An empirical index of Knightian uncertainty

Abstract

We develop an empirical proxy for Knightian uncertainty (ambiguity) regarding the distribution of future stock returns. Stock option implied volatilities across multiple strike prices are used to estimate the mean divergence among implied probability distributions. Our ambiguity measure captures the variation of investor expectations regarding the underlying probability distribution of future stock returns. Portfolios containing stocks in the lowest ambiguity quintile outperform stocks in the highest quintile by 0.80 percent per month, or 10.04 percent annually. We document a negative return-ambiguity relation that cannot be explained by established asset pricing factors.

JEL classifications: G10, G13

Keywords: Knightian uncertainty; Ambiguity; Stock returns; Implied volatility

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1. Introduction

Under the expected utility paradigm, the difference between the certainty equivalent value of a lottery and its expected value is explained by a nonlinear utility function over outcomes (wealth). Expected utility assumes that decision makers process probabilities in a linear manner and suggests that risk and risk aversion are the only parameters that determine expected returns. However, Knight (1921) makes the distinction between "risk" (which can be quantified using probabilities) and "unmeasurable uncertainty". Expected utility does not allow for such a distinction, necessitating a theory of decision making in the absence of objective probabilities. Savage (1954) argues that even when the probabilities are unknown "a person who succeeds in conforming to the principles of coherence" will "behave in accordance with Bayes' theorem as applied to his personal probability measure." As a result the decision maker maximizes expected utility using his own subjective probabilities. Savage's "subjective expected utility" (1954) is challenged in the experimental study of Ellsberg (1961), who finds that decision makers prefer gambles with known risk probabilities over those with ambiguous probabilities. The well-known "Ellsberg paradox" demonstrates that decision makers' preferences may not conform to expected utility theory. Subjective expected utility cannot explain the observed preferences because it is not possible to infer probabilities that are meaningful and consistent for expected utility maximizers.

We examine investors' preferences towards ambiguity using data for a cross-section of stock option implied volatilities on a monthly basis. Investor perceptions regarding future events are estimated using option implied probabilities and realized excess returns are used as a proxy for their preferences. Our empirical proxy for ambiguity is the mean divergence (MD) of probability distributions in the framework of Anderson, Hansen, and Sargent (2003), where ambiguity is measured as the "distance" among the sets of probability distributions.[1]

The main results of the study are as follows. We find that stock market investors experience lower returns on more "ambiguous" stocks. Stocks in the lowest quintiles of our ambiguity measure outperform stocks in the highest quintile by 0.80 percent per month, (10.04 percent annually). Ambiguity is strongly correlated with risk and liquidity factors. Low ambiguity stocks are generally more liquid, supporting the proposition that significant information is impounded through higher trading volumes. Ambiguity is negatively related to volatility and beta, however, indicating that our proxy contains information that distinct from these traditional asset pricing factors. In fact, our results indicate that our ambiguity proxy provides explanatory power even in the presence of a wide range of traditional risk and asset pricing measures. The monthly portfolio CAPM alpha of the lowest ambiguity portfolio is 0.66 percent. We use Fama and MacBeth (1973) cross-sectional regressions to estimate the predictive ability of our ambiguity measures and find further evidence that the ambiguity index are highly related to future returns.

Although there has been extensive research on ambiguity and agents' attitude towards ambiguity, few studies examine the impact of ambiguity on stock returns empirically. Investors in the stock market face a constantly changing information set, so objective probabilities are most likely not available on a regular basis. Thus ambiguity and agents' attitudes towards it play

[1] Recently, Izhakian (2012) proposes a measure for ambiguity which is the square root of the sum of the variance of loss and gain probabilities. We have repeated all of the tests in this paper using Izhakian's measure and find qualitatively similar results. We do not report those results here but they are available upon request.

a significant role in decision making. Brenner and Izhakian (2011) use high frequency trading data to calculate probabilities of loss and gain and find a negative relationship between ambiguity and the return of market portfolio. Diether, Malloy and Scherbina (2002) employ the dispersion in analysts' earnings forecast to proxy for differences of opinion (uncertainty over the mean) and find that stocks with higher dispersion underperform other stocks. Our results are consistent with these studies, and we provide additional contributions to the literature in that we develop a new proxy for ambiguity using all the information available in option implied volatilities across multiple strike prices. Our proxy provides additional precision that is robust to traditional asset pricing factors.

2. Relevant Literature

In the framework of von Neumann-Morgenstern expected utility, investors are theoretically able to derive a numeric utility value from their consumption and investment choices using the corresponding probabilities of potential outcomes. However, in the real world investors cannot assign exact probabilities to all states of the nature. The Ellsberg paradox confirms the appeal of Knightian uncertainty by providing empirical examples where investors' decisions in ambiguous states violate the Savage (1954) "sure thing" principle. He argues that "none of the familiar criteria for predicting or prescribing decision-making under uncertainty corresponds to this pattern of choices. Yet the choices themselves do not appear to be careless or random." Since then, economists have tried to explain decision makers' preferences, the decision making process, and pricing in presence of ambiguity. Psychologists first suggest that attitude towards risk is more complex than simply determining decision makers' feelings regarding wealth; it also depends on how they process probabilities. Kahneman and Tversky (1979) argue that expected utility is not a fully descriptive model of decision making and hence propose Prospect Theory, in which probabilities are replaced with corresponding decision weights. They propose a value function which is concave for gains and convex for losses, as well as steeper for losses than for gains.

Following Ellsberg (1961), the seminal paper of Schmeidler (1989) solves the problem by proposing a rank-dependent utility for uncertainty.[2] He axiomatizes preferences and extends expected utility by using the Choquet (1954) Integral to compute expected utility with respect to nonadditive probability. The Gilboa and Schmeidler (1989) "maxmin" expected utility model (MEU) assumes that the decision maker has a set of prior beliefs, and alternatives are assessed according to their minimal expected utility. Hansen and Sargent (2001) use robust-control theory to deal with decision makers' uncertainty about the probability distribution and extend the maxmin theory of Gilboa and Schmeidler (1989). Anderson, Hansen, and Sargent (2003) introduce "functional multiplier" utility that measures the distance between the approximating model and other models using relative entropy (Kullnack and Leibler, 1951). The decision-maker in this model has a prior guess about the true probabilities of certain events. When making decisions, she takes into account all other possible events and assigns more weight in those probabilities that are closest to her prior guess.

Recent research has been able to achieve a clearer distinction among risk, ambiguity and ambiguity attitude. In the Klibanoff, Marinacci and Mukerji (2005) (KMM) model of preferences, ambiguity about an event is equivalent to uncertainty about the probability of that event. A KMM agent's value function is evaluated by calculating the expectation of a function (φ) of expected utility over each possible distribution; if φ is linear, the agent is ambiguity

[2] See Wakker (2010) for a review on the development of rank dependent utility (RDU) and Prospect Theory.

neutral and uses expected utility, and the concavity of φ determines the degree of ambiguity aversion. This smooth model of decision making provides the decision maker's subjective belief about the uncertainty of the probability and the attitude towards this uncertainty. Further, similar concepts from risk and risk aversion can be used to analyze ambiguity and attitude towards ambiguity. Macherroni, Marrinacci and Ruffino (2011) extend the Arrow-Pratt approximation of certainty equivalence to incorporate ambiguity and find that the ambiguity adjusted certainty equivalence includes an additional term that measures the ambiguity premium. Izhakian (2012) introduces a measure of ambiguity which is four times the variance of probability of loss or gain. Brenner and Izhakian (2011) utilize the measure proposed in Izhakian (2012) to calculate ambiguity and find that market ambiguity and return are negatively correlated.

