Credit Risk for Indian firms

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Classes of models

Two main approaches to estimating probability of default – $E(p)$ – for one bond:

- Models which utilise accounting data to predict probability of defaults or predict ratings using binary choice models.
- Models which utilise stock prices to arrive at probability of default.

In the literature, these remain the dominant approach to pricing credit risk, one bond at a time.

A: Estimating p using binary choice models and accounting data

Modelling default as a binary variable

- Default depends on financial health denoted say by Z_i^*.

$$\begin{array}{lll} \text{If} & Z_i^* \leq 0 & \text{firm defaults on loan} \\ \text{If} & Z_i^* > 0 & \text{firm pays back loan} \end{array}$$

- Financial health depends on a number of factors X_i related to firm characteristics, loan characteristics, macroeconomic factors.

$$Z_i^* = \alpha' X_i + \epsilon_i$$

- Financial health is unobservable. But we observe "outcomes" of financial health.

$$\begin{array}{lll} Z_i = 1 & \text{if} & Z_i^* \leq 0 \\ Z_i = 0 & \text{if} & Z_i^* > 0 \end{array}$$

- Using data on default history and data on X we can estimate (i) default probabilities and (ii) financial health of the firm.

This is more difficult than it appears.

The devil in the details: Modeling issues

- Which universe of companies?
- What time period is used in estimating the model?
- What is the definition of default?
- Which Ratios? - Selection of variables
- Which Model? - Model Selection

Which universe of firms?

- Models of default probability differ across company characteristics.

  For example, Credit Rating Agencies have separate models for public and private companies, or for manufacturing and finance.
- Some types of information are not available for private companies.
- The behaviour of ratios themselves are different across sets of firms.

  For example, leverage for manufacturing firms and financial firms are structurally different.
- Just because a firm has not defaulted does not mean that it is "healthy"!

Our objective: Pick firms that are as alike as possible but one set that has defaulted and one set that has not.
Then care needs to be exercised in which firms are used in the analysis.

Sample time period

- Ideal econometrics: random sample from the universe is selected.

  It is important that the random sample contains some“minimum” number of defaults.
- Data should be spread over a “sufficient” time period for both in-sample estimation and out-of-sample time validation.
- For India, out-of-time validation is tough; even if there is span, there have been lots of regulatory changes that may be external factors affecting firm health.

The definition of default

- Default should be recorded as the first time that a repayment is missed. This is not the definition anywhere.
- In most countries, default is defined as per bankruptcy law or the rules that banks follow to categorise a loan as a non-performing asset (NPA).
- If so, then the model will predict that particular outcome and not precisely default.
- If so, the model will predict different things for different countries!
- For India, lack of data in the past meant that we had to work with “estimates” of default.

  One alternative was to model change in credit rating.
- Today, Indian firms are observed to have an IBC filing in court.

  IBC is the Insolvency and Bankruptcy Code, 2016 of India.

  This data is available only as of January 2017.

Selection of variables

- Data on $X_i = [X_{1i}, X_{2i}, X_{3i}, X_{4i}]$
  - X_{1i} = Firm characteristics (accounting data): Measures of coverage, liquidity, leverage, profitability, dividend history, group affiliation
  - X_{2i} = Firm characteristics (stockmarket data): price-earnings ratio, market capitalization
  - X_{3i} = Loan characteristics: size of loan, collateral, quality of gurantor
  - X_{4i} = Industry characteristics, Macroeconomic factors
- Variable selection: Principal Component Analysis (PCA), elastic net, LASSO, ridge regressions.
- Check for: levels vs. changes; audit quality, accounting practices to standardise variables.

Ratios that are typically used

1. Financial data:
    1.1 Leverage ratios: debt ratio, equity multiplier, liability structure ratio
    1.2 Profitability: EBITD ratio, profit on sales ratio
    1.3 Debt coverage: cash coverage ratio, times interest ratio.
    1.4 Growth: sales growth ratio
    1.5 Activity: trade creditor ratio
    1.6 Productivity: sales to personnel expenses ratio
2. Others:
    - Industry specific factors
    - Macroeconomic factors
    - Management quality

Types of accounting indicators

1. Short–term solvency, or liquidity ratios
2. Asset utilisation or turnover ratios
3. Long term solvency or financial leverage ratios
4. Profitability ratios
5. Market value ratios
6. Other financial variables: mostly related to size

These are typically used in the literature.
In emerging markets, other firm characteristics on corporate governance can matter.

