Measuring and explaining the asymmetry of liquidity

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ABSTRACT

Most measures of liquidity assume that the cost of buying and selling is symmetric. This paper analyses liquidity in an open electronic limit order book exchange where it is possible to directly measure the impact cost of a market order to buy and to sell. There is clear evidence of liquidity asymmetry on the spot market: large market orders to *sell* have higher costs compared with buy orders. In an identical microstructure setting but with no short sales constraints, single stock futures markets show little evidence of liquidity asymmetry. This evidence is consistent with the response of liquidity providers to asymmetric information vis-a-vis informed traders, and implies that short sale constraints are an important source of asymmetry in liquidity.

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I Introduction

Market liquidity is asymmetric when the transactions cost faced by a buyer differs from that faced by a seller, for the same size of transaction. If a market order to buy N shares obtains a price which is $a\%$ away from the midpoint quote (i.e. impact cost of $a\%$), and the same sell order gets a price which is $b\%$ away from the midpoint quote, liquidity is asymmetric when $a \neq b$. If there is such asymmetry in liquidity, it can have important ramifications for the behaviour of traders, by impacting how trading decisions are made, and affecting how traders optimise strategies differently for buying versus selling. Asymmetry in liquidity could influence asymmetry in the distribution of returns (Brennan *et al.*, 2010).

This raises two research questions: Testing for asymmetry, and understanding what drives transactions costs to be different for buyers and sellers. One microstructure feature that could cause asymmetry is short sales constraints, where a sell order faces greater difficulties in delivering shares when compared with buy orders that have to deliver money. At the simplest, this can yield a paucity of limit orders by sellers and inferior liquidity for *buyers* of large sized market orders.

An alternative argument reverses this simple outcome. Miller (1977) argues that when constraints are imposed on short sales, this exacerbates the information asymmetry between informed and uninformed traders in the marketplace and, in turn, causes asymmetry between buy and sell prices in the market. In the standard microstructure model where there are both informed and uninformed traders in the market, the uninformed supplier of liquidity would be *more* wary when a large market order to sell arrives, when there are restrictions against short selling. If short selling is difficult, then a large market order to sell indicates that the selling speculator is confident about a price forecast, and is willing to incur the higher costs required for short selling. This would shape the behaviour of liquidity providers, and induce higher transactions costs for *sell* market orders. Thus, there are two paths through which short sale constraints can induce asymmetry of liquidity, with opposite predictions for the direction of asymmetry.

There are several papers that explore the effect of information asymmetry on *asymmetry of traded prices for buyers* versus sellers of large orders, such as Brennan *et al.* (2010). But relatively little work is done on understanding the link between short sale constraints and liquidity asymmetry. In recent years, many papers have analysed the short sale constraints introduced in 2008 and 2009. Most papers that analyse the impact of these constraints show that liquidity worsened and volatility increased beyond what could be explained by the crisis alone (Helmes *et al.*, 2010; Battalio and Schultz, 2011; Beber and Pagano, 2012). However, these papers do not analyse the potential interplay between asymmetry of liquidity and short sale restrictions.

A major challenge in understanding liquidity asymmetry lies in accessing the data required for a complete analysis of liquidity. Traditional liquidity measures such as the bid-offer spread only measure liquidity for small transactions, and implicitly assume symmetry of liquidity. Information about limit orders is often not observed. Thus, various papers measure liquidity from traded volumes and traded price impact of buy and sell trades (Roll, 1984; Amihud, 2002; Pastor and Stambaugh, 2003) rather than the liquidity that is available in limit orders. This situation on information access has changed in recent years. The dominant form of market organisation, worldwide, is now the open electronic limit order book (OELOB) market, which allows for a substantially improved observation of liquidity. When the entire limit order book is observed, liquidity (defined as the impact cost faced by a market order) can be directly measured for all order sizes. This makes it possible to compare the impact cost for a buy market order versus a sell market order.

In this paper, we analyse an interesting setting where there is an opportunity to test for asymmetry on an electronic limit order book, and obtain insights into the factors shaping asymmetry. These are the markets for equity spot and single stock futures that trade simultaneously at the National Stock Exchange (NSE) in India, which is one of the most active exchanges of the world for both these markets. The paper analyses a unique dataset with 'snapshots' of the complete limit order book on both the spot market and the single stock futures market.

In this market setting, the two markets (spot and single stock futures) have the identical market design in all respects but one: short sale constraints. The spot market settles on a T+2 basis with no formal mechanism for borrowing shares. If a trader wishes to sell short, he has to privately search for a willing lender and borrow shares OTC. In practice, the extent to which this takes place is negligible; these short sale constraints are binding. Simultaneously, the single stock futures markets is cash settled. There is full symmetry between adopting long or short positions.

The empirical analysis proceeds in two steps. First, we test for the asymmetry of liquidity on the spot market using three alternative estimation strategies. For the spot market, all three methods suggest that buy-side impact cost (i.e. the transactions costs faced when buying) is lower than sell-side impact cost. This is consistent with the idea that short sale restrictions interact with asymmetric information and inhibit uninformed liquidity suppliers. Second, we test for asymmetry in the limit order book of the single stock futures using the same three estimation strategies. If short sales constraints are a key source of asymmetry of liquidity, there should be no liquidity asymmetry in these futures, which are cash-settled products, that treat long and short positions symmetrically. The paper finds little evidence of asymmetry of liquidity.

The paper contributes to the existing literature in two ways. Firstly, we harness data from 'snapshots' of the full limit order book to trace out the impact cost of all possible market orders, which yields a new perspective on measuring liquidity. Secondly, the null hypothesis of symmetry is clearly rejected on the spot market, but it is not rejected on the stock futures market. This suggests that short sale constraints may be the source of asymmetry in liquidity. This has many interesting implications. As an example, our results suggest that in addition to hampering market liquidity, a ban on short sales exacerbates asymmetry in liquidity, inducing *bigger price movements* in response to *large sell* orders.