Our study uses options implied volatility data across differing levels of moneyness to construct a proxy for ambiguity and to examine the relationship between ambiguity and returns. Thus, the paper is also related to the literature that examines implied volatility skew. There is significant research into the effects of volatility skew on equity returns. Xing, Zhang, and Zhao (2010) find that stocks with relatively steep volatility skews significantly underperform those with shallower volatility skews, and this effect persists for up to six months. Yan (2011) also finds a negative predictive relation between the slope of implied volatility smile and stock returns. Cao and Han (2013) find that options prices contain a negative volatility risk premium based on underlying stock volatility. Ang, Bali, and Cakici (2012) find that "Stocks with large increases in call implied volatilities tend to rise over the following month whereas increases in put implied volatilities forecast future decreases in next-month stock returns." Finally, Dennis and Mayhew (2002) find that risk-neutral distributions derived from options prices are "more negatively skewed for stocks with higher betas, in periods of higher market volatility, and in periods when the implied density from index options is more negatively skewed." Their study also finds that trading volume and firm size are significant in explaining skew levels, but find no significant effect from leverage. Following their intuition, we examine a number of control variables that have traditionally been linked to theories of asset pricing and how the characteristics of these variables are related to our measures of ambiguity. These studies are similar to this paper in that they provide important contributions regarding the information contained in implied volatility skew. However, our study makes the additional contribution that links this information to an underlying economic phenomenon (Knightian uncertainty, or ambiguity). We also analyze the volatility of skew itself that has not been previously examined, to our knowledge. The findings regarding our measure of ambiguity can be directly related to the economic literature on ambiguity and theories from behavioral finance.

3. Data, and Summary Statistics

3.1. Raw Data

Our initial data set utilizes daily total returns and option implied volatility data from January 2005 to September 2012, obtained via Bloomberg Professional®, and includes all of the stocks included in the S&P 500 Index as of September 28, 2012. The initial sample includes data on returns as well as the constant thirty day maturity option implied volatilities for seven levels of moneyness. Bloomberg uses the mid-quotes of call and put prices, the LIBOR yield curve, and average analyst dividend estimates to derive implied volatilities for American option prices across moneyness and maturities. The mid-quotes are obtained via a "snapshot" taken at 3:45 PM ET to avoid the "noise" that accompanies the traditional 4:00 PM ET closing prices. An implied forward price for the underlying asset is derived using at-the-money options and put-call parity, and European option prices are then calculated using these inputs for a fixed thirty day maturity. The final implied volatility for each maturity and strike price (call/put average) is then obtained by inverting the Black-Scholes formula using the preceding inputs. Options with less than ten days to expiration are excluded from any of the calculations. In order to create the volatility surface, the database uses a non-parametric interpolation (in variance space) across strikes to calculate IVs at fixed moneyness levels, and uses a Hermite cubic spline interpolation in total implied variance space to interpolate in time to maturity.

Bloomberg provides implied volatility for at-the-money options (ATM) and options with strike prices of 80%, 90%, 95%, 97.5%, 102.5%, 105%, 110% and 120% of the underlying stock closing price. We do not use options at the 80% and 120% moneyness levels because those options are relatively illiquid, most likely because the probability of a 20% variation in 30 days is minimal. The seven remaining IVs are used to calculate the risk-neutral probability of each moneyness region (based on the probability of each option expiring in the money). In addition, we eliminate the days in which two or more identical implied volatilities are reported for the option "tails", since those options were most likely not traded that day and Bloomberg interpolates a value from the next closest strike price. Finally, any month with less than 12 daily observations is removed, and we use this final sample to calculate monthly excess returns, ambiguity measures and the control variables.

3.2. Calculation of Probabilities and Ambiguity measures

In the framework of the Black-Scholes option pricing model, the price of a stock is assumed to follow a geometric Brownian motion process with constant drift and volatility. Under the model assumptions, at a given point in time, option volatility should not vary due to differences in strike price and/or maturity. But when the Black-Scholes formula is inverted to derive implied volatility using actual option prices, implied volatilities of options on different strike prices vary with the strike price (at any fixed maturity) and option maturity (at any fixed strike price). Assigning several volatilities to the same underlying asset is a violation of the constant volatility assumption in Black-Scholes formula. However, the observed volatility "smile" (or "smirk" or "skew") may indicate investor expectations of extreme price variation (either because of increased uncertainty in the market or a particular stock). Under this circumstance, when the prices of out-of-money put options imply higher volatility, investors are assigning larger probabilities to the extreme downside events.[3]

In order to estimate the variation in daily implied distributions, we make three main assumptions. First, we assume that investors use some form of the Black-Scholes option pricing formula to price equity options. Second, prices of out-of-the-money options reflect investor concerns about extreme market events. And lastly, shifts in the implied volatility skew (smile) over time are a result of changes in beliefs regarding the probability of extreme events. Implicit in the assumptions above is that there are negligible transaction costs. While transaction costs may impact option prices, they do not affect the general quality of our ambiguity index because we are interested in the variation of the implied volatility skew. We are also concerned about liquidity constraints, so our sample only includes options on firms in the S&P 500 Index that represent the most liquid U.S. equity options. The use of S&P 500 data limits the interpretation of our results to those of the largest firms in the U.S. economy, but these firms represent approximately 70 percent of U.S. market capitalization. The results therefore cannot be

[3] Polkovnichenko and Zhao (2013) find support for overweighting the probability of extreme events by using option prices.

generalized to all listed equities. The sample is limited to options with moneyness levels between 90-110% since options outside of this range are less liquid at the thirty day maturity.

Our approach to the calculation of ambiguity is analogous to the calculations for risk in many empirical studies. Risk (as proxied by realized volatility and/or Beta) is usually calculated using realized returns over a monthly basis. Similarly, we use daily implied probabilities to calculate a monthly measure of ambiguity. For each day, we estimate the implied probabilities between two moneyness levels using the implied volatility (hereafter IV) of the corresponding moneyness levels. At-the-money IV is used to calculate the implied probability of two outcomes, the probability of gaining or losing up to 2.5%:

$$P(0.975 < x < 1) = P(1 < x < 1.025) = \Phi\left(\frac{1.025-\mu}{\sigma_1}\right) - \Phi\left(\frac{1-\mu}{\sigma_1}\right), \qquad (1)$$

where σ_1 is the monthly implied volatility of the at-the-money option (expressed as an annualized standard deviation), μ is mean return that is assumed to be one (expected next month price = current price) and $\Phi(\bullet)$ is the cumulative distribution function for the normal distribution:

$$\Phi\left(\frac{x-\mu}{\sigma}\right) = \int_0^x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (2)$$

We next calculate the probability of the option expiring in the next probability "region" using the next IV; $P(1.025 < x < 1.05)$, $P(1.05 < x < 1.1)$ and $P(x > 1.1)$ are calculated using IVs of options with 102.5 percent, 105 percent and 110 percent moneyness levels, respectively. Likewise, the probabilities on the loss side of the distribution, $P(0.95 < x < 0.975)$, $P(0.90 < x < 0.95)$ and $P(x < 0.90)$ use IVs of the options with 97.5 percent, 95 percent, and 90 percent levels of moneyness, respectively. As a result, the estimated discrete density is a mixture of normal distributions.

At this point, we have eight forward-looking implied probabilities for each day, and these probabilities form our approximate daily implied distributions. The eight calculated implied probabilities provide the inputs to calculate our index of Knightian uncertainty. In our sample of 823,755 daily observations, average total implied probability is 101.70% with a standard deviation of 1.84%. Depending on the shape of the volatility skew, total probabilities may be subadditive or superadditive. The implied probability of loss is on average more than 3 percent higher (52.49 percent probability of loss versus 49.21 percent probability of gain), mainly due to the higher implied probability of extreme losses (volatility skew is generally negatively sloped). Assigning large probabilities to large losses was first documented in Jackwerth and Rubinstein (1996) who find a shift in probability beliefs since the 1987 stock market crash; subsequent to the crash, investors become more concerned with the probability of extreme downside losses and the volatility skew becomes more pronounced. We document similar results surrounding the recent financial crisis of 2007–2008. In our pre-crisis sample, the difference between implied probabilities of losing or gaining more than 10.04 percent over a 30 day period is less than 3 percent, but after the crisis this number gradually increases to more than 5 percent. Finally, it is notable that the summation of extreme probabilities in the "crisis" period from November 2008 to May 2009 accounts for more than 50 percent of total probabilities (versus a mean of 29 percent for the whole sample) and shows that investors were expecting extreme price variations during the crisis. In the same period, the average implied probability of small price variation (between -2.5 and 2.5 percent) drops sharply to just about 10 percent (versus a mean of 23 percent for the whole sample).