Estimation approaches

1. Binary choice estimation approaches such as *probit* or *logit* models.
2. No "standard" model of default for all firms: they have to be modelled separately for different "classes" of firms.
3. Some models use distress proxies as the variable to be predicted (say, net worth is less than 0 or not).

   Advantage: can be applied to all the firms with accounting data.
4. Models with credit ratings as the variable to be predicted (say, the firm's rating is either "A" or not).

   Disadvantage: can be applied to firms that have ratings.
5. Models to predict credit rating migrations.

   Disadvantage: *Hazard* models to predict upgrades/downgrades but only to firms that have ratings.

Model selection

How to do model selection?

- Choose based on Log-likelihood function value
  Traditional econometrics.
- Out-of-sample prediction performance
- Power curves and accuracy ratios

Out of sample model prediction performance

- In sample results will obviously predict the data well.
- Better evidence is how the model performs on firms that was NOT used for model estimation. $\rightarrow$ out-of-sample prediction performance.

Out of sample model prediction performance

- In sample results will obviously predict the data well.
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- Approach: Leave out a set of defaulted firms and non-defaulted firms out of the model estimation and predict $\hat{p}$ for these firms.

Generally, models perform worse on the out-of-sample data compared with the in-sample data.

Validation visualisation tool: Power curves

The power curve is a graph which answers the simple question about model performance:
"How many firms that the model categorised as having a high probability of default have actually defaulted?"

Example of two models and their output

- Dataset: we have 20 firms (named **A**-**T**)
- We observe that four of them have defaulted (**B, J, M, N**).
- Models: we have two candidate models **M1, M2** and their output probability of default for all the firms.

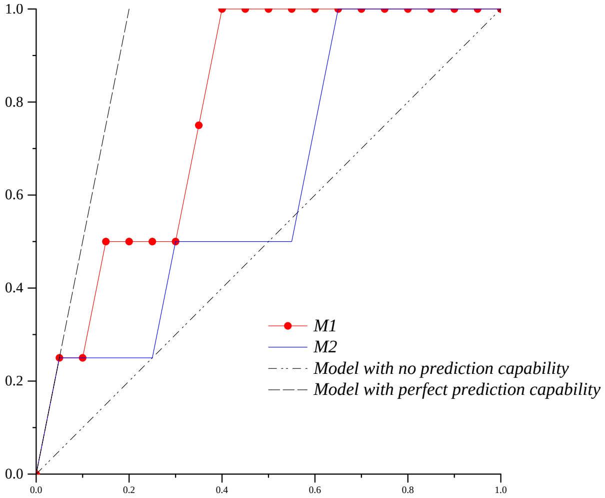
The model outputs

Model	Decreasing order of Probability of Default (PoD)									
	1	2	3	4	5	6	7	8	9	10
M1	*B*	T	*M*	S	R	Q	*N*	*J*	A	C
M2	*B*	S	T	R	Q	*J*	A	C	O	P
	11	12	13	14	15	16	17	18	19	20
M1	O	P	L	D	E	F	H	G	I	K
M2	L	*M*	*N*	D	E	F	H	G	I	K

The power curve

- The x-axis: Fraction of firms in the sample ordered by descending probability of default (falls between 0 and 1).
- The y-axis: ratio of the number of defaulted firms within $X \leq X^*$ to the total number of defaulted firms in the sample.

An example of the power curve



Interpreting the power curve

- The 45^o line is the line of zero prediction.
- The step line close to the y-axis is the line of perfect prediction.
- The closer the model's power curve is to the 45^o line, the worse the model is.
- Accuracy ratio is a relative and quantitative measure of the performance of one model compared to another model.
- The closer the model's power curve is to the 45^o line, the closer the accuracy ratio is to 0.
- The larger the accuracy ratio, the better the model.
- The accuracy ratio varies between 0 for model with no explanatory power to 1 for the model with the highest explanatory power.

