Incentivizing Impact Investing*

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Abstract

The impact investing asset class is gaining popularity among investors. However, it is not well understood what mechanisms are optimal for investment. We propose that investors who choose investments in social projects purchase Social Impact Guarantees (SIGs). SIGs are debt-like securities with par values endogenously determined by realized social output. The repayment design aligns the incentives of commercial and socially-conscious investors, allowing joint investment in social projects. SIGs create a social market within standard market frameworks and exploit well-established market mechanisms to increase social investment efficiency. Furthermore, the pricing of SIGs and residual claims provide valuable information to decision makers.

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1 Introduction

The advent of impact investing as an accepted asset class has been met with mixed reactions. Unlike traditional asset classes in which measured return is a straightforward calculation, the relevant returns for impact investments are less definitive. This inability to settle on a universal definition of performance is attributed to two opposing views: one view in which only financial returns matter and one view in which social good (i.e., social return) is an acceptable substitute for financial returns. Each view point has distinct implications for the businesses that have the ability to undertake an impact investment. Namely, as custodians of shareholder dollars, which type of return should those businesses attempt to maximize?

Milton Friedman (1970), and more recently Aneel Karnani (2010), argue that businesses have a fiduciary duty to maximize financial returns for shareholders. Businesses must therefore refrain from engaging in any activities that might be perceived as socially desirable if such activities would be detrimental to those returns. Social causes are best left for governments, philanthropic individuals, and organizations, they contend. Conversely, Muhammad Yunus, the recipient of the Nobel Peace Prize in 2006, has contended that free market capitalism is not capable of addressing the important social problems facing the world today. Yunus (2008) argues that those businesses with the ability to undertake an impact investment be organized as social entities in which financial returns *must* be sacrificed in the pursuit of *maximizing* social goals.[1]

Despite a lacking consensus regarding the objectives of an impact investment, the asset class is attracting sizable interest. Estimates of the total size of impact investments vary from $50 billion (Monitor Institute, 2009) to nearly $1 trillion (J.P. Morgan, 2010).[2] Indeed, investments in the asset class have outpaced a thorough understanding of the optimal mechanisms to deploy those dollars. Many questions are yet unanswered: Is there a mechanism that enables joint investment by those that value social good and those that do not? In businesses in which managers are rewarded with equity-based compensation, is there a mechanism that mitigates the natural tendency for those managers to pursue profit-maximization at the expense of social objectives? What mechanism allows impact investment dollars to be leveraged and have the largest reach? Indeed, a rigorous microfoundation pertaining to the types of securities that should be utilized for impact investments is conspicuously missing.

[1]A number of states in the U.S. are beginning to introduce a new class of corporation, called the Benefit Corporation, that has a "corporate purpose to create a material positive impact on society"(See benefitcorp.net).

[2]It is important, however, to point out that the relative size of impact investments relative to other asset classes is quite small: the pool of dollars directed towards impact investments pale in comparison to the nearly limitless amount of funds commercially available for investments that generate market returns.

In this paper we satisfy those unanswered questions. We show how it is possible to jointly mobilize socially-conscious and commercial investment in businesses that pursue social objectives. The security and market design we propose aligns the interests of socially-conscious and commercial investors. Furthermore, the security we propose allows investors to write equity-based compensation contracts for managers that induce the managers to pursue well-defined social objectives and at the same time employ scarce resources efficiently.

Consider a project that requires an upfront investment and generates cash profits with a present value less than the investment cost. An agent that cares only about financial returns will not fund this project. But suppose the project also generates some desirable social output that society can measure in equivalent cash units. In a utopian world without financial constraints or agency conflicts, social investors could commit to pay the cash value of realized social output. As such, the project would be considered profitable and it would be undertaken. In reality, however, social capital is scarce: social investors cannot afford extremely successful social outcomes. Indeed, this too could be remedied if social investors subsidized the project's upfront investment and let the project owner undertake the project. In reality, however, agents, e.g., project managers, are limited in their ability to commit: granting ex ante subsidies does not provide incentives to maintain social goals through the project's life. Indeed, the agency conflict associated with limited commitment is at its greatest when social goals are at odds with financial profit maximization.

Both frictions, scarce social capital and limited commitment, are mitigated through the issuance of a Social Impact Guarantee (SIG). A SIG is a security that is sold by the project owner, an agent that cares only about financial returns, to social investors. The proceeds from the security's sale subsidize the project's upfront cost and the security's contractual payments guarantee that project owner and social investors' incentives are aligned throughout the project's life. The SIG, which can alternatively be thought of as a receive-money-back-for-failure security, provides the much needed incentive alignment component that is lacking among the securities currently used for impact investments.[3]

The security's repayments have a debt-like structure. If observable and verifiable output mea-

[3]Social Impact Bonds (SIBs), also called pay-for-success bonds, have been proposed and introduced in the U.K., the U.S. and Australia. Social Impact Bonds are sold to investors and funds raised are used by organizations to accomplish some well-defined social objectives. If the objective is met, the government - that might value the social benefit to the society - pays the investors after the success has been demonstrated and independently verified (see Liebman, 2011). A SIB may be useful for raising funds but *does not* resolve the conflict of interest among different investors. Investors holding the SIB would prefer the organization to pursue the social goal but their motives might continue to be in conflict with other commercial investors who would rather have the firm pursue profit-maximizing opportunities.

sures suggest that the project was underfunded, the project owner must pay the project's cash profits to the SIG holders up to some *par value*.[4] Conversely, if outputs are indicative of sufficient investment, the commercial investor pays the *par value* of debt and retains all residual cash generated from the project and the security's sale. We emphasize *par value* because, unlike standard debt contracts, it is endogenously determined by the realized social output. If social output is large (small), the *par value* is small (large). We also show that our security design is not perfect: a SIG may involve some inefficiency and perfect incentive alignment may not always be possible. The optimal design trades off inefficiency caused by differences in preferences between the project owner and social investors and the limited availability of funds from social investors.