The paper is organised as follows. Section II reviews the potential explanations for liquidity asymmetry from the existing literature. Section III describes the unique setting of the dataset. Section IV proposes three methods to measure asymmetry in liquidity on a limit order book market. These methods are applied to the spot market in Section V and to the single stock futures market in Section VI. Section VII concludes.

II Asymmetry in transactions costs for buying versus selling

In the quest for better understanding about market liquidity, one question that arises is whether buyers and sellers face the same transactions costs, when placing the same size of order. The early literature suggests that the traded price associated with a large sell trade has a higher price impact compared with a similarly large buy trade (Kraus and Stoll, 1972). This asymmetry in price impact is attributed to trades by institutional investors, who are assumed to have lower information asymmetry about the firm, and whose sell orders signal negative news about the firm. However, this is not a sufficient explanation for why large buy orders have a different liquidity premium.

Miller (1977) presents two factors that have implications for asymmetry of liquidity: the asymmetry of information between informed traders and uninformed owners of shares, and the effect of short sale constraints on liquidity outcomes. This paper suggests that given the setting where the shares available for trade is less than the funds available for trade, *and* there is asymmetry of information between informed and uninformed traders, the introduction of short sale constraints exacerbates asymmetry between buy and sell liquidity.

Several papers develop models of asymmetry of information between informed traders and liquidity intermediaries in the market (such as dealers or market makers) to explain asymmetry in traded prices at sale or purchase (Ho and Stoll, 1981; Subrahmanyam, 1991; Chan and Lakonishok, 1993; Keim and Madhavan, 1996; Brunnermeier and Pedersen, 2009). This literature obtains insights into transactions costs based on the costs suffered by liquidity providing intermediaries. On the one hand, market intermediaries suffer inventory costs of holding shares. On the other hand, there is the fear of information asymmetry when buying large quantities from an informed seller. Thus, they are likely to offer relatively easy terms to large buy orders to reduce inventory costs, but impose a higher premium on large sale orders to adjust for tangible losses that are involved if a large order is purchased at an adverse price relative to the true price. The effect of information asymmetry is demonstrated effectively by Michayluk and Neuhauser (2008) in an analysis using prices of newly listed internet and technology stocks where the information asymmetry is higher which can exacerbate liquidity asymmetry. The paper finds that liquidity asymmetry in the traded prices of these securities is indeed greater.

Short sale constraints can also play a role in inducing asymmetry of liquidity. One line of thought suggests that since short sale constraints interfere with how limit orders to sell are placed, *buyers* will end up paying larger transactions costs than sellers for the same large sized order. However, Miller (1977) emphasises an explanation based on asymmetry of information. When an informed speculator has to face higher frictions owing to short sale restrictions, and yet chooses to place a large sell market order, there is a greater

Table I India's NSE in global rankings

In spot and in single stock futures trading, the Indian NSE is one of the largest exchanges in the world.

(a) Spot market

	Exchange	Shares (million)
1.	NYSE Euronext (US)	931.16
2.	NASDAQ OMX (US)	759.87
3.	Shanghai SE	758.84
4.	NSE	705.58
5.	Shenzhen SE	548.84

(b) Single stock futures market

	Exchange	Contracts (million)
1.	NYSE Liffe Europe	161.75
2.	EUREX	142.00
3.	NSE	84.41
4.	Johannesburg SE	28.61
5.	Korea Exchange	24.10

Source: World Federation of Exchanges, first half of 2011

chance that the counterparty (traders with limit orders to buy) will lose. In equilibrium, liquidity providers will thus be more wary of large sell market orders as compared with large market orders to buy. This will give rise to asymmetry in impact cost with a bigger impact cost on *sell* market orders (Brennan *et al.*, 2010; Nguyen *et al.*, 2010).

The first efforts on analysing liquidity asymmetry use datasets that are limited to the bid offer spread, where (by definition) the impact cost for buying versus selling is identical when compared against the midpoint quote. In recent years, the bulk of exchanges worldwide have moved to the open, anonymous, electronic, limit order book markets. The measurement of buy and sell impact cost is easily done on a limit order book market, given the full observability of the limit order book. The asymmetry question consists of exploring differences between the transactions costs faced by market orders to buy versus market orders to sell. In contrast with research based on estimates of price impact reconstructed from the time-series of quotes or traded prices, the entire limit order book is observed in electronic limit order book markets, thus permitting direct measurement of impact cost when buying or selling large orders.

Several models of limit order book markets exist, some that build on models of information asymmetry developed by Kyle (1985) in this new market setting (Glosten, 1994; deJong *et al.*, 1996; Biais and Weill, 2009) and others that move away from it (Rosu, 2009). For instance, Rosu (2009) explicitly models multiple agents with the ability to place and cancel different orders at various points in time. The paper derives implications about the dynamic behaviour of the bid-ask spread, price impact of transactions and the evolution of the entire limit order book over time. However, it does not address the question of asymmetry between the buy and sell sides. Hedvall *et al.* (1997) explicitly addresses the question of asymmetry, present in the order flow as a set of demand and supply curves, and argues that the general buy and sell side price impact is likely to be symmetric in these markets.

III The setting

The National Stock Exchange (NSE), in India, is one of the more active exchanges in the world in trading equity. Table I shows NSE as the 4th largest exchange in terms

Table II Summary statistics of spot market liquidity

The table presents summary statistics for liquidity of the sample. The statistics are presented for both the overall sample as well as subsets of firms categorised in size quintiles, from *S-big* (largest market capitalisation) to *S-small* (smallest).
The *bid-ask spread* is the relative spread, measured as the ratio of the spread as a percentage of the mid-quote price. The *inside depth* is the sum of the quantities available for trading at the bid and the ask, measured as number of shares. The *buy (sell) side depth* is the total number of shares available for buying (selling).
For each security, the median value is calculated across all order book snapshots. Across all securities in a given category, the sample mean of the medians is reported. The cross-sectional standard deviation (of the medians) is presented in parentheses.

