unleveraged*

• *Index products* also provide:
  ✓ ability to specialise exposure by
    - *geographical* region and *industry*
    - credit *quality*
  ✓ ability to *hedge* single name credit *against* movement in *market* spread
  ✓ high *liquidity*
Tranch Products
Tranched Products – Example: Cash CDO

• **Assets**
  ✓ *homogeneous* portfolio of “average” quality loans / bonds (e.g., A/BBB)

• **Liabilities**
  ✓ series of “tranches” that have *unequal exposure* to portfolio losses
  ✓ *senior tranche* – bears losses only if losses exceed 30% (say) of portfolio value
  ✓ *equity tranche* – bears initial losses (up to limit)

“repackaging”
Structure of Collateralised Loan and Bond Obligations

Underlying Assets (e.g., A, BBB)
Bank Loans (CLO)
Bonds (CBO)

Senior Tranche (e.g., AAA)

Mezzanine Tranche (e.g., BB)

Junior Tranche / Equity (NR)
Investor preferences and Repackaged/Tranched Structures

- **Cash flow** and **risk** characteristics of underlying assets may not fit easily into investor portfolios
  - **Cash flow frequency** (MBS* - monthly vs. 6-monthly)
  - **prepayment exposure** (tranched MBS redistribute prepayment risk)
  - **credit risk**: may be better market for large volume of AAA and small volume of lower grade than 100% A/BBB

*Note: mortgage backed securities*
Other Features of Tranched Structures

• **Regulation**: banks reduce their capital requirements by moving credit risk off balance
  ✓ tranched *cash CLOs* have been used to do this for many years (i.e., *selling loans*)
  ✓ with *CDS* … credit risk can be moved off balance sheet by *buying credit protection* rather than selling loan (synthetic CDO/CLO)
Funded vs. Unfunded Credit Exposure I

Selling protection via CDS

• **Selling protection via CDS** is approximately same as *long position* in *credit risky bond* and *short position* in *riskless bond*
  - i.e., 100% leveraged position in risky bond

• Provides *unfunded exposure* to credit risk:
  - receives premium (equivalent to spread)
  - in default, seller must pay LGD (net)

• **Implication**: because payment of LGD in default not paid up front, credit protection provided by CDS is *unfunded*
Funded vs. Unfunded Credit Exposure II

Holding risky bond

• **Bondholder** receives spread (equivalent to CDS premium)

• **In default**, bond holder “pays” LGD (net) by accepting recovery amount in exchange for giving up claim on par

• Implication: *protection is funded* – in default only payment is due from *purchaser* of protection (bond issuer) who pays recovery to *seller* of protection (holder)

• **In contrast**: CDX indices (tranched and untranch) and single name CDS provide unfunded exposure
Tranched Dow-Jones CDX Structure

Credit Default Swaps

- **CDS**
  - Notional x [ ] bp p.a.
  - Credit Risk of XYZ

Notional: USD 10MM

Delivery of 10MM Principal
XYZ Senior Unsecured Obligation

$10MM Cash
Occurrence of a Credit Event (Physical Settlement)

Index Products Dow Jones CDX

125 equally weighted names

Next Generation Products Tranched Dow Jones CDX

- Senior Tranche: 30–100%
- Junior Super Senior Tranche: 15–30%
- AAA Tranche: 10–15%
- AAA Tranche: 7–10%
- Junior Mezzanine (BBB): 3–7%
- Equity Tranche: 0–3%
Tranched 125 Name DJ.CDX.NA.IG Series 5  
(Illustrative Pricing 16 Feb, 2006)

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Estimated Rating</th>
<th>Market Quote (bp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15% - 30%</td>
<td>AAA (junior super senior)</td>
<td>4/5</td>
</tr>
<tr>
<td>10% - 15%</td>
<td>AAA (junior super senior)</td>
<td>12/13</td>
</tr>
<tr>
<td>7% - 10%</td>
<td>AAA (junior super senior)</td>
<td>26/26</td>
</tr>
<tr>
<td>3% - 7%</td>
<td>BBB-</td>
<td>108/110</td>
</tr>
<tr>
<td>0% - 3%</td>
<td>Not rated</td>
<td>35.4% / 35.9% + 500 bp</td>
</tr>
</tbody>
</table>

*Source*: Morgan Stanley
Attachment and Detachment Points

• Each tranche is defined in terms of its attachment ($\beta_A$) and detachment ($\beta_D$) points
  ✓ these are measured in terms of losses as percent of total face value of basket

• The attachment point defines the limit below which the tranche bears none of the loss

• The detachment point defines the limit above which the tranche loss does not increase
Tranche Loss Payments

- If total losses (as a percent of the total nominal portfolio value) are $L$, then for a tranche with attachment and detachment points $\beta_A$ and $\beta_D$ the tranche loss payment is:

  \[
  \text{Tranche Loss} = \begin{cases} 
  0 & L < \beta_A \\
  L - \beta_A & \beta_A \leq L \leq \beta_D \\
  \beta_D - \beta_A & L > \beta_D 
  \end{cases}
  \]
Tranche Loss Payments: Equity and Senior Tranche

- **Equity tranche** loss is *concave* in portfolio loss: expected loss on tranche *decreases* (and *value of tranche increases*) with variance of portfolio loss.

- **Senior tranche** loss is *convex* in portfolio loss: expected loss on tranche *increases* (and *value of tranche decreases*) with variance of portfolio loss.
Effect of Correlation on Loss Distribution and Tranche Values

<table>
<thead>
<tr>
<th>Attachment</th>
<th>Equity</th>
<th>Mezz</th>
<th>Senior</th>
<th>Total*</th>
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</thead>
<tbody>
<tr>
<td>3%</td>
<td>0%</td>
<td>3%</td>
<td>7%</td>
<td>0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Detachment</th>
<th>3%</th>
<th>7%</th>
<th>100%</th>
<th>100%</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Correlation</th>
<th>5%</th>
<th>Equity</th>
<th>Mezz</th>
<th>Senior</th>
<th>Total*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.94%</td>
<td>2.39%</td>
<td>0.72%</td>
<td>6.0%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.29%</td>
<td>1.64%</td>
<td>2.12%</td>
<td>6.0%</td>
<td></td>
</tr>
</tbody>
</table>

Acharya and Schaefer: Correlation Products
CDS Spreads on Indices and Tranches

<table>
<thead>
<tr>
<th>CDS index spreads¹</th>
<th>Tranches²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Master investment grade indices</td>
<td>North America master investment grade.</td>
</tr>
<tr>
<td></td>
<td>³7-10%</td>
</tr>
<tr>
<td></td>
<td>10-15%</td>
</tr>
<tr>
<td></td>
<td>15-30%</td>
</tr>
</tbody>
</table>

¹ On-the-run five-year swap spreads, in basis points. ² North America master investment grade. ³ Source: Amato and Gyntelberg, “CDS Index Tranches and the Pricing of Credit Risk Correlations”, BIS Quarterly Review, March 2005
Tranche Spreads and Correlation

*Note*: number of names = 100; CDS spread =100 bps; LGD = 0.6;