FIGURE 2   Evolution of asset values and default points for Compaq and Anheuser-Busch

Source: Moody’s-KMV
FIGURE 3  Corresponding evolution of the annual default probabilities

Acharya and Schaefer: Structural Models 2  
Source: Moody’s-KMV
What do we learn from these plots?

- The volatility of a firm’s assets is a major determinant of its risk of default

- But how do we estimate it?

- We need to use Merton model more powerfully than just use distance to default computations...
Implementing Structural Models: Estimating Value and Volatility of Firms Assets
Firm Value and Volatility

• Structural models depend on value and volatility of firm assets
  ✓ neither is directly observable

• Value of equity = stock price x number of shares

• Problem is the value of debt
  ✓ which debt to include? (e.g., should some short-term debt be netted against short-term assets)
  ✓ what is value of debt? only part of total debt will be traded.
  ✓ total borrowings observed only in periodic accounts.
Unobservability of firm values

• The value of equity, viewed as a call on the firm, depends on $V$ and $\sigma_V$ (as well as the observable variables $B, T, r$)

• Letting $f$ denote the call pricing function, and suppressing dependence on the observable variables, we write

\[ E = f(V, \sigma_V) \]

• Under the Black-Scholes assumptions,

\[ E = f(V, \sigma_V) = V \cdot N(d_1) - e^{-rT} B \cdot N(d_1 - \sigma \sqrt{T}) \]

where

\[ d_1 = \frac{\ln(V/B) + (r + \frac{1}{2} \sigma_V^2) T}{\sigma_V \sqrt{T}} \]

• But we have two unknowns, so one equation is not enough
Unobservability of firm volatility

• Since equity is an option on firm value, the volatility of equity, denoted $\sigma_E$, is also a function of $V$ and $\sigma$:

$$\sigma_E = g(V, \sigma)$$

• For example, in a Black-Scholes world, we have

$$\sigma_E = g(V, \sigma) = \sigma \cdot V \cdot \frac{f_V}{f} = \sigma \cdot V \cdot \frac{N(d_1)}{E}$$

where $f$ is the call pricing function and $f_V$ is the partial derivative of $f$ with respect to $V$. It is essentially the “delta” of equity with respect to firm value, $N(d_1)$.
Two equations, two unknowns

• Expressions for $E$ and $\sigma_E$ give us two (non-linear) equations for two unknowns $V$ and $\sigma_V$ in terms of
  - Equity Value $E$
  - Equity volatility $\sigma_E$
  - Other observable variables ($B$, $T$, $r$).

• Since equity values are observable and equity volatility may be computed, we can use these three expressions to solve for the unknowns $V$ and $\sigma_V$.

• The associated spreadsheet provides a numerical illustration of this procedure using spreadsheet.
The MKMV Model

• The MKMV model uses a four-step procedure to track changes in credit risk for publicly-traded firms:

1. Identify the **default point** \( B \) to be used in the computation.
   - Set to Short-term liabilities + 0.5 * Long-term liabilities

2. Use the default point in conjunction with the firm’s equity value and equity volatility to identify the firm value \( V \) and the firm volatility \( \sigma_V \).

3. Given these quantities, identify the number of standard deviation moves that would result in the firm value falling below \( B \). This is the firm’s **distance-to-default**.

4. * Use its database to identify the proportion of firms with distance-to-default who actually defaulted within a year. This is the expected **default frequency** or EDF.
Distance-to-Default and EDF

• In principle, we should be able to compute default frequencies using
  - the distance-to-default, and
  - the probability distribution governing the evolution of \( V \).
• However, it turns out that under the usual assumptions, this method *underpredicts* defaults by a large margin.
  - It is typical to assume *normality* in returns and value distribution.
  - reality is *fat-tailed* (leptokurtic) and extreme events are far more likely.
• Using the empirical database improves default predictions enormously.
Distance-to-Default and EDF (Cont’d)

• For instance
  - For distance-to-default of 4.3, the EDF using KMV’s database was 0.03%. Had we used normality, it would have been 0.00086%!
  - For distance-to-default of 3.2, the EDF using KMV’s database was 0.25%. Using normality, it would have been 0.069%!

• In fact, the default probabilities predicted by the Merton Model—which is based on normality—would imply that well over 50% of all US companies are AAA or better!

• So MKMV uses Merton model to rank credit risk of firms in a relative sense but does not use its absolute probability
A direct alternative method

- Alternatively
  - use market value for equity ($E$) and equity volatility
  - book value for total debt ($D^*$) and market value of traded debt for debt volatility
  - calculate $V = E + D^*$ and calculate leverage ratio
  - Calculate asset volatility from debt and equity volatility, correlation and leverage:

$$R_V = \frac{E}{V} R_E + \frac{D^*}{V} R_D$$

$$\sigma^2_V = \frac{E}{V} \sigma^2_E + \frac{D^*}{V} \sigma^2_D + 2 \frac{E}{V} \frac{D^*}{V} \text{cov}(R_E, R_D)$$
The Volatility of Corporate Assets

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<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
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**Quasi-Market Leverage**

Mean

Std.Dev.