	Market cap.	Bid-ask spread	Inside depth	Sell-side depth	Buy-side depth
	(Rs. billion)	(%)	(Number of shares)		
Overall sample	97.32	0.15	4270	254700	392100
	(332.01)	(0.04)	(11120)	(384290)	(711410)
S-big	516.72	0.11	1670	217550	272190
	(473.09)	(0.02)	(1480)	(130290)	(185240)
S2	164.30	0.13	1930	204710	269840
	(53.82)	(0.02)	(1840)	(161010)	(237250)
S3	97.37	0.16	3440	285180	463270
	(13.30)	(0.04)	(6930)	(492720)	(897420)
S4	60.61	0.18	3600	330730	577400
	(12.22)	(0.03)	(5810)	(591150)	(1083280)
S-small	32.43	0.20	10580	233460	371810
	(8.72)	(0.03)	(22250)	(340490)	(686130)

of the number of shares traded on the spot market, and the 3rd largest in terms of the of contracts traded of stock futures futures, in the first half of 2011. A unique feature of the single stock futures traded on the NSE is that they are cash settled rather than physically settled.

All trading on the exchange is done through an anonymous open electronic limit order book. At any given point in time, the best five prices to both buy and sell are visible to all traders, along with the aggregate quantity available at these prices. The quantity that is available at each price is the amount aggregated over all the limit orders placed at that price, anonymously by traders. This aggregation is done automatically without the intervention of specialists or market makers, neither of which are present in this market.

In this paper, we analyse the liquidity of the 100 largest stocks by market capitalisation in 2009. The dataset consists of snapshots of the entire limit order book (LOB) at four different times of the day - 11 A.M., 12 P.M., 1 P.M. and 2 P.M. - and is the source for all the liquidity measures used in the rest of the paper. In all, the dataset comprises of 194,400 limit order books, each of which pertains to one stock at one point in time.

Table II presents descriptive statistics about spot market liquidity in the sample, including traditional measures of liquidity such as the *bid-ask spread* and *depth* for the buy-side and sell-side of the market separately. The table reports the median liquidity for the overall sample and size-based quintiles with the standard deviation in parentheses showing the variation of the median within each group. The bid-ask spread systematically increases across firm size, showing that the better liquidity is provided for larger firms compared to smaller ones. Depth does not show a similarly consistent pattern. Smaller firms may show a higher depth in terms of shares, but may still be less liquid in terms of transactions costs.

IV Measurement of asymmetry in liquidity in the LOB market

It is possible to estimate the exact price per unit share, P_Q, that would be paid for a market order of size Q using the full limit order book. The degree of illiquidity of the security would depend upon how far P_Q is from the mid-point quote, $\bar{P} = (bid + ask)/2$. The percentage degradation of P_Q compared with $\bar{P}$ is called the *impact cost* (IC) of the trade.

For any given transaction size Q, each LOB snapshot would yield an estimate of the impact cost for a *market order* of size Q:

$$\mathrm{IC}_Q = 100(P_Q - \bar{P})/\bar{P}$$

When IC_Q is calculated for all trade sizes from $Q = 1 \ldots \bar{Q}_{max}$, the entire *liquidity supply schedule* (LSS) can be traced out, which describes the impact cost faced for all possible buy or sell orders.[1] In the dataset, the LSS is observed for all securities at all order book snapshots. An example of the $\mathrm{LSS}_{\mathrm{sell}}$ and $\mathrm{LSS}_{\mathrm{buy}}$ of the LOB for a given security can be seen in Figure 1. We see that the IC_Q is a weakly monotonic function in Q.

Based on this limit order data and the impact cost, three measures of liquidity are calculated to obtain evidence about the asymmetry of liquidity between sell-side and buy-side in the electronic limit order book market:

- Probability of full execution of an order of size Q, for a fixed set of sizes.
- Difference in the estimated impact cost to buy versus to sell ($\mathrm{IC}_{(\mathrm{sell},Q)}$ - $\mathrm{IC}_{(\mathrm{buy},Q)}$)
- Difference in the estimated parameters of a parametric model of the LSS for a given security, on the sell-side versus the buy-side.

We now discuss each of these measures.

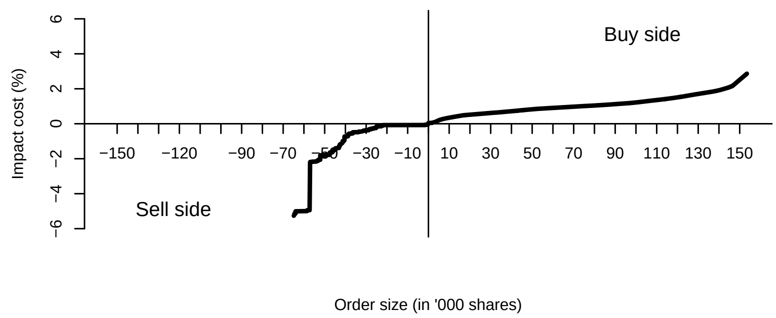
A Probability of full market order execution

From the limit order book (LOB), we can calculate whether a single market order of size Q can be fully executed within the LOB or not. If the observed depth in the LOB is less

[1] This graph can be related to what Chacko *et al.* (2008) terms the 'quantity structure of immediacy prices'.

Figure 1 The liquidity supply schedule: An illustration

This figure shows one example of the liquidity supply schedule from the limit order book of Infosys Technologies, at 12 noon on 8^{th} June 2009. On the y-axis is the impact cost faced by market orders at continuous order sizes. The impact cost is positive for buy limit orders: the larger the order to buy, the higher the price. As an example, a market order to buy 100,000 shares has an impact cost of 1.8%. On the sell side, price impact is negative for sell limit orders, with the sale price being less than the midquote price.
In this example, there is asymmetry in liquidity for all quantities from 70,000 to 150,000 shares: the impact cost of buy oders is smaller than impact cost of sell orders.



than the size of the market order, the order cannot be fully executed. This is a simple measure, which can be calculated for any given order size Q, calculated separately for the sell-side and the buy-side of an LOB market.

