Correlation - Modelling

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Credit Risk Elective
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Valuation of CDO Tranches

Tranched 125 Name DJ.CDX.NA.IG Series 5 (Illustrative Pricing 16 Feb, 2006)

Tranche	***Estimated Rating***	***Market Quote (bp)***
15% - 30%	AAA (junior super senior)	4/5
10% - 15%	AAA (junior super senior)	12/13
7% - 10%	AAA (junior super senior)	26/26
3% - 7%	BBB-	108/110
0% - 3%	Not rated	35.4% / 35.9% + 500 bp

Source: Morgan Stanley

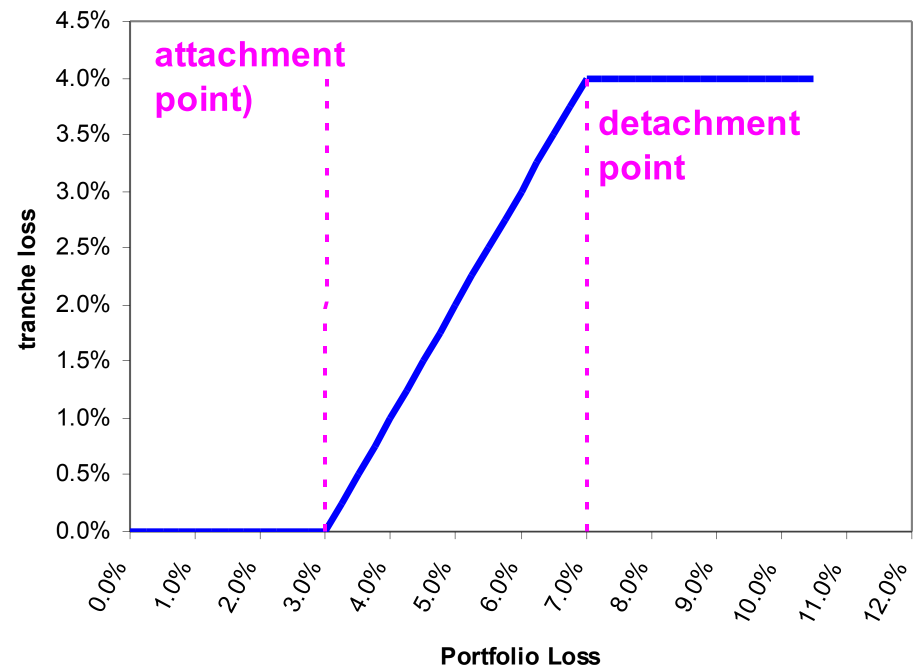
Attachment and Detachment Points

- Each tranche is defined in terms of its ***attachment*** (β_A) and ***detachment*** (β_D) points
  - ✓ these are measured in terms of losses as percent of total face value of basket
- The ***attachment*** point defines the limit ***below which*** the tranche bears ***none*** of the ***loss***
- The ***detachment*** point defines the limit ***above which*** the tranche loss ***does not increase***