Having estimated risk-neutral implied probabilities using option implied volatilities, we find motivation for our empirical proxy of ambiguity in the studies of Hansen and Sargent (2001) and Anderson, Hansen, and Sargent (2003). These studies introduce “multiplier utility” (MU) by incorporating robust-control theory to decision making in the presence of model uncertainty. Strzalecki (2010) introduces a system of axioms regarding MU, whereby agents use robust-control theory when they face model uncertainty and assess the following value function:

$$V(x) = \min_p \int u(x)dp + \theta R(p \parallel q) \ , \tag{3}$$

where ambiguity aversion increases with $\theta^{-1}$and $R(p \parallel q)$ is relative entropy (Kullback-Leibler (1951) divergence) of p (other plausible probabilities) with respect to q (agents’ best guess), and measures the distance between two probability distributions. At $\theta = \infty$, the agent has full confidence in her reference guess and sets $p = q$, thus she uses expected utility. The MU agent is always averse to ambiguity, but our main goal is to test investors’ preferences towards ambiguity. Consequently, we do not assume that decision makers use the above value function to assess prospects; we simply utilize Kullback and Leibler’s divergence to capture the divergences among implied distributions as a proxy for ambiguity. Specifically, because the implied distributions in our sample are discrete, we calculate the divergence between two implied distributions by computing:

$$\mathrm{J}_{\mathrm{P(s)_t}}(\mathrm{P(s)_{t-1}}) = \sum_{\mathrm{s}} \mathrm{P(s)_t} \ln \frac{\mathrm{P(s)_t}}{\mathrm{P(s)_{t-1}}} + \sum_{\mathrm{s}} \mathrm{P(s)_{t-1}} \ln \frac{\mathrm{P(s)_{t-1}}}{\mathrm{P(s)_t}}$$

$$= \sum_{\mathrm{s}} (\mathrm{P(s)_t} - \mathrm{P(s)_{t-1}})(\ln \mathrm{P(s)_t} - \ln \mathrm{P(s)_{t-1}}) \ , \tag{4}$$

which is symmetric and non-negative. The above equation measures the distance between two distributions and is correlated with variations in variance and all higher moments. We take the average of these daily distances to form our proxy for monthly ambiguity (*mean distance or divergence)*, hereafter MD:

$$\mathrm{MD} = \frac{1}{\mathrm{D}} \sum_1^{\mathrm{D}} \mathrm{J}_{\mathrm{P(s)_t}}(\mathrm{P(s)_{t-1}}) \tag{5}$$

where D is the number of days in a month. Our construction of the MD index is comparable to the calculation of volatility in a time-series setting: the more returns vary over a time period, the higher is volatility over that period. Similarly, the higher the variation in probabilities over a time period, the greater are the distances calculated using equation (4) and the larger is the MD index. Volatility captures 'return variation' while the MD index captures 'distribution variation' and can be interpreted as "realized ambiguity." If investors consistently assign the same implied return distribution to a certain stock, the MD value will be zero, but if investor perception about return distributions change each day, the MD index will be a positive number and the size of the index depends on the amount of change in each part of the distribution. Distribution variation can be attributed to projected changes in risk, skewness and all of the higher moments of expected return.

Because ambiguity deals with the difficulty that investors face in estimating the probabilities of different outcomes, the question becomes: "Is it really harder for investors to estimate the return distribution for stocks with a higher MD index?" The answer to this question is similarly difficult as the answer to the parallel question regarding risk "Are stocks with higher realized volatility more risky?" This important question has led to various measures of risk, including "physical" or "realized" volatility, beta, upside and downside risk, Value-at-Risk, daily price range (Parkinson's Extreme Value Method, 1980), and more recently the Economic Index of Riskiness (Aumann and Serrano, 2008) and the Operational Measure of Riskiness (Foster and Hart, 2009). The proposed index of Knightian uncertainty in this study satisfies one main requirement to qualify as a measurement of ambiguity; the more the probabilities vary over time, the higher is the MD index. The relation between realized ambiguity and actual ambiguity is beyond the scope of this study.

The top panel of Figure 1 displays the distribution of our ambiguity measure. Note that the MD index can be any positive number. For example, if the probability of a part of the distribution drops sharply to zero, MD will tend to infinity. The bottom panel of Figure 1 shows that the proxy for ambiguity increases sharply during sudden shocks to the stock market. The first sharp increase occurs at the peak of the financial crisis (October-November 2008) when Lehman Brothers collapsed and AIG was bailed out. The next two sudden increases occur in June 2010 when the Euro plunged to a four-year low because of concerns over rising sovereign debt and slow growth, and then during September-October 2011 when investors were facing the uncertainty surrounding the European debt crisis. The European Union announced an agreement on October 27th, 2011 in response to debt and banking problems, and the proxy declines sharply in the month following this announcement.

3.3. Control variable sorts and expected excess return

In later sections, we test the impact of ambiguity on returns to examine return predictability (expected returns). Thus, we define expected excess return as the following month return less the risk free rate, which we obtain as the one-month T-bill rate supplied by the St. Louis Federal Reserve Bank.

In this section we calculate monthly risk variables and other factors known to be empirical anomalies of the Sharpe (1964), Lintner (1965 a, b) and Mossin (1966) Capital Asset Pricing Model (CAPM). We will use these pricing factors in our cross-sectional tests to examine whether the return-ambiguity relationship can be explained by these variables. These variables include beta, volatility, size, book-to-market, momentum, short-term reversal, skewness, kurtosis and turnover. As in Section 3.1, the raw data for these calculations is obtained from Bloomberg

Professional® and includes all of the stocks included in the S&P 500 Index for which the data is available.

Beta is estimated by regressing weekly returns on the S&P 500 Index using the prior two years of data on a rolling basis. Volatility is computed using the methodology of French, Schwert, and Stambaugh (1987): $V_{pt} = \sum_{d=1}^{D_t} r_{pd}^2 + 2\sum_{d=2}^{D_t} r_{pd} r_{pd-1}$, where r_{pd} is the daily return and D_t is the number of days in month *t*. It is well known that returns on small stocks are higher than large stocks, and our measure of firm size is the natural logarithm of each stock's market capitalization, in the manner of Banz (1981), Fama and French (1992), and Fama and French (1996). The ratio of book value to market capitalization (book-to-market) is known to be positively correlated with returns, as in Rosenberg et al (1985), Fama and French (1992), and Fama and French (1996). Jegadeesh and Titman (1993) find that buying winners and selling losers (based on last year's returns) produce abnormal returns over the following three to twelve months (momentum). We include the momentum effect by calculating the prior twelve month return from time *t-12* to *t-1*. Jegadeesh (1990) finds a short term reversal (previous month's return) in stock returns. The relationships among skewness, co-skewness, and returns are studied in several papers, including Rubenstein (1973) and Kraus and Litzenberger (1976), who extend the mean-variance capital asset pricing model to include the impact of skewness (they find a preference for positive skewness) on expected returns. Harvey and Siddique (2000) show that systematic skewness is associated with a risk premium. In a recent study, Xu (2007) finds that the skewness of stock returns is positively correlated with contemporaneous returns. We calculate skewness (the third order centralized moment) using:

$E(x-\mu)^3 / \sigma^3 = \sum_{i=1}^{n} \frac{(x_i-\bar{x})^3}{n} / \hat{\sigma}_x{}^3$, where $\bar{x}$ and $\hat{\sigma}_x$ are the sample mean and standard error of daily returns. Following Xu (2007) we use a one year window for stocks with at least 40 observations. Fang and Lai (1997) note that investors are compensated for bearing kurtosis risk via excess returns. We calculate kurtosis using $E(x-\mu)^4 / \sigma^4 = \sum_{i=1}^{n} \frac{(x_i-\bar{x})^4}{n} / \hat{\sigma}_x{}^4$, where the variables are defined as above. Amihud and Mendelson (1986) argue that investors are concerned with returns net of transaction costs, thus illiquid assets should have higher returns. Datar et. al. (1998) confirms the return-liquidity relationship using stock turnover. Their proxy for liquidity is the number of shares traded in a month divided by the number of shares outstanding. We use this ratio of monthly total stock turnover to total shares outstanding to proxy for liquidity in that month.