If liquidity is symmetric, then there should be an *equal probability of full execution* on the sell-side of the LOB and the buy-side, for a market order of a given order size Q.

B Difference in the estimated impact cost to buy versus to sell

As discussed above, on the limit order book market, it is possible to measure the buy-side and sell-side impact cost associated with an order of size Q across all order book snapshots. We analyse our data for four order sizes:

- Rs.25,000 – which is the average size of trade on the spot market,
- Rs.250,000 – which is the average size of trade on the derivatives market,
- Rs.1 million and Rs.10 million – which correspond to large orders.

When market orders for these four transaction sizes are simulated, missing data arises when the limit order book is not able to fill the required order size. This can arise when buying or selling or both. Combining information across multiple order book snapshots requires special care in addressing this problem of missing data.

Table III shows the fraction of times that a trader would have been able to execute a market order of size Q across the numerous order book snapshots of the dataset. The

Table III Probability of complete execution of market orders, spot market

The table presents the probability of full execution of market orders, for both buying and selling, for a set of order sizes. The probability of execution is the fraction of limit order book snapshots for which full execution is obtained for a market order of the stated size.
This execution probability is computed for each security and the overall average is shown in the table. The standard deviation across firm-means is reported in parenthesis. As an example, when executing market orders of size Rs.1 million, there is an 84% probability of getting a full execution (averaging across all firms). The cross-sectional standard deviation of the execution probability, across firms, is 0.08.

	Probability of full execution			
Q (in Rs. million)	0.025	0.25	1	10
Overall	1.00	0.95	0.84	0.68
sample	(0.00)	(0.05)	(0.08)	(0.30)

table shows that only a market order of $Q = 25000$ can be fully executed for all securities in the sample, for all available LOB snapshots. In other words, there would be no missing data about impact cost, when buying or when selling, for only one case: $Q = 25000$. For the three other transaction sizes, the true impact cost is sometimes unobserved.

When full order execution of a given size is less than certain, this has important implications for the statistical analysis. Consider IC_Q for a given security, which is observed across many order book snapshots, subject to the problem that in certain order book snapshots, a single market order of size Q could not be executed. This induces missing data. At the same time, this missing data represents non-random censoring: IC will be unobserved when there is illiquidity, i.e. when the cost of transacting is very high. Hence, a sample mean computed using only observed values will induce a downward bias. The sample mean of IC_Q computed in this fashion is a biased estimator. There are two approaches to address this:

A location estimator of impact cost Suppose IC_Q is observed for more than half of the order book snapshots (with failure in execution for the remainder). We assume that the true IC for unobserved values is a larger number that is unobserved. Then, the sample median constitutes a good location estimator of IC_Q. It is insensitive to the specific value adopted for missing data. If we believe that missing values correspond to a large IC_Q, the sample median is a robust location estimator.

As an example, suppose there are five order book snapshots, and the IC_Q observed for a buy market order of Q =1000 shares are (0.5, 0.6, 0.7, NA, NA) where the last two observations are missing because the order book was not able to support a single buy order for 1000 shares. The sample mean of non-missing data, 0.6, is biased downwards since liquidity is poor in the two snapshots where IC_Q is unobserved. If the two NAs are viewed as large values, the sample median, 0.7, is a sound location estimator.

An estimator of the gap between buy and sell impact cost We can observe liquidity asymmetry using the *differences* of IC_Q for the buy-side and the sell-side, subject to the requirement that *both* these are observed. Here, for a given security i, the asymmetry is denoted as $\text{dIC}_{(Q,i)}$, and is measured as:

$$\mathrm{dIC}_{(Q,i)} = \mathrm{IC}_{(\text{sell-side},Q,i)} - \mathrm{IC}_{(\text{buy-side},Q,i)}$$

dIC is only observed when both buy and sell IC are observed; the sample mean of dIC is then uncontaminated by missing data.

Both these approaches are less vulnerable to the problem of missing data.

C A parametric model of the LSS

The non-parametric approach consists of an examination of a few specific points on the liquidity supply schedule, and about the depth present in the book. Alternatively, the full LSS can be modelled by a parametric function as follows:

$$\mathrm{IC}_{\text{sell/buy},Q} = f(Q_{\text{sell/buy}})$$

where $\mathrm{IC}_{\text{sell/buy},Q}$ is the price impact of a market order to sell or buy Q shares.

Theoretical models of the price impact cost have been proposed to describe the form of the LSS functions, but there has been little consensus so far. Kyle (1985) assumed that impact is both linear in the traded volume and permanent in time. Bertimas and Lo (1998) assumed a linear permanent price impact while deriving dynamic optimal trading strategies that minimise the expected cost of trading Q over a fixed time horizon. Kempf and Korn (1999) modeled the price impact using a neural network model and found a non-linear relation between net order flow and price changes. Gatheral (2010) assume a no dynamic arbitrage principle which implies that the expected cost of trading should be non-negative so that price manipulation is not possible.

Empirical studies broadly conclude that the price impact of trades is an increasing, concave function of trade size ($\sqrt{Q}$) (Evans and Lyons, 2002; Gabaix *et al.*, 2003; Hasbrouck, 1991; Kempf and Korn, 1999; Plerou *et al.*, 2002; Potters and Bouchaud, 2003). A minority of recent studies find no significant deviation from linearity (Engle and Lange, 2001; Breen *et al.*, 2002; Korajczyk and Sadka, 2004). Almgren *et al.* (2005)) rejects the common square root model in favour of a *3/5 power law* function across the range of trade sizes considered. Ting and Warachka (2003) and (Huang and Ting, 2008) use intraday trade data to find support for a S-curve model as best capturing liquidity supply curves in terms of parameter t-statistics and adjusted R^2 performance. Most recently, Rosu (2009) starts from a structural model where agents place orders into the market continuously, and show that the shape of the LSS can vary between a quadratic and an exponential, or a mixture of the two.