The security's contractual payments are continuous in both outputs, and, as such, minimize the incentive to manipulate observed outputs for financial gain. Furthermore, the contractual payments rely crucially on well-defined measurable benchmarks. In addition to cash flow performance measures that are more readily available, we also require reliable indicators of social performance. There are several recent developments that make this possible. For example, the Benefit Corporation requires "reports on its overall social and environmental performance using recognized third party standards." (See benefitcorp.net). Impact Reporting & Investment Standards (IRIS : http://iris.thegiin.org/) and GIIRS Ratings and Analytics for Impact Investing (http://giirs.org/) are examples of organizations that also help establish independent standards similar to the role played by credit rating agencies in providing useful default information on corporate bonds.

We also consider that a SIG may trade in the secondary market. The prices of the SIG and other firm securities will aggregate investor information that might be useful both for managers as well as investors.[5] For example, a rise in the secondary price of the SIG would indicate that the firm is less likely to meet its social objective benchmark. If the SIG is senior to a firm's other claims, the SIG does not need to trade to provide useful information via prices: the prices of all junior claims will provide indirect information about the SIG's performance. Furthermore, in a dynamic model in which the firm repeatedly raises funds, secondary pricing of securities provides useful information to managers about which social objectives will likely be valued more highly by investors (and society) in the future. This allows them to build capacities for future expansion in desirable social activities.[6]

Our paper provides a novel addition to the security design literature by considering how to

[4] Chan (2011) develops a model with a similar insight.

[5] See Subrahmanyam and Titman (1999).

[6] See Subrahmanyam and Titman (2001) for an argument on how stock prices provide useful information on firms' future cash flows.

incentivize an agent to invest in a non-rival, public good that is only valued by a subset of investors. Innes (1990) provides the most similar framework to ours. In his model, a financially constrained entrepreneur sells a security to cover a project's upfront cost and the entrepreneur subsequently makes an unobservable effort choice. The entrepreneur's effort choice directly influences the project's cash output which is observable and contractible. Innes shows that the optimal security construct, with the constraint that the repayments be non-decreasing, is a debt contract. We add to his findings by considering that output has two dimensions: cash profit and social good. Furthermore, we demonstrate the the par value of the debt contract is endogenously determined by the realized social output.

The design we propose necessarily partitions the project's cash flows between contractual claim holders and residual claim holders. Boot and Thakor (1993) suggest that the partitioning of a firm's cash flow is attributed to creating an "informationally sensitive" and an "informationally insensitive" component. The two distinct components encourage investors to acquire information, which enhances firm value. In our setup, the partitioning of cash flows induces incentive alignment between those that hold the contractual claim and those that hold the residual. Allen and Gale (1988) consider a setup in which security issuance is costly and different groups of investors assess different values to the same security. They show that the optimal security allocates each state-contingent cash flow according to which investors value the proceeds in that state. Our model, however, assumes that all investors value cash profit in the same manner in every state. Investors do differ with regard to whether or not they value social good.[7]

2 The Project, the Owner, and the Social Investor

Consider a social project that produces both cash profit and social good. Undertaking the project requires a fixed upfront cost equal to k. If the project is undertaken, for some additional level of investment $i \geq 0$ over the upfront cost k, the cash profit is a random variable $x \geq 0$ with conditional density $f(x|i)$ and the social good produced is a random variable $s \geq 0$ with conditional density $g(s|i)$. We assume that conditional on investment, cash profit and social good are independent so that the joint density function is simply the product of the two conditional densities $f(x|i)g(s|i)$.

The project's ownership rights are endowed to a regular profit-maximizing agent who only values cash profit (we refer to the agent as the "owner" hereafter and we will use superscript π to characterize him). There also exists a social investor who values both cash profit and social

[7]Madan and Soubra (1991) consider the model of Allen and Gale (1988) with the addition of marketing costs.

good (we will use superscript ψ to characterize this investor). We assume that the project owner is unconstrained and has unlimited access to capital for positive net present value projects. We assume the owner operates in a competitive market place and expects to earn zero profits.[8] The social investor, however, is constrained and has a maximum capital budget $a > 0$ and a is less than the fixed cost of the investment k. This assumption is made to capture an important feature of impact investing — the funds available from socially-minded investors are limited so that many social projects are forgone because they cannot be solely funded by investors who are willing to accept a smaller rate of cash return on their invested capital. Investment occurs in period 1, and the payoffs occur in period 2.

Both $f(x|i)$ and $g(s|i)$ satisfy a monotone likelihood ratio property (MLRP):

$$\frac{\partial}{\partial x}\left[\frac{f_i(x|i)}{f(x|i)}\right] > 0, \tag{1}$$

and

$$\frac{\partial}{\partial s}\left[\frac{g_i(s|i)}{g(s|i)}\right] > 0, \tag{2}$$

where $f_i(x|i) \equiv \frac{\partial f(x|i)}{\partial i}$ and $g_i(s|i) \equiv \frac{\partial g(s|i)}{\partial i}$. These conditions imply that greater investment i makes higher realizations of cash-profit and social good more likely. Our setup so far is redolent of Innes (1990). As such, we make the following assumption in the same spirit,

Assumption 1. *There exists a finite i^{max} such that*

$$\lim_{i \to i^{max}} \int_0^\infty x f(x|i)\,\mathrm{d}x + \int_0^\infty s g(s|i)\,\mathrm{d}s - i - k < 0. \tag{3}$$

Assumption 1 implicitly requires that the cash profit and social good returns on invested capital are finite and that they are negative as i approaches i^{max}. The assumption allows us to focus on the investment choice set $[0, i^{max}]$ without loss of generality. We now make an additional assumption to guarantee uniqueness throughout our analysis,

Assumption 2. *Both $\int_0^\infty x f(x|i)\,\mathrm{d}x$ and $\int_0^\infty s g(s|i)\,\mathrm{d}s$ are concave in i.*

Assumption 2 is a regularity condition that allows us to focus on unique solutions in the problems we consider hereafter. The assumption is natural as well — the social project demonstrates diminishing returns to investment.

[8] Shortly, we will introduce a security that is traded between the project owner and the social investor. The assumption that the project owner is competitive is natural and allows us to pin down a unique price for the security.