Table 1 presents the average cross-sectional (Spearman) correlations as well as the summary statistics for MD, excess returns and the control variables. All of the variables in our sample (except implied probabilities) are truncated at the extremes by 0.5 percent to remove the impact of outliers, resulting in 31,480 firm-month observations. The mean value for MD is 0.42% with a standard deviation of 0.37%. MD is negatively correlated with next month returns; in fact, it has the highest absolute correlation with future returns among all of the control variables, although all of these correlations are quite low. MD is also negatively correlated with volatility, beta, and turnover, indicating that it captures incremental information regarding future returns in the presence of these asset pricing factors.

Our first set of analyses involves sorting the stocks in the sample into five quintiles based on MD, then examining the characteristics of these portfolios relative to the control variables. The first set of control sorts is presented in Table 2. Each month the sample is sorted into five portfolios based on low (portfolio 1) to high (portfolio 5) ambiguity, as proxied by MD, and next month return is calculated for each of the five equally-weighted portfolios. Next month excess returns of the Low minus High ambiguity portfolios are 0.80 percent per month, indicating that the returns of low ambiguity portfolios are 0.80 percent higher per month during our sample, or 10.04 percent on an annual basis. The results for the volatility characteristics of the portfolios are consistent with this result, since the lower ambiguity portfolios coincide with those of higher risk, as measured by volatility. Lower ambiguity stocks also seem to possess higher sensitivity to market returns, as evidenced by their higher Betas, which is parallel to the results based on total risk presented previously. When considering liquidity, as proxied by ratio of shares traded to shares outstanding (turnover), the lowest ambiguity stocks generally have higher turnover. If information is imparted through trading activity, ambiguity should be attenuated through larger trading volumes, and Table 2 demonstrates that low ambiguity stocks experience higher turnover. Several theoretical and empirical studies find that information is imparted through trading activity,[4] and information quality is recognized to be an important determinant of ambiguity in Epstein and Schneider (2008), who find that investors dislike assets with poor information quality. Note that in Table 1, the variables that are most highly correlated with the ambiguity index are turnover and beta. In line with the high correlation between liquidity and ambiguity, Table 2 shows that liquidity changes significantly among different ambiguity quintiles, consistent with the prediction of Epstein and Schneider (2008). Moreover, higher liquidity is known to be associated with higher volatility.[5]

Low ambiguity stocks are also generally smaller in size, and this may be a result of the fact that even though the firms are smaller, they exhibit higher turnover that mitigates the size effect. Size and turnover are negatively correlated, and the most ambiguous firms are the large illiquid ones. We note, however, that all of the firms in our sample (S&P 500) are large

[4] Glosten and Milgrom (1985) , Admati and Pfleiderer (1988), Malinova and Park (2010).
[5] See, for example, French and Roll (1996), Haugen (2010), Avramov, Chordia, and Goyal (2006), and Malinova and Park (2011).

capitalization companies. Consistent with our prior results based on risk, the sorts confirm that low ambiguity stocks are generally less leptokurtotic and more positively skewed, consistent with prior literature showing preferences for low kurtosis and positive skewness.[6] Book to Market is not significantly different for the extreme portfolios.

In order to preclude the possibility that we are simply picking up effects unrelated to ambiguity, we form three-way portfolio sorts using MD and the six variables that demonstrate the highest correlation with MD in Table 1. Because MD has the highest correlations with beta and turnover, Panel A of Table 3 includes average next month returns in three-way sorts by risk (Beta), liquidity, and ambiguity (MD) to rule out the possibility that the relationship between MD and returns can be explained by risk and liquidity. Stocks are first sorted into three terciles based on their beta; within each beta portfolio stocks are further divided into three portfolios by turnover, and finally each of these nine portfolios are sorted into three more portfolios using ambiguity. We find that the differences between low and high ambiguity terciles are positive and significant indicating that we are not capturing a risk or liquidity effect. In addition, the results indicate that the effect of ambiguity is strongest for high turnover stocks with smaller betas. Thus investors receive a higher premium for low ambiguity stocks in the presence of lower risk and higher illiquidity. Similarly, Panel B presents three way sorts based on size, book-to-market and ambiguity to test whether size and/or value effects are driving the results. Once again, the low-high return difference is positive for all categories, indicating the incremental contribution of ambiguity in the explanation of excess returns. Finally, Panel C categorizes portfolios by skewness, kurtosis and ambiguity and we find a positive difference between returns of the stocks in the lowest ambiguity quintile and those in the highest quintile which indicates that our MD

[6] See, for example, Friend and Westerfield (2012), Harvey and Siddique (2000), Kane (1982), Kraus and Litzenberger (1976), and Scott and Horvath (2012).

measure is not simply capturing higher moments of the return distribution. Overall, Table 3 demonstrates that average return increases as ambiguity decreases and that we are capturing incremental ambiguity effects in the prediction of future returns over and above those explained by other factors.

4. Methodology and Results

4.1. Ambiguity and asset pricing model alphas

In order to assess whether or not the return difference among portfolios formed based on ambiguity is significant in traditional asset pricing models, we conduct Fama and French (1996) and Carhart (1997) regressions for each of our five equally weighted portfolios ranked based on the level of MD index. Fama and French (1996) demonstrate that CAPM is enhanced using a three-factor model to capture certain empirical anomalies. Carhart (1997) adds another term (momentum, or prior one year returns), specifically the difference between returns of a portfolio of previous winners minus return of a portfolio of previous losers ("up minus down," or UMD), and accounts for the momentum effect of Jegadessh and Titman (1993). Monthly returns of each portfolio are used in three asset pricing models: CAPM, the Fama-French three-factor model and the Fama-French-Carhart four-factor model. We seek to test whether the alphas in these equations are significantly different from zero across ambiguity quintiles. The additional monthly Fama-French (1996) factors for SMB, HML, as well as Carhart's (1997) momentum are obtained from the website of Kenneth French.[7]

[7] *http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html*

The results in Panel A of Table 4 show that the alphas for all three models using returns on portfolios sorted by MD are significantly different from zero for the lowest ambiguity quintiles. The monthly CAPM alpha of the lowest ambiguity portfolio is 0.66 percent, or 8.2 percent annually. These alphas are also significant for the second lowest ambiguity quintile, although the economic and statistical significance declines. The CAPM betas and the coefficients on SMB are strongly statistically significant for all of the quintiles, while those on HML and UMB are only sporadically significant. Additionally, as shown in Panel B, the difference in alphas between the low and high ambiguity portfolios is between 0.54 and 0.62 percent monthly (6.67 and 7.81 percent annually). The table provides evidence that portfolios sorted on the ambiguity index exhibit an increasing trend in alphas, showing significant excess returns in the lower ambiguity quintiles that is not explained by traditional asset pricing models.

4.2. Ambiguity and asset pricing variables

In order to examine the relation between ambiguity and other stock characteristics, we implement Fama and MacBeth (1973) monthly cross-sectional regressions of MD as the dependent variable at time *t* on other pricing factors at time *t-1*. We regress the ambiguity proxy on each of the control variables described in the previous section. The results are presented in Tables 5. In these predictive regressions, the average distance between implied distributions over the prior month (MD) is negatively correlated with both volatility and Beta (coefficients of -0.038 and -0.003, respectively, both significant at the one percent level) and volatility and Beta explain up to nine percent of the variation in MD.