A limitation of the existing literature lies in the use of *trade* data to estimate price impact and its relation to trade size. In comparison, this paper uses the data observed on the orders placed in the LOB to directly calculate the price impact of the market order at any stated trade size Q. This offers a rich dataset for estimation of functions expressing the LSS for any given security.

Once an empirical form for the LSS is estimated, testing for asymmetry can be carried out by comparing estimated parameter values. Similarly, the impact of short sale constraints on liquidity can be done by calculating the difference between the parameters of the buy-side and sell-side LSS functions for the *spot* market, and then testing whether there is a comparable difference of parameters for the SSF market or not.

The existing literature has proposed the following functional forms for the LSS:

1. Linear polynomial: $\text{IC}_Q = \alpha + \beta Q$
2. Quadratic polynomial : $\text{IC}_Q = \alpha + \beta Q + \gamma Q^2$
3. Exponential : $\text{IC}_Q = exp^{\alpha + \beta Q}$
4. Stretched exponential : $\text{IC}_Q = exp^{(\alpha + \beta Q + \gamma Q^2)}$

Each of these have a common feature of being monotonically increasing in Q. Each of these models has an *intercept* term and one or more *slope* coefficients, which captures how the price impact cost changes for larger order sizes. In all cases, Q is the log of the transaction size expressed in rupees.

These functions are all estimated using the calculated IC_Q for the full LOB for every security, separately for the buy-side and the sell-side. For each security, the average *adjusted* R^2 is calculated for all the buy-side LOB and the sell-side LOB observations. Then, the average of these adjusted R^2 are calculated for the sample and reported in Table IV. The model with the highest adjusted R^2 is chosen as the best representation of the LSS. The results in Table IV strongly suggest that the *stretched exponential* (Model 4) is the best model according to the adjusted R^2.

As an illustration of the extent to which Model 4 works well, Table V compares actual values against model predictions for one security from *S-big* and one security from *S-small*. We see that the IC estimates from the model compares well against the actual IC measured from the LOB for the different trade sizes for these two securities. Under this model,

$$\frac{\partial \text{IC}}{\partial Q} = (\beta + 2\gamma Q)\text{IC}_Q$$

As Q increases, IC_Q worsens not just by β and IC_Q, but also by γ and Q itself, which is a sharper increase of IC_Q compared to Models 1-3. This, in turn, implies that when market liquidity of a security is measured by the the LSS takes the functional form of a stretched exponential, liquidity tends to be more sensitive to changes in Q than if it is linear in Q, or follows a exponential form.

The estimated parameter values of this parametric model of the LSS – $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ – can be used to test for asymmetry, by examining the following hypotheses:

- Is $\hat{\alpha}_S > \hat{\alpha}_B$?
- Is $\hat{\beta}_S > \hat{\beta}_B$?
- Is $\hat{\gamma}_S \neq \hat{\gamma}_B$?

Table IV Adjusted R^2 of alternate functions for the spot market LSS

The table reports the *adjusted* R^2 of the regression for the functional candidates, Model 1-4, for the LSS on both the buy and sell side of the limit order book. **Model 1** is the *linear* model. **Model 2** is the *quadratic* model. **Model 3** is the *exponential* model. **Model 4** is the *stretched exponential* model. The average adjusted R^2 is reported for each quintile with the standard deviation in parentheses. *S-big* is the quintile of securities with the highest market capitalisation and *S-small* has the lowest market capitalisation. The values in boldface represents the models which have the best fit in terms of adjusted R^2.

	Sell side				Buy side			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
S-big	0.53	0.81	0.85	**0.90**	0.51	0.79	0.85	**0.98**
	(0.16)	(0.13)	(0.13)	(0.06)	(0.16)	(0.10)	(0.12)	(0.05)
S2	0.54	0.80	0.88	**0.97**	0.59	0.80	0.90	**0.91**
	(0.13)	(0.10)	(0.09)	(0.03)	(0.14)	(0.11)	(0.08)	(0.03)
S3	0.57	0.83	0.88	**0.97**	0.59	0.83	0.90	**0.90**
	(0.13)	(0.10)	(0.10)	(0.04)	(0.13)	(0.10)	(0.09)	(0.04)
S4	0.57	0.84	0.89	**0.98**	0.56	0.82	0.89	**0.92**
	(0.13)	(0.09)	(0.10)	(0.04)	(0.13)	(0.10)	(0.09)	(0.03)
S-small	0.58	0.85	0.89	**0.97**	0.57	0.83	0.90	**0.90**
	(0.13)	(0.09)	(0.10)	(0.03)	(0.13)	(0.10)	(0.09)	(0.03)

Table V An illustration of Model 4 estimated IC versus the IC measured from the LOB for two securities

The values presented are the estimated IC ($\hat{IC}$) and the actual IC observed in the LOB (IC) for a market order of Q = Rs 0.025 million and Rs 1 million. This is done for a large market capitalisation stock, *S-big* and a small market capitalisation stock, *S-small*.

Trade Size	*S-big*				*S-small*			
(Rs Mn.)	$\hat{IC}_{buy}$	$\hat{IC}_{sell}$	IC_{buy}	IC_{sell}	$\hat{IC}_{buy}$	$\hat{IC}_{sell}$	IC_{buy}	IC_{sell}
Q = 0.025	0.066	0.066	0.065	0.060	0.146	0.129	0.202	0.144
Q = 1	0.087	0.094	0.121	0.102	1.432	1.868	1.771	2.012

Table VI Probability of full execution on spot LOB

For each security, the fraction of limit order book observations where a market order of size Q can be fully executed is computed. This is the probability of full execution shown below, for firms in quintiles by market capitalisation, going from *S-big* (the biggest) to *S-small* (the smallest). Values in boldface indicate where the probability of execution on one side of the book is statistically higher at a 5% level of significance. As an example, among the smallest quintile of stocks, a single buy order for Rs.1 million is executed with a 0.72 per cent probability when it is a sell order, but a statistically significantly higher probability of 0.80 when it is a buy order.
For large sized orders across all firm sizes, there is a higher probability of being able to execute a single large order on the buy side. This shows the presence of asymmetry between buying and selling; sell side liquidity is worse than buy side liquidity.