We begin by considering the welfare maximizing level of investment in a frictionless economy, that is, no agency conflicts or financial constraints exist. In this case, the welfare function is given by,

$$W(i) \equiv E_x\left[x|i\right] + E_s\left[s|i\right] - i - k, \tag{4}$$

and a social planner's problem is,

$$\max_i W(i). \tag{5}$$

The solution, i^{FB}, to the "first-best" choice problem is characterized by the first-order condition

$$0 = \int_0^\infty x f_i(x|i^{FB})\,\mathrm{d}x + \int_0^\infty s g_i(s|i^{FB})\,\mathrm{d}s - 1, \tag{6}$$

so long as $W(i^{FB}) \geq 0$. We assume that the $W(i^{FB}) \geq 0$ hereafter to focus on the case in which the social project should be undertaken.

Now, recall that the project owner cares only about cash profit. His level of investment is the solution to the following optimization,

$$\max_{i\geq 0} E_x[x|i] - i - k \tag{7}$$

$$\text{s.t. } E_x[x|i] - i - k \geq 0. \tag{7.1}$$

Let i^π denote the solution if the participation constraint is slack. Then i^π is implicitly defined by,

$$\int_0^\infty x f_i(x|i^\pi) = 1. \tag{8}$$

Assumption 2 implies that $i^\pi < i^{FB}$. In other words, the project owner will underinvest in this social project. We further assume that $E_x[x|i^\pi] - i^\pi - k < 0$ so that he will choose not to invest in the social project at all. This captures the central problem with impact investing — regular profit-maximizing agents do not find it profitable to invest in projects that may be socially beneficial because they are not profitable enough to recoup their cost of capital.

In this paper, we consider the design of a security that allows the social project to be undertaken via joint investment by the project owner and the social investor. The security is sold by the project owner to the social investor and its construct simultaneously accomplishes a dual role. First, the security's repayment schedule improves the incentives of the project owner to invest in the social project, and, second, the security's upfront price is affordable for the social investor, i.e., the security leverages the social investor's limited capital.

2.1 The Social Impact Guarantee

The investment above and beyond the upfront fixed cost is unobservable and thereby non-contractible. Both outputs and the initial capital outlay k, however, are observable and contractible.[9] Consider a traded security offered by the project owner to the social investor. The security has price p^{ψ} and pays $y^{\psi}(x,s)$ in period 2. It is important that both variables, x and s, be ex post observable and contractible.

The ex ante payoff to the project owner is equal to the expected cash from for-profit investment minus the expected security repayment to the social investor plus the cash raised from the initial sale of the security,

$$E_x[x|i] - i - k - E_{x,s}[y^{\psi}(x,s)|i] + p^{\psi}. \tag{9}$$

When considering the design of the security, we assume (i) $y^{\psi}(x,s) \leq x$: the project owner cannot be required to pay more than the profits produced to the social investor; (ii) $y^{\psi}(x,s) \geq 0$: the social investor's liability is limited to his initial investment; and (iii) $y^{\psi}(x,s) \leq y^{\psi}(\hat{x},s)$ for all $x \leq \hat{x}$: the security payoff is non-decreasing in cash profit.

Both (i) and (ii) are standard limited liability constraints. We motivate (iii) by considering the possibility for the project owner to report greater earnings than those produced by the investment project.[10] Given our assumption on the cost of the capital, the project owner can always supplement cash output with costless borrowing to make it appear as if he had undertaken more upfront investment. A non-decreasing contract eliminates any incentive for this type of manipulation. Since social output is measured by an independent third party, we do not restrict the contract to be non-decreasing in s. However, as we will show shortly in Lemma 2, the optimal contract is continuous in social output. Any gains from a slight manipulation in social output will therefore be relatively small.

The investment decision is made after the security has been sold to the social investor. Thus, the project owner does not internalize the impact of his investment choice on the price of the security. Consequently, the level of investment chosen by the project owner is the solution to the following optimization,

$$\max_i E_x[x|i] - E_{x,s}[y^{\psi}(x,s)|i] - i. \tag{10}$$

[9] The ability to observe the initial capital outlay k and the inability to observe incremental investment i captures the notion that undertaking a project is observable but investment in the project is not (e.g., one can observe if a factory was built, but cannot observe how much investment has been made into it).

[10] The addition of a monotonicity constraint is standard in the security design literature, e.g., Innes (1990) and Gangopadhyay et al. (2005).

Since the social investor benefits from the investment in the social project, he internalizes the impact of the security on the expected level of social output. Thus, the ex ante payoff from owning the security is equal to the expected security payoff plus the expected social good minus the upfront cost of the security,

$$E_{x,s}[y^{\psi}(x,s)|i] + E_s[s|i] - p^{\psi}. \tag{11}$$

Recall that the project owner is competitive, and, necessarily, his individual rationality constraint binds at zero, i.e., his expected payoff (9) is equal to zero. The binding constraint implies the security's price is,

$$p^{\psi} = i + k - E_x[x|i] + E_{x,s}[y^{\psi}(x,s)|i], \tag{12}$$

where i is the equilibrium level of investment. A substitution of this explicit form of the price into the social investor's payoff yields,

$$E_x[x|i] + E_s[s|i] - i - k, \tag{13}$$

which is equal to the social welfare function from (4).

All surplus generated from the interaction between the project owner and the social investor accrues to the social investor. This is because the project owner is competitive. The contract that maximizes the social investor's expected payoff, and as result total welfare, solves the following optimization problem,

$$\max_{\{i, y^{\psi}(.)\}} E_x[x|i] + E_s[s|i] - i - k \tag{14}$$

$$\text{s.t. } E_x[x|i] + E_s[s|i] - i - k \geq 0 \tag{14.1}$$

$$i \in \arg\max_{i'} E_x[x|i'] - E_{x,s}[y^{\psi}(x,s)|i'] - i' \tag{14.2}$$

$$y^{\psi}(x,s) \geq 0 \;\; \forall \; (x,s) \tag{14.3}$$

$$y^{\psi}(x,s) \leq x \;\; \forall \; (x,s), \tag{14.4}$$

$$p^{\psi} \leq a \;, \tag{14.5}$$

$$y^{\psi}(x,s) \leq y^{\psi}(\hat{x},s) \;\; \forall \; x \leq \hat{x}, \tag{14.6}$$

where (14.1) is the social investor's individual rationality constraint, (14.2) is the incentive compatibility constraint of the project owner, (14.3) is the social investor's limited liability constraint, (14.4) is the project owner's limited liability constraint, (14.5) requires that the security's price is less than a, and (14.6) requires that the security's payoff is non-decreasing in realized profit.