Information flow is closely related to trading activity, and there is a strong negative correlation between ambiguity and turnover. Liquidity is one of the most important determinants of the ambiguity index, and the strong negative ambiguity-liquidity coefficients in Tables 5 (-0.008) indicate that the quality of information is higher for liquid stocks and information flows through trading, which results in lower ambiguity. In the last columns of Tables 5, we include all of the factors and this estimation explain seventeen percent of future variation in MD.

4.3. Ambiguity and returns

Although the previous sections have provided information that ambiguity provides incremental information regarding stock returns, we are interested in how investors evaluate ambiguity and impound this information (or lack thereof) into stock returns. We use Fama and MacBeth (1973) cross-sectional regressions to estimate investors' attitude towards ambiguity. Because the ambiguity measure is calculated using thirty day maturity implied volatilities (which ostensibly contain information regarding future thirty day volatility and returns) all of the independent variables are lagged one period, so the return in month *t* is explained by ambiguity and other independent variable(s) in month *t-1*.

Table 6 examines the predictive ability of our two ambiguity measures in the presence of other factors known to predict returns. We first regress excess returns on the ambiguity index alone, and then add the control variables described in Section 2.3. The MD coefficients are negative and statistically significant using Newey-West (1987) autocorrelation corrected standard errors. The coefficient in the base model (1) implies that a one standard deviation increase in MD corresponds to a decrease in annual returns of 6.34 percent, which is an economically significant result. Including other risk factors and other variables that are documented as anomalies generally does not impact the statistical significance of ambiguity-return relationship; the coefficients remain negative and significant when all of the control variables are included in the regression. In fact, only ambiguity and size retain predictive ability in this estimation. The regression results confirm the initial evidence provided by the control sorts that ambiguity is a priced factor such that firms with lower ambiguity tend to experience higher future returns.

4.5. Robustness checks

In this section, we confirm the relation between stocks excess returns and ambiguity through a series of robustness checks. We divide our sample into three time periods to test whether the observed return-ambiguity relationship holds in all sub-periods. The three time period are pre-financial crisis (2005 to 2007), crisis (2008-2009) and post-crisis (2010-2012). The results in Table 7 demonstrate that the average return of low minus high ambiguity portfolio is economically significant in all periods. The portfolios ranked using MD show a low-high annual premium of 9.64 percent in the pre-crisis period, 11.48 percent during the financial crisis and 9.38 percent in post-crisis period. Although the results are not statistically significant in the crisis period, the economic significance of the sort results is highest during this time. We also examine the ambiguity return relationship using Fama and MacBeth (1973) regressions for each period and the results are qualitatively similar, hence we do not report those results in the interest of brevity.

We also estimate the risk-ambiguity-return surface using a non-parametric regression by estimating the bandwidth using the modified Akaike information criterion (AIC) approach of Hurvich, Simonoff and Tsai (1998). First we build 25 portfolios based on beta and ambiguity and then calculate the return of each equally-weighted portfolio over the next month. The process yields 2,300 observations that are used in a non-parametric estimation and presented in Figure 2.

In line with estimations in Tables 6, the non-parametric results show a weak but positive return-beta relationship (median coefficient < 0.001). MD is strongly related to future excess returns (median coefficient = -1.70), which is even larger than our previous estimations. However, the graph also implies a weak positive return-ambiguity relation when the MD index is approximately between 0.007 and 0.015, but only about 15 percent of our observations lie in this region.

Finally, we estimate quintile (median) regressions to further control for outliers as well as a panel-fixed effects regression to control for unobserved factors that vary between firms. We omit the full results in the interest of brevity, but the relationship between returns and the proposed ambiguity index is always negative and significant in all specifications. We conclude that the ambiguity index calculated and tested in this study includes certain information that is not explained by risk or other asset pricing factors.

4.6. Possible explanations

Several studies in the literature find inconsistencies in the way people process probabilities, risk and ambiguity. For instance, people seem to overweight small probabilities (certainty and possibility effects) and underweight large probabilities. Moreover, the weighting process changes from losses to gains. Abdellaoui (2000) and Abdellaoui, Bleichrodt and L'Haridon (2008) suggest that people overweight probabilities for probabilities less than one third and from there the weighting function changes to underweighting. In Tversky and Kahneman (1992), individuals are risk seeking for gains and risk averse for losses of low probability, but they are risk averse for gains and risk seeking for losses of high probability, suggesting an inverse S-shaped weighting probability function and an S-shaped utility function.

Tversky and Fox (1995) and Gonzalez and Wu (1999) find similar results regarding the non-linearity of the probability weighting process. One of the earliest studies on the variation of risk attitude is Markowitz (1952) who hypothesizes a utility function with three inflection points, one at the origin and two on both sides of the wealth axis, suggesting that risk attitude changes four times on the wealth domain.

Likewise, studies have found mixed results regarding decision makers' preference towards ambiguity. In Ellsberg (1961), people prefer to gamble on the prospects with known probabilities, indicating ambiguity aversion. Also, procedures based on the worst case scenario such as Value at Risk and max-min optimization are widely used in practice and research. In these cases people are concerned about the worse outcome (extreme ambiguity aversion) and they maximize utility based on the worse scenario. On the other hand, most recent studies confirm that people demonstrate ambiguity seeking behavior under certain conditions. For example Bier and Connell (1994) find that people are ambiguity seekers when probabilities are positively framed and ambiguity neutral when negatively framed. They also find evidence that optimistic people show greater ambiguity seeking. Shyti (2013) finds that overconfidence leads to ambiguity seeking among corporate managers. Ambiguity attitude may depend on whether people see the situation as cooperative or competitive (Kuhberger and Perner 2003). In Pulford (2009), ambiguity-seeking depends on the degree of optimism; low optimism subjects are ambiguity averse and high optimism subjects are ambiguity lovers. He argues that "It is the presence of optimism and not the absence of pessimism that reduces ambiguity aversion," concluding that highly optimistic people feel that luck is on their side, even in the presence of a competitor. In another experimental study, Dimmock, Kouwenberg, and Wakker (2012) examine the relation between ambiguity and participation in the stock market and find that ambiguity aversion has a negative relation to participation for people who believe that returns are highly ambiguous. The level of probability of loss is documented to be correlated with ambiguity preference in Hogarth and Kunreuther (1989). The study finds that people are averse to ambiguity when the probability of loss is small but they exhibit less aversion as the chance of losing increases; they become ambiguity seekers for large probabilities of loss. Chakravarty and Roy (2009) find that people are ambiguity neutral over gains and ambiguity seeking over losses. Abdellaoui, Vossman and Weber (2005), Di Mauro and Maffioletti (1996), and Einhorn and Hogarth (1986) also find evidence of ambiguity seeking attitude and our results support these findings. In a study closely related to ours, Brenner and Izhakian (2011) document the same negative ambiguity-return relationship found here.

5. Conclusion and Implications

Under expected utility theory, investors evaluate utility based on their consumption and investment choices using the corresponding probabilities of potential outcomes. In reality, however, representative agents cannot assign exact probabilities to all states of the nature, thus Knight (1921) develops the concept of uncertainty regarding probabilities of future events (ambiguity). We calculate an empirical index for ambiguity to examine investors' preference towards ambiguity regarding future stock returns using data for a cross-section of stock option implied volatilities on a monthly basis over the period from 2005 to 2012.

The study is one of a few papers that examine the impact of ambiguity on stock returns in an empirical setting. Investors in the stock market face a constantly changing information set, so ambiguity and agents' attitudes towards it play a significant role in decision making. We find a significant negative relation between levels of ambiguity and future stock returns. Stocks in the lowest quintiles for our ambiguity measures outperform stocks in the highest quintile by 0.80 percent per month (10.04 percent annually). The monthly CAPM alpha of the lowest ambiguity portfolio is 0.66 percent (8.2 percent annually). Low ambiguity firms are generally riskier (in terms of volatility) but may experience better information quality via higher levels of trading. High ambiguity stocks are less volatile but illiquid, supporting the proposition that significant information regarding ambiguity is impounded through higher trading volumes. Fama-MacBeth regression estimations using a variety of control variables confirm initial evidence provided by control sorts. Investors are concerned about "immeasurable uncertainty" and ambiguity is a priced factor and provides explanatory power for returns even in the presence of traditional risk and asset pricing measures. We do not find a positive return-ambiguity relation in any of our specifications and our results are economically significant in several different sub-periods and robust to alternative statistical techniques. The findings are consistent with the experimental literature suggesting that under certain conditions investors demonstrate ambiguity seeking behavior, as well as with concurrent research regarding the negative relationship between ambiguity and stock returns.