	Sell-side Q (Rs. Mln.)				Buy-side Q (Rs. Mln.)			
	0.025	0.25	1	10	0.025	0.25	1	10
S-big	1	1	0.90	0.78	1	1	**0.98**	**0.91**
S2	1	1	0.88	0.71	1	1	**0.96**	**0.82**
S3	1	0.90	0.80	0.65	1	**0.96**	**0.90**	**0.70**
S4	1	0.88	0.80	0.50	1	**0.92**	**0.86**	**0.62**
S-small	1	0.80	0.72	0.39	1	**0.87**	**0.80**	**0.55**
Overall sample	1	0.92	0.78	0.27	1	1	**0.84**	**0.40**

If the estimated sell-side parameters are individually and jointly lower/higher than the estimated buy side parameters, we would conclude that the sell-side liquidity is better/worse than buy-side liquidity. We use the Kolmogorov-Smirnov (KS) test on the distributions of estimated α_B, α_S, β_B, β_S, γ_B, γ_S to establish whether there are asymmetries in liquidity.

V Testing for asymmetry of liquidity in the spot market

We present results of our analysis of liquidity asymmetry in the spot market through the three different testing procedures listed in the previous section.

A Evidence on probability of full execution of market orders

Table VI shows that there is a higher probability of executing large orders on the *buy-side* compared to the *sell-side*. It is easier to buy large amounts of shares rather than to sell large amounts of shares. The evidence in favour of asymmetry is striking: all the cells in the table, with values other than 1, have statistically significant differences between buying and selling.

B Evidence about buy-side versus sell-side IC

Table VII present the average dIC_Q for a given sample of securities, where $\mathrm{dIC}_{(Q,i)}$ is calculated as the median value of the difference between sell-side IC and buy-side IC for

Table VII Average difference between buy-side and sell-side liquidity in the spot market

In the dataset, a large number of snapshots of the limit order book are observed. For each security, in each snapshot, a single market order of size Q is attempted. We discard snapshots where a full execution was not obtained for either buy or sell. For the remainder, we compute $\mathrm{dIC}_{(Q,i)} = \mathrm{IC}_{(\text{sell-side},Q,i)} - \mathrm{IC}_{(\text{buy-side},Q,i)}$, the extent to which sell impact cost (in per cent) is bigger than buy impact cost (in per cent).

For each security, the median value across multiple snapshots is utilised. Each cell of the table shows the sample mean of the values across all securities. The values in brackets are sample standard deviations. The values in boldface indicate instances when $\mathrm{dIC}_{(Q)}$ are different from zero and statistically significant at a 95 per cent level. In these values, sell side liquidity is worse than buy side liquidity.

As an example, this shows us that in the smallest quintile of firms, for transactions of Rs.1 million, on average, sell impact cost was worse than buy impact cost by 0.39 percentage points. This difference was statistically significant at a 95 per cent level.

	Q (Rs. Mln.)			
	0.025	0.25	1	10
S-big	0.01	0.02	0.04	**0.52**
	(0.00)	(0.03)	(0.12)	(0.81)
S2	0.01	0.04	0.04	**0.78**
	(0.01)	(0.06)	(0.25)	(1.37)
S3	0.02	0.09	0.10	**1.20**
	(0.02)	(0.15)	(0.50)	(1.23)
S4	0.02	0.11	0.12	**0.97**
	(0.01)	(0.09)	(0.29)	(1.92)
S-small	0.03	0.04	**0.39**	**2.68**
	(0.02)	(0.27)	(0.61)	(3.36)
Overall	0.02	0.04	0.05	**0.95**
sample	(0.02)	(0.16)	(0.43)	(1.69)

Table VIII LSS function estimates for the spot market LOB

The values presented are summary statistics of the parameter estimates of the stretched exponential model – α, β, γ – for the sample.
The table shows average values of the median parameter estimate for the securities in the overall sample, as well as, for market capitalisation based quintiles, $S1 - S5$. *S-big* securities having the highest market capitalisation and *S-small* having the lowest. These parameters are calculated separately for the sell-side and the buy-side.

	$\hat{\alpha}^S_S$	$\hat{\alpha}^S_B$	$\hat{\beta}^S_S$	$\hat{\beta}^S_B$	$\hat{\gamma}^S_S$	$\hat{\gamma}^S_B$
S-big	**2.93**	2.11	**0.14**	0.12	**3.44**	2.62
S2	**3.58**	2.14	**0.13**	0.10	**3.14**	2.41
S3	**4.63**	2.67	**0.12**	0.10	**2.92**	2.36
S4	**4.22**	2.90	**0.38**	0.33	**3.29**	2.95
S-small	**6.50**	3.84	**0.60**	0.52	**3.38**	2.38
Overall	3.63	2.42	0.20	0.16	3.26	2.44

all the LOB observations for a security i where both are observed. The standard deviation of the sample average is reported in parentheses as well. The results show that for small Q, liquidity is symmetric for buyers and sellers. As Q becomes larger, $\mathrm{dIC}_{(Q,i)}$ becomes positive and significant. This implies that it is more difficult to *sell* large quantities than it is to *buy* the same Q. For example, in the overall sample, $\mathrm{IC}_{\mathrm{sell}}$ for $Q \geq$ Rs.1 million is, on average, 1.5 times higher than the value of $\mathrm{IC}_{\mathrm{buy}}$ for the same Q.

The evidence for the behaviour of liquidity asymmetry persists across all quartiles of securities by market capitalisation. For small Q, buy-side and sell-side liquidity is symmetric, and asymmetric for larger Q with a larger premium on the sell-side.

C Evidence from a parametric model of the LSS

The liquidity measures in this section are the parameter estimates of the *stretched exponential model*, which are α, which expresses the base level of liquidity and the parameters β and γ, which express the rate of change of liquidity with Q. The β parameter can be interpreted as a β% change in log IC in response to a 1% change in log Q where Q was quantity measured in Rs. This is equivalent to saying that the elasticity of the LSS is equal to β. The γ parameter can be expressed as the change in log IC in response to a 1% change in square of log Q where trade size Q was measured in Rupees.