Before we proceed to solving the social investor's optimization in (14) we define $h(x,s,i)$ as,

$$h(x,s,i) \equiv \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)}, \tag{15}$$

which is the likelihood function for joint output. The assumption that the marginal distributions satisfy MLRP implies that $h()$ is an increasing function in x and s for all i. Furthermore, MLRP dictates that the distribution $f(x|\hat{i})$ first-order stochastically dominates $f(x|\tilde{i})$ for all $\hat{i} > \tilde{i}$, i.e., density is shifted from lower states to higher states. It is also true that $\int_0^\infty f_i(x|i)\,\mathrm{d}x = 0$ for all i; that is, shifting the density is a zero-sum game. The expression we call $h(x,s,i)$ in (15) is a normalization: the changes in density are divided by the densities themselves, $f_i(x|i)/f(x|i)$ and $g_i(s|i)/g(s|i)$. The normalization provides a measurement of relative density increases (or decreases) across states, where each state is a unique combination of x and s. A positive value of $h(x,s,i)$ implies a relative increase in density in that state. From an incentive perspective, providing the entire cash flow, x, to the project owner when $h(x,s,i)$ is positive and forcing him to pay it out when the function is negative maximizes ex ante investment incentives. That is, an all-or-nothing security construct with a threshold of $h = 0$ maximizes the project owner's incentive to invest in the social project. An all-or-nothing security construct, however, violates our requirement that the security's repayment be non-decreasing in x. We refer the reader to Appendix B, where we consider the optimal security construct in the absence of that constraint. The following two lemmas characterize the optimal security construct that solves the social investor's optimization in (14).

Lemma 1. *For any given s, the security which maximizes total welfare (given constraints 14.1-14.6) and which is non-decreasing in x is a debt contract with respect to x.*

Lemma 2. *The optimal security which satisfies constraints 14.1-14.6 and which is non-decreasing in x is a debt-like security in which the par value of debt decreases in s. This security is continuous in s, and the par value of debt, $\overline{x}(s,i,h^-)$, in any cross section of s is implicitly defined by,*

$$h(x,s,i)|_{x=\overline{x}(s,i,h^-)} = h^-, \tag{16}$$

for some $h^- < 0$.

According to Lemma 1, for any cross section in (x,s) in which s is fixed, the optimal contract resembles a debt contract. That is, for low values of x below some threshold (the so called "par value" of debt which depends on s), the security pays all cash flows to the investor. Above this threshold, the security pays the par value to the investor with the owner retaining the residual cash flow. In Appendix B we consider the optimal non-monotonic security and demonstrate that optimal investment incentives are achieved by rewarding the project owner for output realizations which are indicative of high initial investment and punishing the project owner otherwise. The same intuition leads debt to be optimal here. For a given realization of s, low realizations of x indicate low initial investment. Given limited liability, the debt contract penalizes the project owner as much as possible when x is below the par value by transferring all project profits to the social investor. When x is above the par value, the optimal security rewards the project owner as much as possible without violating monotonicity. That is, the security repays the social investor just the par value and assigns all residual cash flows to the project owner.[11] Furthermore, because our non-decreasing constraint applies only to x, the par value of debt for each cross section of s can be evaluated in pointwise fashion.

Additionally, Lemma 2 dictates that the threshold is non-positive. A lower threshold makes the contract more affordable by allowing the project owner to keep a greater share of x in more states. This is valuable due to the social investor's budget constraint, as it necessarily lowers the security's upfront price. However, lowering the threshold may also has an adverse effect on ex ante investment incentives: it allows the project owner to keep a greater share of the project's realized cash, even if outputs are suggestive of low investment. As the threshold decreases, the contract becomes increasingly affordable, but investment incentives may be tapered.

Definition 1. *We define the security characterized in Lemmas 1 and 2 as a Social Impact Guarantee (SIG). That is, a SIG is a debt-like contract sold by a project owner to a social investor. The security's repayment schedule requires that the par value of the contract be decreasing with the measured social output.*

Definition 1 is the heart of our paper: a Social Impact Guarantee, as defined, better aligns the incentives of the project owner and social investors, and mobilizes social capital. The security is non-decreasing in cash profit x, and, as mentioned previously, is continuous in realized social output s. As such, the incentive to manipulate measured cash profit is suppressed and any small manipulation of realized social production has a small effect on repayment. It is also important that a SIG's par value decrease with the realized social output. If it did not, that is, if it resembled

[11] In general, any non-debt-like contract which satisfies limited liability, monotonicity, and is a revenue neutral (i.e., the upfront price is unchanged) necessarily includes a smaller penalty for sufficiently low realizations of x and a smaller reward for sufficiently high realizations of x when compared with our debt-like contract. Innes (1990) considers a one dimensional setup where the project's only output is cash. He demonstrates that any revenue neutral deviation (i.e., equal expected repayment to investors) from a debt contract to another monotonic security induces lower ex ante incentives.

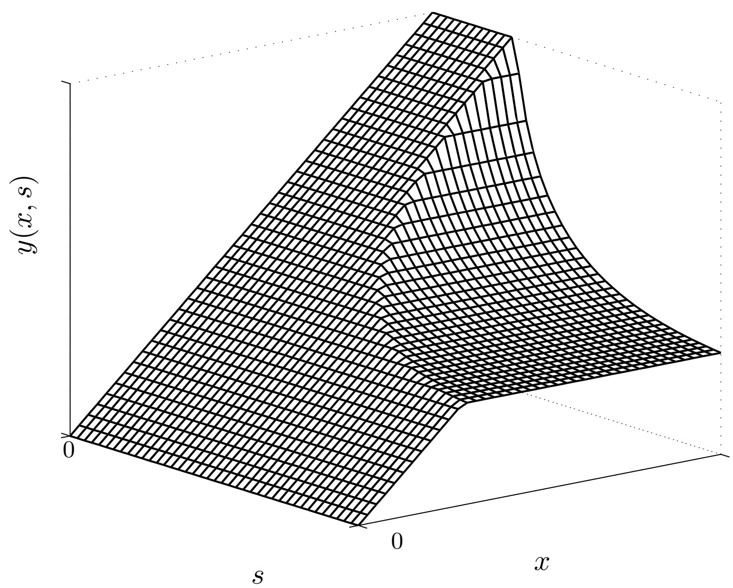

Figure 1: The Social Impact Guarantee is non-decreasing in project profit and is a debt-like security in which the par value of debt declines as measured social output increases.

a standard debt contract with a fixed par value, the contract would adversely affect incentives. In fact, the project owner's incentive to invest in the project is lower with a standard debt contract than in the absence of it. A visual depiction of the SIG is illustrated in Figure 1: for each cross section of s, the contract resembles a debt contract and the par value is decreasing with s.