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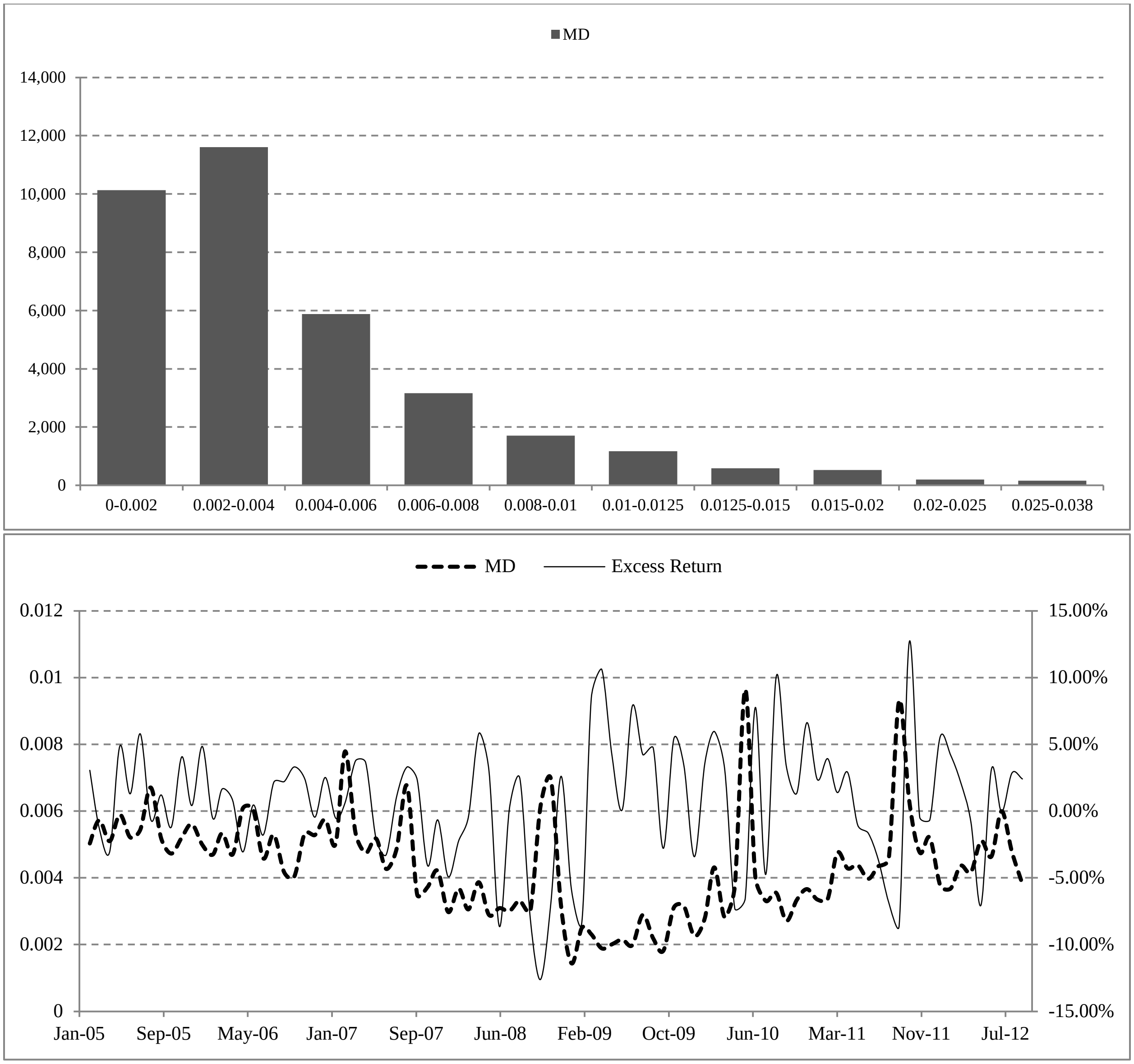

Fig 1. Ambiguity index (MD) distribution and time trend. This figure shows the distribution and time-variation of ambiguity measure for all stocks in the S&P 500 Index between 2005-2012 (for which data is available). MD is the monthly average distance (MD) between probability distributions for the sample. In the lower graph MD is drawn on the primary axis and excess returns on the secondary axis. Excess return is the dividend adjusted excess return on an equal-weighted portfolio of the stocks in S&P 500.

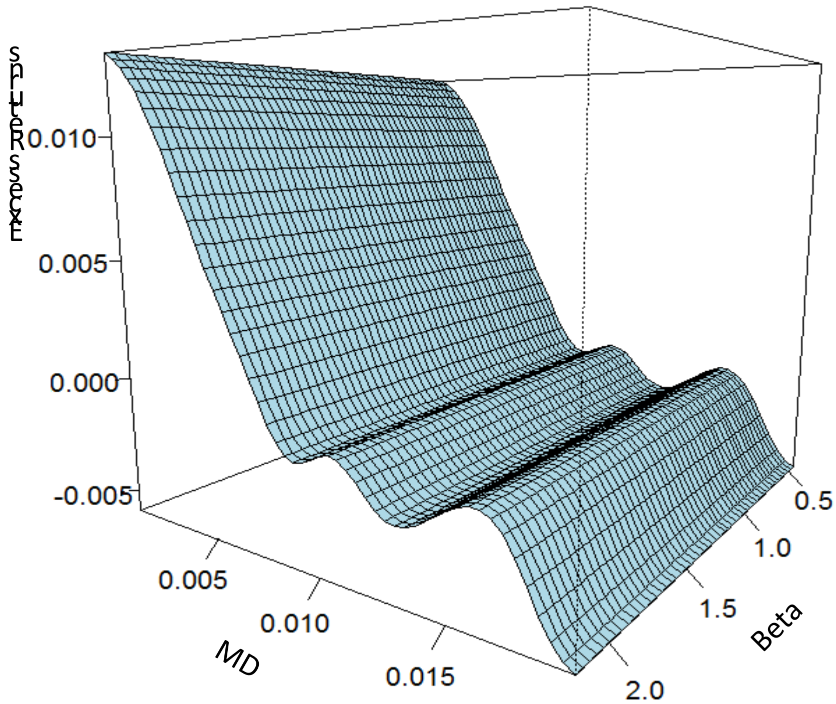

Fig 2. Risk, ambiguity and return surface. This graph demonstrates a nonparametric estimation of excess returns as a function of Beta (systematic risk) and the MD index. Bandwidth is selected using the modified Akaike information criterion (AIC) approach of Hurvich, Simonoff and Tsai (1998). Each month, the sample is divided into 25 portfolios based on Beta and MD and next month's excess return of each equally-weighted portfolio is calculated. This sample is then used to estimate the MD-Beta-Return surface. The surface demonstrates the generally negative relation between ambiguity and future returns, with an ambiguity-neutral relation over the central portion of the sample.

Table 1

Descriptive statistics and average cross-sectional correlations (Spearman) for MD and control variables.

This table contains summary statistics and Spearman cross-sectional correlations for the variables used in the paper. MD is the monthly average distance between probability distributions of two consecutive days [$Ave\left(J_{P_t}(P_{t-1})\right)$]. Excess return is the cumulative monthly returns less the risk free rate. Volatility is monthly standard deviation of price and is calculated using the methodology of French et. al. (1987). Beta is computed by regressing each stock's weekly return on the S&P 500 market portfolio using the previous two years of data. Size is the natural logarithm of market capitalization on the last day of each month. Book-to-market is the ratio of last reported book value to market capitalization. Short-term reversal is each stock's prior month return. Momentum is the cumulative stock return over the months from *t -12 to t – 1*. Kurtosis is the prior year fourth-order centralized moment of daily stock returns. Skewness is prior year third-order centralized moment of daily stock returns. Turnover is the ratio of the total number of shares traded over the last month to total number of shares outstanding. Variables in percentage (multiplied by 100) are distinguished by *. The number of firm-month observations is 31,840.