These estimates are presented in Table VIII as $\hat{\alpha}^S, \hat{\beta}^S, \hat{\gamma}^S$ where the superscript S indicates that these are spot market estimates. In each size quintile, the mean is shown as a location estimator of parameter values observed across different stocks. All the parameter estimates are positive which is consistent with the observation that liquidity worsens for larger order sizes. The sell-side estimates are consistently higher than the buy-side for the securities in the case of all three parameters. This implies that the drop off in liquidity is worse for sellers compared with buyers in the market.

D Summary

We have examined buying and selling on the spot market using three different estimation strategies. Across all three methods, there is striking evidence of asymmetry in liquidity: buying is easier than selling on the spot market.

VI Testing for asymmetry of liquidity in single stock futures

One potential explanation that could shape differences in liquidity between buying and selling lies in short sale constraints, since these constraints are innately asymmetric and merit exploration. In order to assess the role (if any) of short sale constraints, we exploit a remarkable feature of the setting in the paper: the presence of a single stock futures (SSF) market alongside the spot market. Trading in spot and the SSF for the identical securities takes place with the identical market rules. The participants of both markets are also quite similar. There is only one major microstructural difference which has an asymmetric impact: the spot market has no formal borrowing mechanism for shares, and requires delivery of shares on date T+2.[2] In contrast, the SSF market is cash settled; buying and selling is fully symmetric. The fact that both the SSF (where long and short positions are symmetric) and spot market (selling is difficult for a speculator who does not happen to already own the shares) trades simultaneously, with the identical market design, offers an opportunity to understand the extent to which short sale constraints are a source of asymmetry in liquidity.

The same methodologies are used to to measure and test for liquidity between sell-side and buy-side on the SSF markets. We compare these results with that of the spot market. If both markets show similar asymmetry in liquidity, we can conclude that it is the information asymmetry between informed and uninformed traders that are the reason for liquidity asymmetry, rather than the microstructure different of short sale constraints. If the SSF market displays no asymmetry in liquidity, we may conclude that short sale constraints are an important source of liquidity asymmetry.

A Evidence on probability of full execution of market orders

Table IX shows the probability of full execution of a market order, when buying and selling, on the SSF market. This should be compared and contrasted with the identical evidence for the spot market in Table VI.

[2]The sell position at the end of each trading day is required to induce delivery of shares on date $T + 2$. When speculators have a negative view about a price, they have two choices. One is that they can coincidentally own the stock and thus sell it off. Alternatively, they can borrow shares OTC from informal networks. The absence of a formal stock borrowing / margin trading mechanism (as was the case in India during the period of the study in this paper) amounts to an important constraint on short sale.

Table IX Probability of full execution of market orders in the SSF market

The values reported in the table are the fraction of the LOB data for a given security where a market order of size Q gets immediate and complete execution.
This probability of full execution is presented as the average for the overall sample, as well as for size quintiles, going from *S-big* (the biggest) to *S-small* (the smallest). Values in boldface indicate that the probability of execution on one side of the book is statistically higher at a 95% level of significance.
As an example, among the stocks in the smallest quintile, a single order to sell Rs.1 million has a 98% probability of full execution while a buy order has a 99% probability of full execution. None of the values in the table are in boldface; i.e. in no case is there a statistically significant difference in execution probability between buying and selling.

	Sell-side Q (Rs. Mln.)				Buy-side Q (Rs. Mln.)			
	0.025	0.25	1	10	0.025	0.25	1	10
Overall sample		1	1	0.82		1	1	0.79
S-big		1	1	0.98		1	1	0.98
S2		1	1	0.95		1	1	0.93
S3		1	0.99	0.82		1	1	0.81
S4		1	1	0.74		1	1	0.70
S-small		1	0.98	0.64		1	0.99	0.55

The probability of full execution drops for larger transaction sizes in the futures market, just as it did in the spot market. The probability of full execution decreases when we go from large firms to small firms. However, in the full dataset, and across all size quintiles, the difference in the execution probability between buying and selling is not statistically significantly different from 0. This is the first piece of evidence that the SSF market has no asymmetry in liquidity.

B Evidence about buy-side versus sell-side IC

The second measure of liquidity asymmetry is the difference in the impact cost on the buy-side and the sell-side ($\mathrm{dIC}_{Q,i}$) for each observed LOB in the SSF market. This is calculated using the SSF market LOB for the same range of Q sizes of Rs.$0.25, 1, 10$ million as was done for the spot market.

Table X shows the median IC difference calculated for the overall sample as well as for the size based quintiles of the sample securities. This should be compared and contrasted with Table VII which shows the same evidence for the spot market.

There is only one case (4th quantile, for transactions of Rs.10 million) out of the 18 estimates shown in the table, where there is evidence of asymmetry. This is in contrast with the corresponding table for the spot market, where 7 of the 18 values showed evidence of asymmetry. This is the second piece of evidence that indicates that there is little evidence of asymmetry in liquidity on the SSF market.[3]

[3]When the null hypothesis is true, a test at the 95% level of significance falsely rejects 5% of the time. If the null is always true, and 18 tests are conducted, it is not surprising to find one rejection of the null.

Table X Average difference between sell-side and buy-side liquidity in the SSF market

The table presents the difference between the $IC_{(\text{sell-side},Q)}$ and $IC_{(\text{buy-side},Q)}$ on the SSF market. Snapshots where a full execution was not obtained for either buy or sell were discarded. $dIC_{(Q,i)}$ is computed as $IC_{(\text{sell-side},Q,i)} - IC_{(\text{buy-side},Q,i)}$. The median value is calculated for each security, and the mean of firm-medians is reported for the overall sample as well as for the quartiles by size.
The values in brackets are cross-sectional standard deviations across the firm-medians. The values in boldface indicate instances when $dIC_{(Q)}$ are different from zero and statistically significant at a 95 per cent level. For these values (that are in boldface), sell side liquidity is worse than buy side liquidity.