3 Concluding Discussion

We conclude by discussing considerations for the practical implementation of the SIGs.

3.1 SIGs and Atomistic Investors

Social good is a non-rival public good, and, as such, free-riding potentially impedes individual social investors from participating in the sale of a SIG. In our earlier analysis, we informally treat social utility as if there is a single social investor whose value for social output is equal to the total social surplus generated. In practice, this simplification is problematic since individual social investors do not generally internalize the benefit of social output to all other investors. Albeit, the contract derived in Section 2.1 may still incrementally improve social welfare if social investors are willing to pay, at least in part, for the increase in social investment that results from their security. A social investor is only willing to pay if she believes that the equilibrium level of social investment depends critically on her purchase of the SIG. This, however, prohibits the sale of SIGs to small social investors.

If investors are indeed atomistic, each investor recognizes that his purchase of the SIG has a negligible impact on the project owner's investment incentives. Since each individual contract does not improve incentives for social investment, each investor is unwilling to pay upfront for any expected increase in social output. Consequently, the contracts will fail to transfer any social utility from social investors to the project owner, prohibiting the project owner's zero-profit constraint from being satisfied. As a result, for the security to successfully improve welfare, it must be sold to large social block holders who internalize (and pay for) their direct impact on social investment, e.g., the Bill and Melinda Gates Foundation and the J. Paul Getty Trust. It is worthwhile to mention that the case for large social block holders is further motivated by our analysis in Appendix B. In that Appendix, we allow the SIG holder to pledge her residual social capital, $a - p$, as an ex post payment to the project owner. We show that ex post payments from the investor to the project owner increases investment incentives and leads to a security construct that weakly dominates our design from Section 2.1.

3.2 SIGs and Renegotiation

The existence of a secondary market for trading SIGs critically depends on whether or not the contract is renegotiable. If the project owner's incentives remain unchanged when the security is resold, i.e., the contract cannot be legally renegotiated or investment has already been made (and is consequently fixed), a secondary market can exist without compromising the security's intent. This is because the original social investor continues to enjoy the benefits of greater social investment even if she resells the security. Furthermore, the secondary market value of the security to those that value social good and to those that do not is the same (the expected cash flow). If, however, the security is not renegotiation proof, i.e., either investment has not been made or investment can be liquidated at low enough cost, secondary market trading may be limited. This is evident by considering a sale to a different investor that only cares about cash profit: once the security is sold to this investor, the expected utility from owning that security is $E_{x,s}[y(x, s)|i]$. Recall that the expected profit from investment for the project owner is $E_x[x|i] - E_{x,s}[y(x, s)|i] - i - k$. Thus, the sum of utilities to the project owner and security holder is equal to $E_x[x|i] - i - k$, which is the project owner's profit maximization problem under sole-ownership in (7). Here, total profit is maximized by letting $i = i^\pi$, which can be obtained by renegotiating the security to the null contract. However, if the initial social investor anticipates that the contract will be renegotiated and that social investment will subsequently be reduced, the security's intent unravels. Thus, resale of the security to investors that do not value social good is not possible.

3.3 SIGs and the Prices of Residual Securities

The sale of a SIG may provide useful information to market participants through prices, even if a secondary market for the SIG does not exist. Security prices play an important role in aggregating investors' diverse private information (Hayek 1945). In this way, the introduction of a new security with cash flows tied directly to achievement of social output provides a new channel for firms, investors, and policy makers to acquire information related to the value of social good production. If the SIG is not renegotiable and trading is liquid, its market price provides information directly. Conversely, if a secondary market is not supported for reasons previously discussed or the SIG's price is considered stale due to infrequent trade, information may still be available. In particular, if the social project is held by a publicly traded firm, the firm's stock price provides an indirect measure of social good production: recall that the residual profit which flows to project owners is net of the repayment of the SIG. Furthermore, analyst reports on earnings forecasts and target prices may provide additional resources for teasing out the portion of firm value which reflects social good versus expected earnings.

Appendix A

Proof of Lemma 1:

To understand the optimal security that is non-decreasing in x, it is useful to first understand the optimal security construct in the absence of that constraint. In Appendix B, we provide that analysis and refer to the findings hereafter.

We now provide an intuitive proof. For a formal derivation of optimal debt, see Innes (1990). For a given realization of social output $\overline{s}$ and level of investment i, the function $h(x, \overline{s}, i)$ depends only on profit and is increasing in profit. From the proof of Lemma B1, the security which optimally trades off investment incentives and affordability maximizes repayment when $h()$ is below some threshold and minimizes repayment when $h()$ is above this threshold. When faced with the constraint that repayment be non-decreasing in x, the minimum payment above a threshold is simply the par value of debt.

■

Proof of Lemma 2:

Let $\overline{x}(s, i, \hat{h})$ denote the par value of debt for a given s, level of investment i, and threshold $\hat{h}$, and consider a contract such that $\overline{x}(s_1, i, \hat{h}) - \overline{x}(s_2, i, \hat{h}) > 0$ for some $s_1 > s_2$. Given this contract, repayment is strictly higher for all $x > \overline{x}(s_2)$ when $s = s_1$ than when $s = s_2$. However $h(x, s_1, i) > h(x, s_2, i) \quad \forall x$, so following the intuition in Lemma B1, investment incentives can be improved without impacting affordability by lowering the $\overline{x}(s_1, i, \hat{h})$ and increasing $\overline{x}(s_2, i, \hat{h})$ such that expected repayment remains the same. Thus any security in which the par value of debt increases in social output is weakly dominated by an alternative contract in which the par value of debt is decreasing in s.