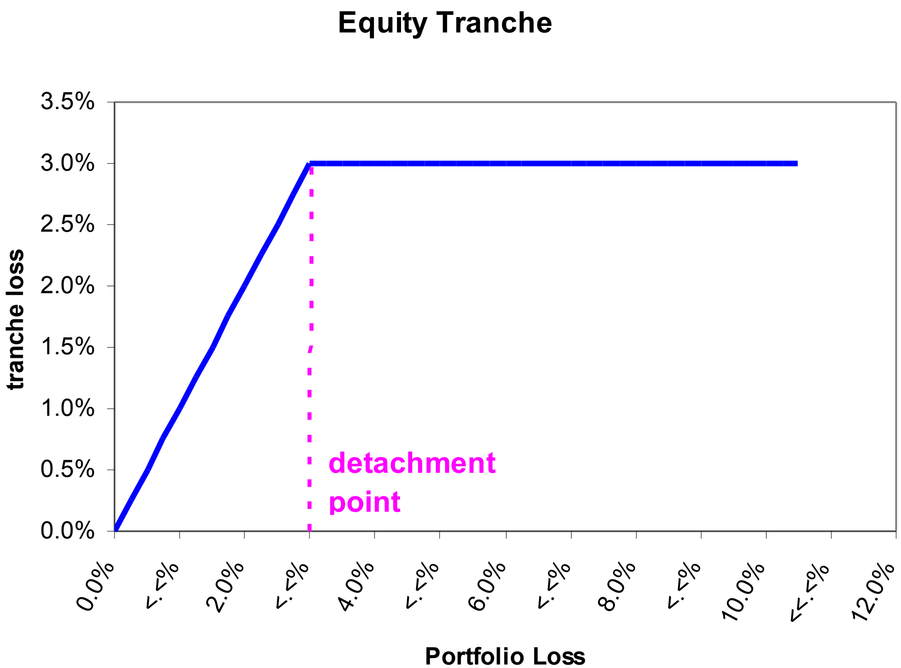
Tranche Loss Payments

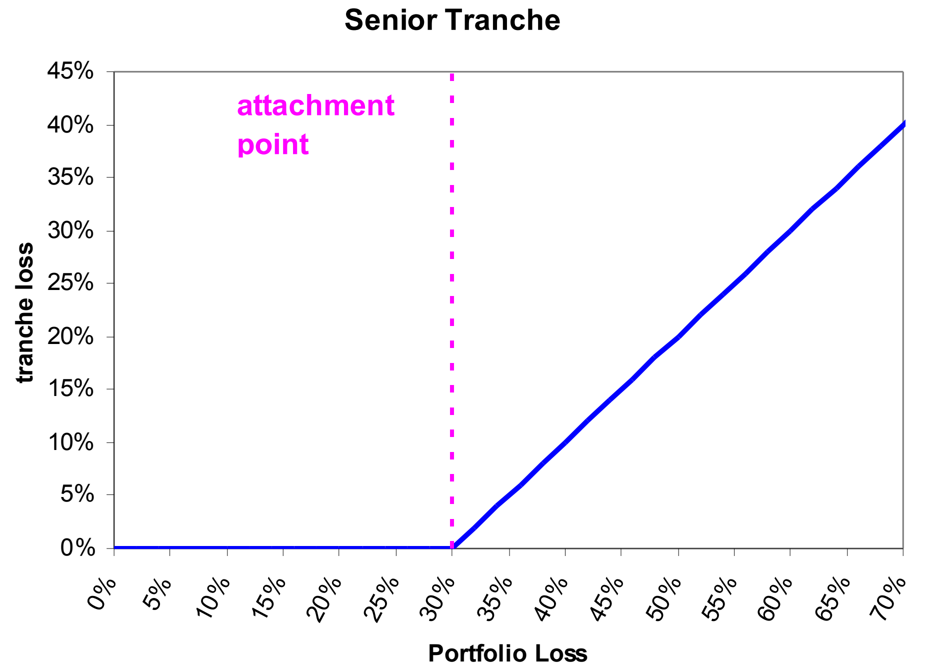
- If total losses (as a percent of the total nominal portfolio value) are L, then for a tranche with attachment and detachment points β_A and β_D the tranche loss payment is:

$$\text{Tranche Loss} = \begin{cases} 0 & L < \beta_A \\ L - \beta_A & \beta_A \quad L \quad \beta_D \\ \beta_D - \beta_A & L > \beta_D \end{cases}$$



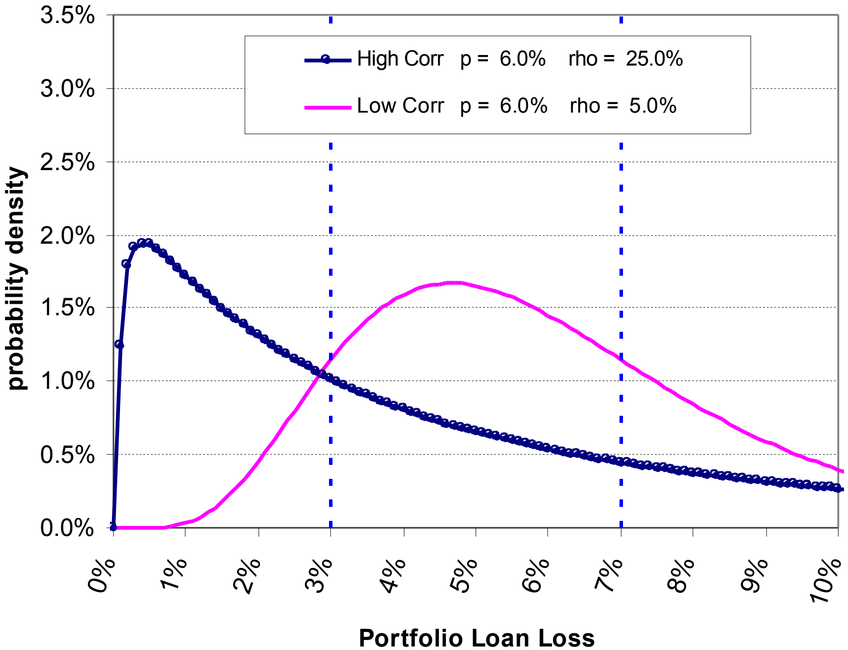
Tranche Loss Payments: Equity and Senior Tranche