References to start with

- E. Altman, “Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy,” Journal of Finance, September 1968, pp. 589-609.
- E. Altman, J. Hartzell, and M. Peck, “Emerging Markets Corporate Bonds: A Scoring System,” Salomon Brothers Inc, New York, 1995.
- http://www.defaultrisk.com/
- https://www.rmicri.org/en/

One example of $E(p)$ model performance for Indian firms, Sarkar Thomas 2002

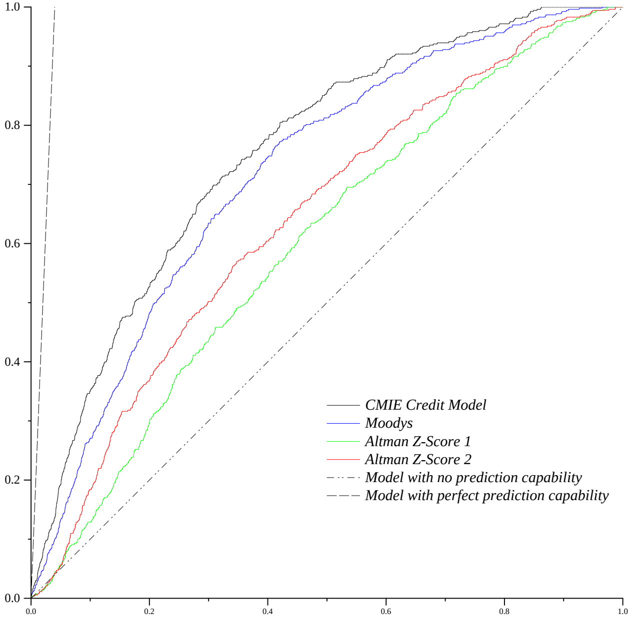
- Data period: 1990-2001.
  - In-sample: 1995-1999
  - Out-of-sample: 2000-2002
- Number of firms:

  584 default episodes and 17981 non-default events.
- Probit model including:

  Debt equity ratio, Times interest ratio, Fixed assets turnover, Cash ratio, Productivity ratio.

  Industry dummies, business house characteristics, threshold probability variable.

Comparing power curves



B: Estimating p from equity prices

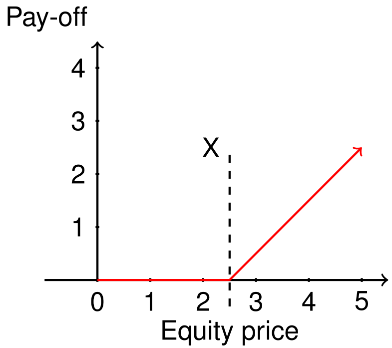
Estimating the prob(default) using call option pricing

- Merton (1974) presents that there is a call option payoff *in every firm*.
- If this is true, then we can use the well-established formula to price a call option (Black-Scholes options pricing model (1973), Merton (1974)) to estimate the probability that a firm will default on payments.
- How?

What is a call option?

- A call option gives the investor the right (not obligation) to buy a share at a pre-determined price (X) within a pre-determined period of time (T).

  For this right, the investor pays C.
- If the price of the share rises much higher than X during T, the investor gets to buy the share at X.
- If the price drops below X, the investor has the choice to not buy.



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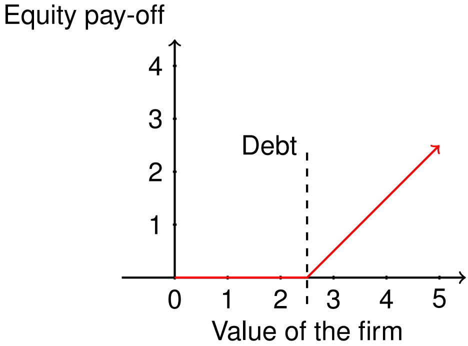
Recall: what is a firm

- The firm is constructed out of a combination of Equity + Debt
- Equity shareholders have control over the firm. Debt holders have the promise that they will be paid (a fixed amount).
- When shareholders cannot pay the debt, they lose control to the debt holders. This is called Default.
- As long as shareholders can pay the debt, they have the upside. The upside is like a call option on the value of the firm.