The next step is to show that the monotonic (in x) security is continuous in s. First note that it suffices to show that the par value of debt is continuous in s. Since we have already established that the par value is decreasing in s, we must rule out the possibility of a discontinuous downward jump in $\overline{x}(s, i, \hat{h})$ for some s, investment i, and threshold $\hat{h}$. Consider a security with such a discontinuity at s^*. Since $h(x, s, i)$ is continuous in x and s for any i, there exists $\epsilon > 0$, x^+, and x^- such that $y(x_1, s^* + \epsilon) < y(x_2, s^*)$ and $h(x_1, s^* + \epsilon, i) < h(x_2, s^*, i)$ for all $x_1 < x^+$ and all $x_2 > x^-$. Thus, there exists a revenue neutral deviation which improves investment incentives by lowering the par value of debt for $s = s^*$ and increasing the par value for $s = s^* + \epsilon$.

■

Appendix B

B.1 Optimal Non-Monotonic Security

In this subsection we consider the setup from Section 2.1 and we solve for the optimal non-monotonic security to establish intuition for the proofs in Appendix A. Consider the social investor's problem outlined in (14) without the requirement that the payoff be non-decreasing in x. The social investor's problem is,

$$\max_{\{i,y^{\psi}(.)\}} E_x[x|i] + E_s[s|i] - i - k \tag{B1}$$

$$\text{s.t. } E_x[x|i] + E_s[s|i] - i - k \geq 0 \tag{B1.1}$$

$$i \in \arg\max_{i'} E_x[x|i'] - E_{x,s}[y^{\psi}(x,s)|i'] - i' \tag{B1.2}$$

$$y^{\psi}(x,s) \geq 0 \;\; \forall \; (x,s) \tag{B1.3}$$

$$y^{\psi}(x,s) \leq x \;\; \forall \; (x,s), \tag{B1.4}$$

$$p^{\psi} \leq a \; , \tag{B1.5}$$

where (B1.1) is the social investor's individual rationality constraint, (B1.2) is the incentive compatibility constraint of the project owner, (B1.3) is the investor's limited liability constraint, (B1.4) is the owner's limited liability constraint, and (B1.5) requires that the security's price is less than a.

Lemma B1. *The security which maximizes total welfare (given constraints B1.1-B1.5) is an all-or-nothing contract of the form*

$$y^{\psi}(x,s;h^-) = \begin{cases} x & \forall \; h(x,s,i) < h^- \\ 0 & \forall \; h(x,s,i) \geq h^- \end{cases} \tag{B2}$$

where $h^- \leq 0$ and $h(x,s,i)$ is an increasing function of x and s for any i.

Proof of Lemma B1:

Consider the constrained optimization and replace the project owner's incentive compatibility constraint with the first-order condition. The constraints and objective function are combined to form the following Lagrangian,

$$
\begin{aligned}
L(y^{\psi}, i, \kappa, \mu, \theta, \eta, \lambda) = \int_0^{\infty}\int_0^{\infty} \Bigg[& (1+\kappa)\,(x+s-i-k) \\
& + \mu \left((x - y^{\psi}(x,s)) \left(\frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - 1 \right) \Bigg] f(x|i) g(s|i) \text{ d}s \text{ d}x \\
& + \int_0^{\infty}\int_0^{\infty} \Big[\theta(x,s) y^{\psi}(x,s) + \eta(x,s)(x - y^{\psi}(x,s)) \Big] f(x|i) g(s|i) \text{ d}s \text{ d}x \\
& + \lambda \left(a - \int_0^{\infty}\int_0^{\infty} \Big[y^{\psi}(x,s) - x \Big] f(x|i) g(s|i) \text{ d}s \text{ d}x - i - k \right). \qquad \text{(B3)}
\end{aligned}
$$

We now focus on solving for the optimal security construct with point-wise maximization. The first-order conditions with respect to the security repayment are

$$
\left[-\mu \left(\frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - \lambda \right] + \theta(x,s) - \eta(x,s) = 0 \qquad \forall (x,s) \qquad \text{(B3.1)}
$$

Due to non-negativity and complementary slackness conditions for $\theta(x,s)$ and $\eta(x,s)$, condition (B16.1) yields

$$
-\mu \left(\frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - \lambda > 0 \implies y^{\psi}(x,s) = x \qquad \text{(B3.1)}
$$

$$
-\mu \left(\frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - \lambda \leq 0 \implies y^{\psi}(x,s) = 0 \qquad \text{(B3.2)}
$$

Since $i^{\pi} < i^{FB}$, project payoffs are improved by incentivizing additional social investment in excess to that which the project owner undertakes on his own. As a result, the multiplier μ is strictly positive. Let $h(x,s,i) \equiv \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)}$. Then,

$$
-\mu \left(\frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - \lambda \leq 0 \iff h(x,s,i) \geq \frac{-\lambda}{\mu} \equiv h^{-} \qquad \text{(B4)}
$$

and by MLRP, $h(x,s,i)$ is increasing in x and s for all i.

■

According to Lemma B1, all produced cash flows remain with the project owner when the function $h(x,s,i)$ exceeds the threshold h^{-}. Conversely, when x and s are small enough such that $h(x,s,i)$ falls below h^{-}, all cash flows are paid to the social investor.