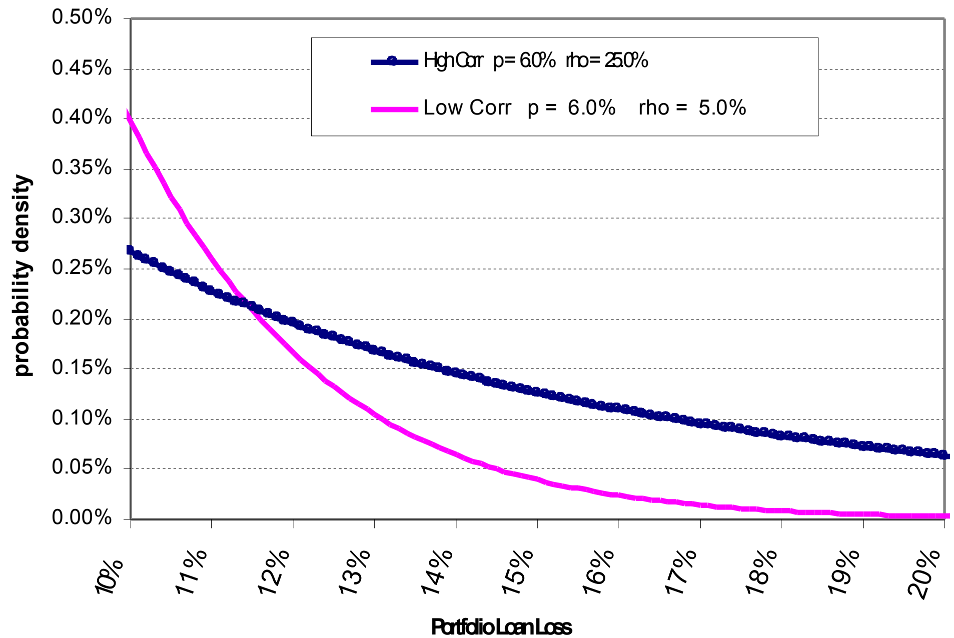




- ***Equity tranche*** loss is ***concave*** in portfolio loss: expected loss on tranche ***decreases*** (and ***value of tranche increases***) with variance of portfolio loss
- ***Senior tranche*** loss is ***convex*** in portfolio loss: expected loss on tranche ***increases*** (and ***value of tranche decreases***) with variance of portfolio loss

Effect of Correlation on Loss Distribution and Tranche Values





	Equity	***Mezz***	***Senior***	***Total****
Attachment	0%	3%	7%	0%
Detachment	3%	7%	100%	100%
Correlation	***Expected Loss***			
5%	**2.94%**	**2.39%**	**0.72%**	**6.0%**
25%	**2.29%**	**1.64%**	**2.12%**	**6.0%**

Valuation Method: the Idea

- ***Three*** steps:
  - ✓ valuation of the ***protection*** leg
  - ✓ valuation of the ***premium*** leg
  - ✓ Calculation of tranche ***running spread*** as the value that equates the value of the premium and protection legs.

Protection Leg Valuation

- Need to calculate distribution of portfolio losses:
  - ✓ Suppose portfolio contains ***125 names***
  - ✓ Use Monte Carlo and (e.g.) Gaussian copula to compute drawings from the ***joint distribution of default times*** for 125 names
  - ✓ For each name (k) and each trial (j) in simulation, if simulated default time is $\tau_{k,j}$ calculate default event indicator $Def_{k,j}$:

$$Def_{k,j} = \begin{array}{ccc} 0 & \text{no default} & \tau_{k,j} \quad Maturity \\ 1 & \text{default} & \tau_{k,j} < Maturity \end{array}$$

Protection Leg Valuation: Portfolio Loss Percentage

- Now, for each trial, j, count the number of defaults and divide by 125 to give the loss frequency (as a percentage) and then multiply by LGD to give the portfolio loss as a percentage of total portfolio face value:

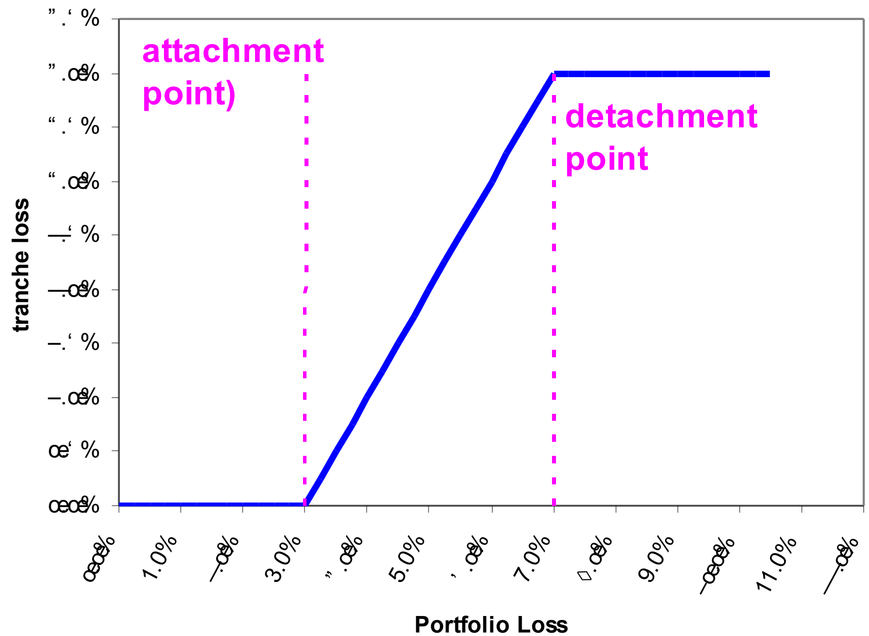
$$L_{Port,j} = LGD \quad \frac{\sum_{k=1}^{125} Def_{k,j}}{125} = LGD \quad \text{Loss Frequency (\%)}$$

Tranche Loss Percentage

- Using the relation between the tranche loss and the portfolio loss, compute the ***tranche loss*** $TL(L_{port,j})$ for this trial
- In other words, for tranche *m* and trial *j* compute:

$$TL_{m,j} = T_m(L_{Port,j})$$

- Where the function $T_m(L_{port,j})$ – *shown at right for the 3%-7% tranche* – gives the tranche loss as a function of the total portfolio loss for tranche *m* (both as a percent of total portfolio face value)



Value of Protection Leg

- To calculate the value of the protection leg $V_{m,Prot}$ (again as a percentage of the portfolio face value) we now simply calculate the discounted average value of the tranche loss over the N Monte-Carlo trials and discount this to the present*:

$$V_{m,\mathrm{Prot}} = e^{-rT} E\left(TL_{m,j}\right)$$

$$= e^{-rT} \frac{1}{N} \sum_{j=1}^{N} TL_{m,j}$$

**Note*: this assumes that all loss payments are made at maturity, T.

Calculating Tranche Losses from Total Portfolio Loss

- The calculation of tranche losses is illustrated in the three tranche example below:
  - ✓ from the portfolio loss the losses attributed to the three tranches are calculated by reference to the attachment and detachment points
  - ✓ the ***expected value*** of each tranche loss (previous slide) is then just the average of the column of losses for that tranche

Trial (j)	Portfolio Loss	Equity (0% - 3%)	Mezz (3%-7%)	Senior (7%-100%)
1	5%	3%	2%	0%
2	1%	1%	0%	0%
3	2%	2%	0%	0%
4	10%	3%	4%	3%
…	…	…	….	….
Average				

Premium Leg Valuation

- We ***assume*** that ***full premium*** is received until ***end of contract irrespective*** of number of defaults.
  - ✓ in fact, if one (say) of the 125 names defaults, the premium received across all the tranches will be the reduced by 1/125th from that time.
- Ignoring this, the present value of the premium leg is simply the ***value of an annuity*** paying the spread, ***S*** up to contract maturity
- This is expressed as a ***percentage of the total exposure*** of the tranche which is the face value, F, multiplied by (β_D - β_A), the ***width of the tranche***.

$$PV_{\mathrm{Prem}} = S \times \underbrace{(\beta_D - \beta_A) \times F}_{\text{tranche exposure}} \times \underbrace{\frac{1}{r}\left(1 - \exp(-rT)\right)}_{\text{annuity factor}}$$