	Ret*	**MD***	**Vol***	**Beta**	**Ln(S)**	**B/M**	**S.T.R.***	**Mom***	**Kurt**	**Skew**	**Turn**
Excess Returns	1										
MD	-0.06	1									
Volatility	-0.01	-0.21	1								
Beta	0.01	-0.28	0.38	1							
ln(Size)	-0.04	0.14	-0.21	-0.20	1						
Book to Market	0.01	0.01	0.09	0.15	-0.07	1					
Short-term reversal	-0.02	-0.07	-0.16	0.01	0.03	-0.09	1				
Momentum	-0.01	0.02	-0.14	0.01	0.10	-0.24	0.21	1			
Kurtosis	-0.02	0.10	0.03	-0.16	-0.09	-0.12	-0.04	-0.10	1		
Skewness	0.00	-0.09	0.03	0.05	-0.04	-0.09	0.06	0.10	0.10	1	
Turnover	0.01	-0.28	0.62	0.37	-0.38	0.07	-0.07	-0.09	0.03	0.00	1
Mean	**0.75**	**0.42**	**8.26**	**1.13**	**23.3**	**0.45**	**0.90**	**10.60**	**6.30**	**0.12**	**0.23**
Standard deviation	**8.33**	**0.37**	**4.70**	**0.43**	**1.00**	**0.30**	**8.19**	**32.10**	**3.85**	**0.67**	**0.15**

Table 2
Control variable sorts.

Our sample of stocks is first sorted into five portfolios based on MD and we calculate the average of each control variable for each quintile. Columns 1-5 report the calculated average for each variable. Next, every month we regress the difference between the averages of the asset pricing variables in the lowest and highest ambiguity quintiles on a constant (one), the coefficient of this constant is reported in the 6th column (Low−High), the corresponding *t*-statistics is reported in the last column and is Newey-West corrected. The coefficients that are significant at 10 percent level are presented in **bold** face. Variables expressed in percentage are distinguished by *.

	Portfolio						
	Low	2	3	4	High	Low−High	*t*-statistic
MD	0.14	0.23	0.33	0.48	0.94	-0.81	
Excess Returns*	1.16	0.90	0.71	0.62	0.36	**0.80**	2.30
Volatility*	9.90	8.79	8.12	7.60	7.08	**2.82**	12.70
Beta	1.34	1.22	1.12	1.03	0.92	**0.43**	16.70
ln(Size)	23.00	23.38	23.40	23.50	23.51	**-0.51**	5.85
Book to Market	0.43	0.45	0.46	0.44	0.45	-0.01	-0.63
Short-term reversal*	1.16	1.09	0.83	0.76	0.51	**0.66**	1.89
Momentum*	15.86	12.00	10.30	8.35	6.75	**9.11**	2.67
Kurtosis	6.19	6.14	6.15	6.26	6.77	**-0.58**	3.74
Skewness	0.17	0.13	0.12	0.10	0.10	**0.07**	1.91
Turnover	0.30	0.25	0.21	0.19	0.18	**0.12**	47.30

Table 3
Mean Portfolio Returns.

In Panel A, each month we sort the stocks into three portfolios based on the level of turnover, which is measured by the total shares traded in the prior month to the total number of shares outstanding. Stocks in each turnover portfolio are further divided into three portfolios based on their beta, measured by regressing weekly stocks returns on market return (S&P 500) from *t-24* to *t-1*. Stocks in each of the nine liquidity-risk portfolio are sorted into three additional groups based on the level of MD. In Panel B, each month we sort the stocks in three portfolios based on the level of market capitalization on the last day of the previous month. Stocks in each size portfolio are then divided into three portfolios based on the level of market-to-book ratio, measured by the last reported book-value to market capitalization of the last month. Stocks in each of the nine liquidity-risk portfolio are sorted into three additional groups based on the level of MD. In Panel C, each month we sort the stocks in three portfolios based on the level of third-order centralized moment (Skewness) of daily returns from *t-12* to *t-1*. Stocks in each Skewness portfolio are further divided into three portfolios based on their fourth-order centralized moment (Kurtosis) of daily returns from *t-12* to *t-1*. Stocks in each nine Skewness-Kurtosis portfolio are then sorted into three additional groups based on the level of MD. In each panel, for each of the 27 portfolios, we report the excess return (equal weighted) over the next month. Next for the nine combinations of the two control variables we generate a new portfolio by going long the low ambiguity portfolio and shorting the high ambiguity portfolio. The return of this portfolio is then regressed on a constant (which is 1) to find the Low – High return and *t*-statistics. The reported *t*-statistics is corrected using the Newey-West (1987) procedure. The Low – High differences with significance of greater than 10% level are presented in **bold** face.

Mean Returns									
Panel A. Beta, Turnover and MD									
	Small Beta			Medium Beta			Large Beta		
Ambiguity quintiles	Low Turn	Average Turn	High Turn	Low Turn	Average Turn	High Turn	Low Turn	Average Turn	High Turn
Low	0.56	0.78	1.09	0.94	0.77	1.46	1.01	1.28	0.92
Medium	0.41	0.60	0.74	0.75	0.55	0.73	0.64	0.99	0.85
High	0.13	0.48	0.49	0.53	0.18	0.55	0.42	0.59	0.67
Low−High	**0.43**	0.30	0.59	**0.40**	0.18	**0.91**	**0.59**	0.69	0.25
t-statistic	2.55	0.84	1.39	1.94	0.85	3.68	1.88	1.63	0.56
Panel B. Size, Book-to-Market and MD									
	Low Book-to-Market			Medium Book-to-Market			High Book-to-Market		
Ambiguity quintiles	Small Cap	Medium Cap	Large Cap	Small Cap	Medium Cap	Large Cap	Small Cap	Medium Cap	Large Cap
Low	1.97	0.86	0.61	1.70	0.87	0.48	1.06	0.82	0.65
Medium	1.24	0.26	0.82	0.96	0.60	0.50	0.89	0.80	0.46
High	0.64	0.73	0.39	0.75	0.34	0.40	0.52	0.41	0.31
Low−High	**1.33**	0.13	0.22	**0.96**	**0.53**	0.09	0.54	0.41	0.34
t-statistic	3.73	0.30	0.98	2.62	1.84	0.40	1.52	0.88	0.94
Panel C. Skewness, Kurtosis and MD									
	Small Kurtosis			Medium Kurtosis			Large Kurtosis		
Ambiguity quintiles	Small Skewness	Medium Skewness	Large Skewness	Small Skewness	Medium Skewness	Large Skewness	Small Skewness	Medium Skewness	Large Skewness
Low	1.66	1.20	1.07	1.04	0.76	0.80	0.73	0.63	1.22
Medium	0.80	0.90	0.57	0.71	0.70	0.77	1.01	0.41	0.68
High	0.65	0.69	0.56	0.19	0.40	0.23	0.24	0.60	0.69
Low−High	**1.01**	0.51	0.51	**0.85**	0.37	**0.57**	0.50	0.03	0.52
t-statistic	2.43	1.30	1.60	2.87	1.02	1.70	1.41	0.08	1.40

Table 4
Tests of CAPM, three and four factor models for equally weighted portfolios sorted on MD.

This table displays alphas and factor loadings for five portfolios sorted by MD. The portfolios are formed as in Table 4. We run CAPM, Fama-French three factor model and Fama-French-Carhart four factor model by estimating: $r_{it} - r_{ft} = \alpha + \beta(r_{Mt} - r_{ft}) + sSMB_t + hHML_t + mUMD_t$ for every month from February 2005 to September 2012. r_{it} is the return on the equally weighted portfolio, r_{Mt} is the return on the value weighted market portfolio (S&P500). SMB, HML and UMD are downloaded from Kenneth French's website. Newey-West t-statistics (12 month lag) are reported in parenthesis. We report the Low minus High alpha for the three models and the corresponding t-statistics in Panel B. The coefficients that are significant at 10 percent level are presented in **bold** face.