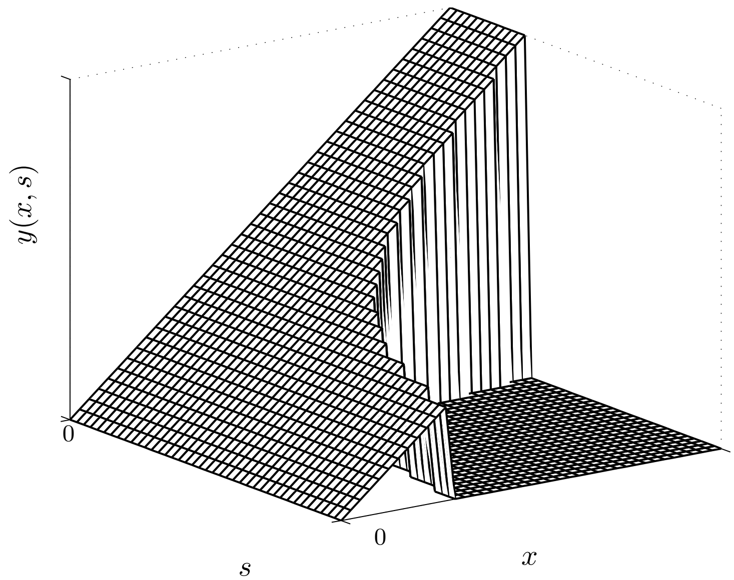

Figure 2: Optimal security is an all-or-nothing contract which pays all cash flows to the investor when profit and social output are low and retains all cash flows with the project owner when output is high.

B.2 Pledgable Assets

Our base model in Section 2 assumes $0 \leq y^{\alpha}(x,s) \leq x$: the project owner owner cannot be required to pay more than the profits produced and the social investor's liability is limited to his initial investment. However as discussed in Section 3, implementation of a SIG might require the social investor to be sufficiently large so that his investment has a direct (and non-trivial) impact on the firm owner's incentive for social investment. When SIGs are sold primarily to large social block holders, limited commitment from the point of view of social investors need not apply. That is, large foundations and investment trusts likely have adequate reputational concerns (and sufficient seizable financial assets) such that they can be expected not to renege on payments promised after the output realizations. With this in mind, we relax the social investor's liability constraint in this section, and allow her to pledge any residual social capital, $a - p^{\alpha}$, as a contractual payment to the project owner,

$$p^{\alpha} - a \leq y^{\alpha}(x,s) \leq x. \tag{B5}$$

We do not allow the security price to fall below zero ($p^{\alpha} \geq 0$). Indeed, if the price was negative, the interpretation would be that the security provides a single period loan from the project owner to the social investor.

With the exception of the conditions just outlined, we adopt the model assumptions from Section

2. The project owner's individual rationality constraint binds and the security's price is given by,

$$p^{\alpha} = i + k - E_x[x|i] + E_{x,s}[y^{\alpha}(x,s)|i], \tag{B6}$$

and the social investor's expected payoff is,

$$E_x[x|i] + E_s[s|i] - i - k, \tag{B7}$$

The contract that maximizes the social investor's expected payoff, and as result total welfare, solves the following optimization problem,

$$\max_{\{i, y^{\alpha}(.)\}} E_x[x|i] + E_s[s|i] - i - k \tag{B8}$$

$$\text{s.t. } E_x[x|i] + E_s[s|i] - i - k \geq 0 \tag{B8.1}$$

$$i \in \arg\max_{i'} E_x[x|i'] - E_{x,s}[y^{\alpha}(x,s)|i'] - i' \tag{B8.2}$$

$$y^{\alpha}(x,s) \geq p^{\alpha} - a \;\; \forall \, (x,s) \tag{B8.3}$$

$$y^{\alpha}(x,s) \leq x \;\; \forall \, (x,s), \tag{B8.4}$$

$$y^{\alpha}(x,s) \leq y^{\alpha}(\hat{x},s) \;\; \forall \, x \leq \hat{x}, \tag{B8.5}$$

$$p^{\alpha} \leq a \ , \tag{B8.6}$$

$$p^{\alpha} \geq 0 \ , \tag{B8.7}$$

here (B8.1) is the social investor's individual rationality constraint, (B8.2) is the incentive compatibility constraint of the project owner, (B8.3) is the investor's limited liability constraint, (B8.4) is the owner's limited liability constraint, (B8.5) requires that the security's repayment is non-decreasing with x, (B8.6) requires that the security's price is less than a, and (B8.7) requires that the security's price is weakly greater than zero.

Lemma B2. *If the social investor is permitted to pledge residual capital as payment to the project owner,*

$$y^{\alpha}(x,s) \geq p^{\alpha} - a, \tag{B9}$$

then the incentives induced by the contract y^{ψ} derived in Lemma 1 can be replicated with a zero price contract.

Proof of Lemma B2: Consider an alternative problem to the one outlined in (14) such that the social investor is compelled to make a payment p^{ψ} (the price of the security constructed in Lemma B1) for all (x,s) to the project owner, while the project owner's payment to the social investor remains based on observed output. Consequently, the security's net payment $y(x,s)$ contains two components: a payment $\hat{y}(x,s)$ from the project owner and the compulsory payment p^{ψ} from social investor,

$$y(x,s) = \hat{y}(x,s) - p^{\psi}. \tag{B10}$$

Now consider the contract which adds the compulsory payment to the optimal contract derived in Lemma B1. Then $y(x,s) = y^{\psi}(x,s) - p^{\psi}$. The price of this security is given from the regular owner's zero-profit constraint,

$$\begin{aligned}
p &= i + k - E_x[x|i] + E_{x,s}[y(x,s)|i] \\
&= i + k - E_x[x|i] + E_{x,s}[y^{\psi}(x,s) - p^{\psi}|i] \\
&= i + k - E_x[x|i] + E_{x,s}[y^{\psi}(x,s)|i] - p^{\psi} \\
&= 0,
\end{aligned} \tag{B11}$$

where the last equality comes from (12). In addition, the level of investment chosen by the project owner is given by

$$i \in \arg\max_{i'} E_x[x|i'] - E_{x,s}[y^{\psi}(x,s)|i'] - i' - p^{\psi}. \tag{B12}$$

Since the compulsory payment p^{ψ} does not depend on the choice of i, the equilibrium level of investment is equal to that chosen under contract $y^{\psi}(x,s)$. Finally, we show that the contract $y(x,s)$ satisfies the social investor's relaxed liability constraint,

$$\begin{aligned}
y(x,s) &= y^{\psi}(x,s) - p^{\psi} \\
&\geq y^{\psi}(x,s) - a \\
&\geq -a \\
&= p - a,
\end{aligned} \tag{B13}$$

where the last equality comes from $p = 0$.