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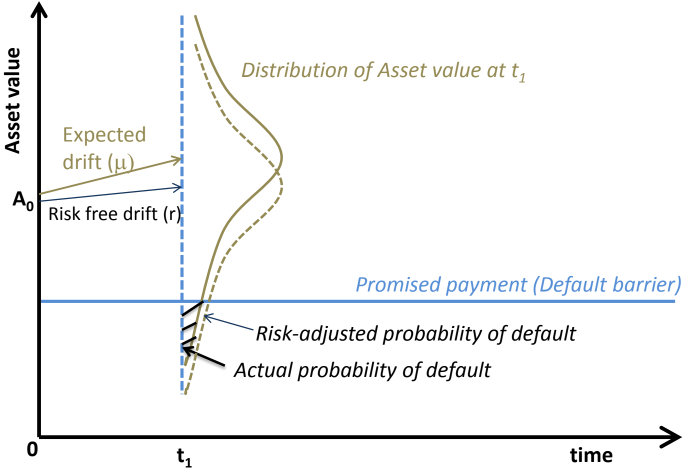
Call option pricing at the heart of estimating prob(default)

- Basic idea:
  1. Equity is a call option on firm's asset value with debt as strike price.
  2. Firm's shareholders hold a call option on the firm's asset.

- Seminal papers:
  1. Merton, R. C., 1974. On the pricing of corporate debt: the risk structure of interest rate. *Journal of Finance* 29, 449-470.
  2. Merton, R. C., 1977. An analytical derivation of the cost of deposit insurance and loan guarantee. An application of modern option pricing theory. *Journal of Banking and Finance* 1, 3-11.

Call option pricing at the heart of estimating prob(default)



Operationalising Merton, 1974

Operationalising estimation of default risk

- What we know:
  1. We see E, the equity market capitalisation of the firm
  2. We see σ_E the equity volatility of the firm
  3. We know the level of Debt, as the book value recorded in the balance sheet.

- What we do not know:
  1. We do not see V, the total value of the firm
  2. We do not know σ_V, the volatility of the total assets of the firm

We can setup two equations in two unknowns – what the KMV model did.

The Kealhofer-Merton-Vasicek (KMV) model

- Debt: homogenous with maturity at time, T
- Capital structure of the firm: $V_A(t) = D(t) + E(t)$
- Perfect markets: no coupons, no dividends, no frictions on trading
- Asset dynamics: assets are traded, and prices follow GBM.

$$dV_A = \mu_A V_A dt + \sigma_A V_A dW$$

where V_A is the value of the asset, σ_A is its volatility, μ_A is the drift and dW is a Wiener process.

KMV methodology

- Using the analogy of the Black-Scholes model,
  1. Equity, E = Call option, C
  2. Debt, D = Strike, K
  3. Value of the firm, V_A = Equity price, S
- Then if the call option pricing formula is:

$$C(t) = S(t)\Phi(d_1) - e^{-r(T-t)}K\Phi(d_2)$$

- Equity, E can be priced as:

$$E(t) = V_A(t)\Phi(d_1) - e^{-r(T-t)}D\Phi(d_2)$$

- Ito's formula is used to show that:

$$\sigma_E = \left(\frac{V_A}{V_E}\right)\left(\frac{\partial V_E}{\partial V_A}\right)\sigma_A$$

KMV model

- To find V_A, σ_A, solve the non-linear system of equations:

$$\begin{aligned} f_1(V_E, \sigma_E) : V_A(t)\Phi(d_1) - e^{-r(T-t)}K\Phi(d_2) - V_E(t) &= 0 \\ f_2(V_E, \sigma_E) : \frac{V_A}{V_E}\Phi(d_1)\sigma_A - \sigma_E &= 0 \end{aligned}$$

- The solution is unique since $\partial f_1 / \partial V_A = \Phi(d_1)$ (analogous to δ in the original B-S).
- f_1 is increasing in $V_A \rightarrow f_1(V_A)$ has a unique solution.
- Similarly, we can see that $f_2(\sigma_E)$ has a unique solution also.