Panel A. Factor loadings

Quintile	alpha (%)	$R_M - R_f$	SMB	HML	UMD
	0.66	**1.13**			
	(1.98)	(14.90)			
Low	**0.58**	**1.08**	**0.53**	**-0.30**	
	(2.17)	(26.19)	(4.89)	(-2.11)	
	0.58	**1.08**	**0.53**	**-0.30**	0.00
	(2.17)	(25.49)	(4.87)	(-2.03)	(-0.04)
	0.42	**1.06**			
	(1.75)	(24.86)			
2	**0.38**	**1.01**	**0.37**	-0.08	
	(1.90)	(26.22)	(5.50)	(-1.25)	
	0.38	**0.99**	**0.37**	**-0.11**	-0.05
	(1.96)	(22.65)	(5.22)	(-1.76)	(-1.94)
	0.21	**1.01**			
	(1.25)	(24.29)			
3	0.17	**0.94**	**0.37**	-0.04	
	(1.35)	(31.81)	(6.71)	(-0.96)	
	0.18	**0.93**	**0.37**	**-0.07**	**-0.05**
	(1.36)	(25.55)	(7.38)	(-1.76)	(-3.47)
	0.20	**0.89**			
	(1.42)	(17.62)			
4	0.18	**0.85**	**0.22**	-0.01	
	(1.56)	(18.30)	(4.32)	(-0.47)	
	0.18	**0.85**	**0.22**	-0.01	0.00
	(1.55)	(17.52)	(4.34)	(-0.46)	(-0.06)
	0.05	**0.70**			
	(0.36)	(16.73)			
5	0.04	**0.67**	**0.14**	0.04	
	(0.32)	(13.46)	(2.16)	(0.53)	
	0.05	**0.69**	**0.14**	0.07	0.06
	(0.42)	(14.10)	(1.99)	(1.09)	(1.54)

Panel B. Difference in alphas

Model	Low minus High Alpha (%)	*t*-statistics
CAPM	**0.62**	1.71
3F	**0.54**	1.83
4F	**0.54**	1.84

Table 5

Fama MacBeth predictive regressions of ambiguity indices on firm characteristics.

We run Fama and Macbeth (1973) monthly cross-sectional regressions from February 2005 to September 2012. The dependent variable is the distance between two implied distributions on two consecutive trading days (MD), averaged over a month(*t*). Beta is measured by regressing weekly stocks returns on market return (S&P 500) from *t-24* to *t-1*. Ln(size) is the natural logarithm of market capitalization on the last day of month *t*. Book-to-Market is the ratio of most recent book value to market capitalization on the last day of month *t*. Short-term reversal is last month's return. Momentum is the cumulative return from *t-12* to *t-1*. Kurtosis is the fourth-order centralized moment of daily returns from *t-11* to *t*. Skewness is third-order centralized moment of daily returns from *t-11* to *t*. Turnover is the ratio of total number of shares traded over the last month to total number of shares outstanding. In All of the independent variables are lagged one period, and the dependent variable is MD. Newey-West t-statistics (12 month lag) are reported in parenthesis. The coefficients that are significant at 10 percent level are presented in **bold** face. Constant terms are omitted.

(*N* =31,480)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(11)	(12)	(13)
Volatility	**-0.038**											**-0.022**
	(-6.42)											(-6.57)
Beta		**-0.003**								**-0.003**	**-0.003**	**-0.001**
		(-7.83)								(-9.34)	(-9.52)	(-8.31)
Ln(size)			**<0.001**							-0.000	-0.000	**<-0.001**
			(1.83)							(-0.41)	(-0.53)	(-1.73)
Book-to-Market				**0.121**						**0.164**	**0.147**	**0.121**
				(2.11)						(4.71)	(4.18)	(5.84)
Short term reversal					**-0.003**							**-0.001**
					(-2.82)							(-2.53)
Momentum						**-0.001**					**-0.001**	**-0.001**
						(-2.45)					(-2.62)	(-1.73)
Kurtosis							**<0.001**					**<0.001**
							(2.87)					(3.52)
Skewness								**-0.001**				**-0.001**
								(-2.70)				(-1.70)
Turnover									**-0.008**			**-0.003**
									(-6.03)			(-5.91)
R-squared	0.091	0.074	0.023	0.008	0.017	0.014	0.009	0.007	0.067	0.107	0.114	0.172

Table 6
Fama-MacBeth regressions of excess return on ambiguity and control variables.

We estimate Fama and MacBeth (1973) monthly cross-sectional regressions from February 2005 to September 2012. The dependent variable is the cumulative returns less the risk free rate at time *t*. MD is the average of all daily distances between two implied distributions on two consecutive trading days in previous month. Beta is measured by regressing weekly stocks returns on market return (SPY return) from *t-24* to *t-1*. Ln(size) is the natural logarithm of market capitalization on last day of previous month. Book-to-Market is the ratio of most recent book value to market capitalization on the last day of previous month. Short-term reversal is last month’s return. Momentum is the cumulative return from *t-12* to *t-1*. Kurtosis is fourth-order centralized moment of daily returns from *t-12* to *t-1*. Skewness is third-order centralized moment of daily returns from *t-12* to *t-1*. Turnover is the ratio of total number of shares traded over the last month to total number of shares outstanding. Constant is not reported. Newey-West t-statistics (12 month lag) are reported in parenthesis. The coefficients that are significant at 10 percent level are presented in **bold** face. Constant terms are omitted.

Panel A. MD

(*N* =31,480)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
MD	**-1.016**	**-0.875**	**-0.829**	**-0.901**	**-0.989**	**-0.944**	**-0.873**	**-1.001**	**-0.993**	**-0.880**	**-0.744**	**-0.655**	**-0.551**
	(-2.01)	(-2.02)	(-2.23)	(-1.85)	(-2.04)	(-1.89)	(-1.79)	(-2.11)	(-2.00)	(-1.98)	(-2.00)	(-1.72)	(-1.69)
Volatility		0.041											0.026
		(1.60)											(1.05)
Beta			0.001								0.000	-0.001	-0.001
			(0.41)								(0.15)	(-0.29)	(-0.60)
Ln(size)				**-0.002**							**-0.002**	**-0.002**	**-0.002**
				(-3.10)							(-3.84)	(-3.89)	(-3.27)
Book-to-Market					-0.209						-0.244	-0.110	-0.092
					(-0.48)						(-0.60)	(-0.34)	(-0.30)
Short term reversal						-0.008							-0.022
						(-0.54)							(-1.38)
Momentum							0.003					0.004	0.003
							(0.54)					(0.70)	(0.65)
Kurtosis								0.000					0.000
								(0.03)					(0.63)
Skewness									0.000				-0.000
									(0.19)				(-0.31)
Turnover										**0.012**			0.001
										(1.98)			(0.21)
R-squared	0.019	0.047	0.063	0.029	0.038	0.042	0.046	0.026	0.024	0.043	0.085	0.105	0.143

Table 7
Sub-period Analysis.

Each month we sort our sample of stocks into five portfolios based on their level of ambiguity and calculate the return of each equally weighted portfolio over the following month. Average return of each portfolio and the difference between returns of the low minus high ambiguity portfolio is reported for each sub-period. All values are calculated using the procedure in Table 2, which is calculating returns for equally weighted portfolios and regressing those return on a constant (one). All returns are in percentages and differences that are significant at 10 percent level are presented in **bold** face.

	Mean Excess Return						
Period	Low	2	3	4	High	Low − High	*t*-stat
2005-2007	1.30	1.01	0.74	0.68	0.53	**0.77**	1.66
2008-2009	0.68	0.25	0.13	-0.03	-0.23	0.91	0.89
2010-2012	1.36	1.22	1.04	0.98	0.61	**0.75**	1.81