■

From Lemma B2 any contract which solves (14) can be replicated with a related contract $y^{\alpha}(x,s)$ which satisfies (B10) and in which $p^{\alpha} = 0$. A similar logic extends this claim to any contract with a positive price and that satisfies (B10). As such, when allowing for ex post payments by social investors, we can restrict attention to only those contracts with a zero price, since any contract with a positive price can be replicated by a zero-price contract with a compulsory payment equal to the previous price.

Lemma B3. *If the social investor is permitted to pledge residual capital as payment to the project owner, there is a zero price contract, $y^{\alpha}(x,s;h^{\alpha})$, that is a debt-like security with a par value that* *is continuous and decreasing in s. The par value for a given s and level of investment i is given by,*

$$\overline{x}(s, i, h^{\alpha}) - a, \tag{B14}$$

and it can be negative for some values of s. Furthermore, the zero price contract weakly dominates $y^{\psi}(x, s; h^{-})$ outlined in Lemma 1.

Proof of Lemma B3: The zero price contract allows the social investor to pledge his entire capital budget a as a contractual payment. The zero price contract requires the addition of an individual rationality constraint for the project owner (in Section 2.1 the project owner's individual rationality constraint was redundant). Therefore, the social investor's problem is,

$$\max_{\{i, y^{\alpha}(.)\}} E_x[x|i] + E_s[s|i] - i - k \tag{B15}$$

$$\text{s.t. } E_x[x|i] + E_s[s|i] - i - k \geq 0 \tag{B15.1}$$

$$E_x[x|i] - E_{x,s}[y^{\alpha}(x, s)|i] - i - k \geq 0 \tag{B15.2}$$

$$i \in \arg\max_{i'} E_x[x|i'] - E_{x,s}[y^{\alpha}(x, s)|i'] - i' \tag{B15.3}$$

$$y^{\alpha}(x, s) \leq y^{\alpha}(\hat{x}, s) \ \ \forall \ x \leq \hat{x}, \tag{B15.4}$$

$$y^{\alpha}(x, s) \geq -a \ \ \forall \ (x, s) \tag{B15.5}$$

$$y^{\alpha}(x, s) \leq x \ \ \forall \ (x, s), \tag{B15.6}$$

where (B15.1) is the social investor's individual rationality constraint, (B15.2) is the project owner's individual rationality constraint, (B15.3) is the incentive compatibility constraint of the project owner, (B15.4) requires that the security's repayment is non-decreasing with x, (B15.5) is the investor's limited liability constraint, and (B15.6) is the owner's limited liability constraint. It is helpful to note that the constraint in (B15.2) almost surely binds because of the project owner is competitive.

In the same way that it was useful to understand the optimal non-monotonic contract for the proof of Lemma 1, consider the constrained optimization outlined in (B15) without the constraint in (B15.4). A replacement of the project owner's incentive compatibility constraint with the first-order condition allows the constraints and objective function to be combined to form the following

Lagrangian,

$$L(y^\alpha, i, \kappa, \mu, \theta, \eta, \nu) = \int_0^\infty \int_0^\infty \Bigg[(1+\kappa)(x+s-i-k)$$
$$+\mu\left((x - y^\alpha(x,s))\left(\frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)}\right) - 1\right)\Bigg] f(x|i)g(s|i)\ \mathrm{d}s\ \mathrm{d}x$$
$$+\int_0^\infty \int_0^\infty \Big[\nu\,(x - y^\alpha(x,s) - i - k)\Big] f(x|i)g(s|i)\ \mathrm{d}s\ \mathrm{d}x$$
$$+\int_0^\infty \int_0^\infty \Big[\theta(x,s)\,(y^\alpha(x,s)+a) + \eta(x,s)(x - y^\alpha(x,s))\Big] f(x|i)g(s|i)\ \mathrm{d}s\ \mathrm{d}x \quad \text{(B16)}$$

We now focus on solving for the optimal security construct with point-wise maximization. The first-order conditions with respect to the security repayment are

$$\left[-\mu\left(\frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)}\right)\right] - \nu + \theta(x,s) - \eta(x,s) = 0 \qquad \forall (x,s) \quad \text{(B16.1)}$$

Due to non-negativity and complementary slackness conditions for $\theta(x,s)$ and $\eta(x,s)$, condition (B16.1) yields

$$-\mu\left(\frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)}\right) - \nu > 0 \implies y^\psi(x,s) = x \quad \text{(B16.1)}$$
$$-\mu\left(\frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)}\right) - \nu \le 0 \implies y^\psi(x,s) = -a \quad \text{(B16.2)}$$

Since $i^\pi < i^{FB}$, project payoffs are improved by incentivizing additional social investment in excess to that which the project owner undertakes on his own. As a result, the multiplier μ is strictly positive. Let $h(x,s,i) \equiv \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)}$. Then,

$$-\mu\left(\frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)}\right) - \nu \le 0 \iff h(x,s,i) \ge \frac{-\nu}{\mu} \equiv h^\alpha \quad \text{(B17)}$$

and by MLRP, $h(x,s,i)$ is increasing in x and s for all i.

A similar discussion to the proofs of Lemma 1 and Lemma 2 imply that the optimal non-decreasing security is a debt-like contract where the par value in any cross section of s is given by $\overline{x}(s,i,h^\alpha) - a$ for some h^α. Furthermore, $\overline{x}(s,i,h^\alpha) - a$ is continuous in s.

■

The optimal contract with the relaxed social investor liability constraint is again a debt-like contract. However in this case, $p^\alpha = 0$, and the social investor subsidizes high output realizations by paying the firm owner the full social budget when $h^{\alpha}(x, s, i) > h^{\alpha}$ and she receives the par value of the contract. It is important to note that the net payment may be negative,

$$\overline{x}(s, i, h^{\alpha}) - a < 0 \text{ for some values of } s \tag{B18}$$

By subsidizing high output, the social investor is able to elicit even greater incentives for social investment. Since the span of contracts considered in the relaxed problem nests those considered in (14), investment incentives are at least as great when the social investor can commit to ex post subsidies as they are when liability is limited to upfront investment.

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