The ultimate prize - Distance from default (DtD)

- Default is the instance when firm value falls below debt or $V_A \leq D$.
- DtD(t) is the distance between the expected firm value and the default point:

$$\mathrm{DtD}(t) = \frac{\log\left(\frac{V_A(t)}{D}\right) + (r - 0.5\sigma_A^2)(T - t)}{\sigma_A\sqrt{T - t}}$$

- Probability of default: substitution into a normal CDF gives:

$$Pr(def)(t) = P[V_A \leq D] = \Phi(-\mathrm{DtD})$$

- When DtD is measured as:

$$\mathrm{DtD} = \frac{V_A - D}{\sigma_A V_A}$$

It is interpreted as the number of standard deviations the firm value is away from the default trigger.

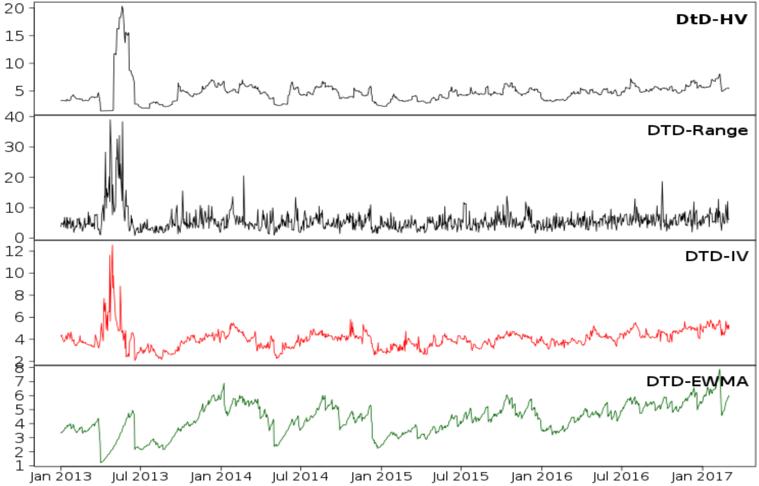
Inputs to the model

- Only applies to listed firms.
- Input (time series of): Market capitalisation (V_E, daily), price volatility (σ_E, daily), Debt (D, annual or quarterly).
- Output (time series of): Value of the firm (V_A), Volatility of the value (σ_A), DtD.
- Optimisation engine to calculate the DtD of a given firm:

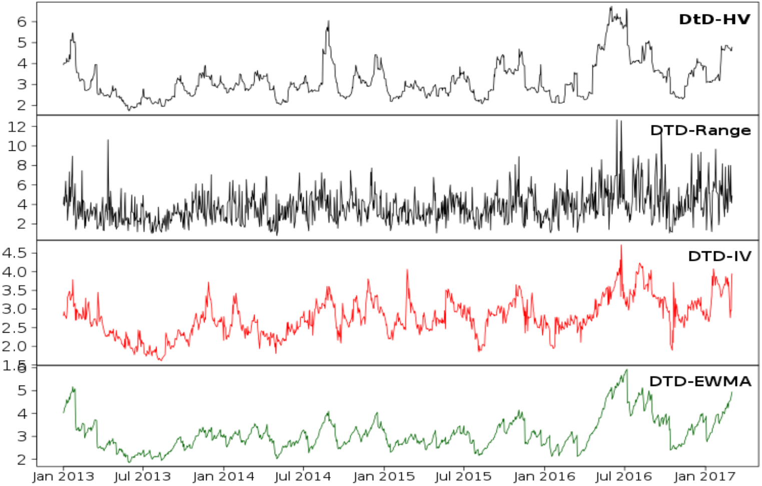
  R package `dtd` at `http://ifrogs.org/systems.html`.

Of these, there is a lot of research in calculating σ_E for a firm.

DtD for Hindustan Unilever, varying σ_E



DtD for Mahindra & Mahindra, varying σ_E



Thank you.

Questions? Comments?

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http://www.ifrogs.org