	Q (Rs. Mln.)		
	0.25	1	10
S-big	0.00	0.01	0.05
	(0.00)	(0.01)	(0.09)
S2	0.00	0.02	0.09
	(0.00)	(0.02)	(0.24)
S3	0.01	0.13	0.02
	(0.04)	(0.32)	(0.27)
S4	0.00	0.07	**0.71**
	(0.01)	(0.06)	(1.14)
S-small	0.00	0.05	0.18
	(0.01)	(0.06)	(0.58)
Overall sample	0.00	0.06	0.14
	(0.02)	(0.15)	(0.61)

C Evidence from a parametric model of the LSS

Lastly, we examine the parameter values for the stretched exponential functions estimated using the SSF LOB, where each estimation has been done separately for the buy-side and the sell-side. This is presented in Table XI as $\hat{\alpha}^F, \hat{\beta}^F, \hat{\gamma}^F$ to denote the parameter estimates for the SSF LSS functions. These results should be compared and contrasted with those in Table VIII for the spot market.

The values of all the parameter estimates are positive for the SSF LSS. This is consistent with the notion that liquidity (as measured by the IC) worsens as the order size Q becomes larger, and similar to the values estimated for the spot LSS functions. However $\hat{\alpha}^F_S$ is consistently *less* than $\hat{\alpha}^F_B$. This is the opposite to what was observed in the case of the spot market in Table VIII where $\hat{\alpha}^S_S > \hat{\alpha}^S_B$. We cannot conclude that the parameters support the hypothesis that the liquidity on the sell-side is worse than liquidity on the buy-side consistently.

VII Conclusion

Economists have long been interested in understanding liquidity. The theoretical literature emphasises the costs faced by liquidity providers, such as dealers and marker makers,

Table XI LSS parameter estimates for the SSF market LOB

The values presented are summary statistics of the parameter estimates of the stretched exponential model – α, β, γ – using the LSS data from the SSF markets of the sample securities.
The table shows average values of the median, minimum and maximum values for the parameters for the overall sample, as well as the market capitalisation based quintiles, $S1 - S5$. *S-big* securities having the highest market capitalisation and *S-small* having the lowest. These parameters are calculated separately for the sell-side and the buy-side.

	α_{sell}	α_{buy}	β_{sell}	β_{buy}	γ_{sell}	γ_{buy}
Overall	0.72	**1.12**	**0.18**	0.12	**1.82**	0.84
S-big	0.76	**0.87**	**0.12**	0.08	**1.05**	0.73
S2	0.87	**1.06**	**0.14**	0.08	**1.58**	0.84
S3	0.64	**1.02**	**0.19**	0.11	**0.81**	0.79
S4	0.74	**1.35**	**0.21**	0.12	**2.00**	1.63
S-small	0.43	**0.73**	**0.26**	0.15	**2.84**	2.64

as the source of impact cost (the divergence from the midpoint quote of the realised price for a market order). The bid-ask spread, which is the workhorse of the bulk of the empirical literature, encourages a symmetric perspective upon the cost faced in buying versus selling.

The bulk of exchanges worldwide have shifted to electronic limit order book markets. The electronic limit order book exchange is also a new kind of setting, when compared with the traditional microstructure literature, in that liquidity provision is the outcome of decentralised decisions by a large number of end-users of the market, rather than market intermediaries such as a set of dealers or a market maker. With full observability of the limit order book, it is now possible to directly observe the impact cost associated with buy or sell market orders of all possible sizes.

The contribution of this paper lies in bringing high quality empirical evidence about asymmetry in liquidity. The impact cost faced by a single market order is directly observed on a limit order book market, thus eliminating the complexities that arise in trying to infer the cost of transacting under other market structures. We draw on evidence about impact cost at various sizes using the limit order books for the top 100 securities by liquidity from one of the most active exchanges of the world. The special feature of this setting is that we observe trading of the equity spot simultaneously with cash-settled single stock futures in the identical microstructure.

The theoretical literature has proposed explanations for asymmetry rooted in the behaviour of market makers, institutional investors, short sale constraints, and asymmetric information. The NSE is a clean setting with only limit orders; there are no market makers or dealers. The role of institutional investors on both the spot and the SSF market is quite small. Hence, theoretical arguments which predict asymmetry based on the behaviour of dealers, market makers and institutional investors are not relevant here.

The evidence presented in the paper, from non-parametric and parametric measures taken together, suggests that there is asymmetry of liquidity on the spot market, but not on the SSF market. Our empirical work has approached the question from three perspectives, harnessing a careful treatment of missing data when market orders are not filled, and the selection of a best-fit parametric model of the liquidity supply schedule. The answers obtained through the three different estimation strategies are consistent, which suggests the findings are robust.

The market design of the spot market and the SSF market is identical in every respect (order matching, time of day, market participants) but one: short sale constraints. There is another difference – SSF markets have leverage while spot markets do not – but this cannot induce asymmetry since the SSF has a linear payoff.

There are two competing mechanisms through which short sale constraints can induce asymmetry. On one hand, short sale constraints can make it harder to place limit orders to sell. This would yield inferior liquidity when *buying*. On the other hand, liquidity providers could be more cautious when facing market sell orders in the presence of short sale constraints. This would yield inferior liquidity when *selling*. Our evidence suggests that the latter phenomenon predominates.

These results give us fresh insights into the recent debates about short selling. Regulators such as the UK FSA banned short selling for some securities, in an attempt at avoiding sharp price declines. The analysis of this paper suggests that short sale constraints reduce liquidity faced by market *sell* orders and thus *enhance* the price response to speculative selling. More generally, when liquidity is asymmetric, idiosyncratic shocks to the order flow should generate asymmetric price responses. Future research will explore whether asymmetry in liquidity induces asymmetries in the distribution of innovations in time-series models of stock returns for open electronic limit order book markets.

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