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**Equity Volatility**

Mean

Std.Dev.

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<tbody>
<tr>
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**Estimated Asset Volatility**

= \( L_{jt} \)  Quasi-market leverage ratio of firm j, time t

Book Value of Debt (Compustat Items 9 and 34)

Book Value of Debt + Market Value of Equity

Estimated asset volatility

\[
\sigma^2_{Ajt} = (1 - L_{jt})^2 \sigma^2_{Ejt} + L_{jt}^2 \sigma^2_{Djt} + 2L_{jt} (1 - L_{jt}) \sigma_{ED,jt}
\]
Bond Prices in the Merton Model
Bond Prices in the Merton Model

• Recall: The Black-Scholes value for a call on the firm assets with exercise price \( B \), i.e., the value of equity, \( E \), is:

\[
E = VN(d_1) - PV(B)N(d_2); \\
d_1 = \frac{\ln(V/B) + (r + \frac{1}{2}\sigma^2_V)T}{\sigma V \sqrt{T}} \\
\text{and } d_2 = d_1 - \sigma \sqrt{T}
\]

• Since the bond value, \( D \), is the firm value minus the equity value:

\[
D = V - E \\
= V - VN(d_1) - PV(B)N(d_2) \\
= V(1 - N(d_1)) + PV(B)N(d_2) \\
= VN(-d_1) + PV(B)N(d_2) \quad \text{since } 1 - N(x) = N(-x)
\]
Understanding the Pricing Formula

\[ D = VN(-d_1) + PV(B)N(d_2) \]

- \( N(d_2) \): is the risk-neutral probability of survival. (To see this, look at the expression for the default probability derived earlier and substitute the riskless rate for the expected return \( \mu \)).

- The payoff in the case of survival (no-default) is \( B \), and its value is therefore the risk-neutral expected payoff \((BN(d_2))\) discounted at the riskless rate, i.e., \( PV(B)N(d_2) \).

- The first term, \( VN(-d_1) \), is the value of the payment in the case of default.
Credit Spreads in the Merton Model

• The promised yield on the bond in the Merton model is

\[ y = r + s \] where \( s \) is the "credit spread" and \( y \) is defined by:

\[ D = e^{-yT}B \quad e^{-(r+s)T}B \quad e^{-sT}PV(B) \]

and \( D = VN(-d_1) + PV(B)N(d_2) \)

• If we now define the “quasi leverage ratio”, \( L \), as \( PV(B)/V \), i.e., the debt-to-firm value ratio, but valuing the debt using the riskless rate, then we can express the spread simply as a function of \( L \) and \( \sigma \sqrt{T} \)

\[ s = -\frac{1}{T} \ln \frac{1}{L} N(-d_1) + N(d_2) \] , where \( d_1 = \frac{\ln(1/L)}{\sigma \sqrt{T}} + \frac{1}{2} \sigma \sqrt{T} \), and \( d_2 = d_1 - \sigma \sqrt{T} \)
The Merton Model, contd.

- The credit spread depends on two variables: \textit{asset volatility} and the \textit{quasi-leverage ratio}: \( L = \frac{PV(B)}{V} \)
- As expected the credit spread is increasing with respect to both variables
- Even in this simple model the term structure of credit spreads can be upward sloping, downward sloping or “hump” shaped

![Graph 1](image1.png)

\( L = 0.65 \)

![Graph 2](image2.png)

\( L = 1.1 \)
Merton model: spreads vs. leverage and volatility

• In the Merton model the spread over riskless rate increases with volatility and leverage ($L$)
The Merton Model: Limitations

• Merton model provides important insight into the problem of pricing default.
• But some limitations...

✓ the value of the firm, $V$, and its volatility hard to estimate (more on this later)
✓ constant risk-free rates … cannot model relation between interest rate risk, asset risk and default risk.
✓ specification of default time restrictive
✓ default is never a surprise: so long as $V > B$ default can never occur in next instant (Enron, Parmalat etc.)
Merton - Summary

• Important first step in modelling default

• Predictions of default are too low

• Predictions of credit spreads appear too low (more later)

• Occurrence of default only at maturity is major limitation
  ✓ BUT, inclusion of early default does not necessarily increase spreads.
Summary

• Merton model
  ✓ basis for all structural models of credit risk
  ✓ default probability and prices in Merton model
  ✓ concept of distance-to-default (used in MKMV is estimating default probabilities)

• Next Session: Models with early default
  ✓ default probabilities
  ✓ empirical evidence