Calculating the Running Spread

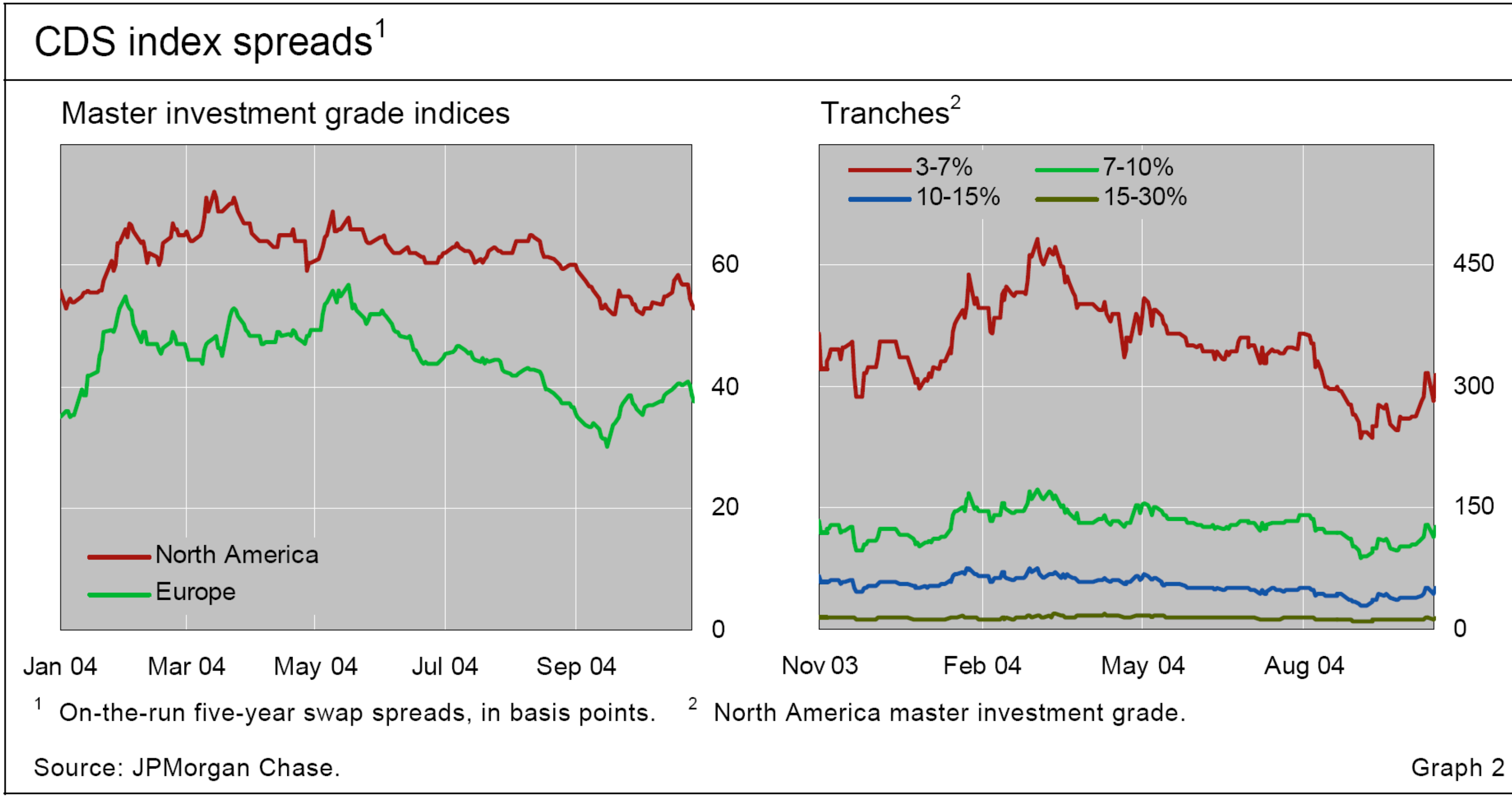
- Finally, we equate the *PV* of the premium leg (expressed as a percentage of the face value of the portfolio) to the value of the protection leg :

$$V_{m,\mathrm{Prot}} = e^{-rT}\frac{1}{N}\sum_{j=1}^{N} TL_{m,j} = S\,(\beta_D - \beta_A)\,\frac{1}{r}\big(1 - \exp(-rT)\big)$$

- And so the running spread is given by:

$$S = \frac{e^{-rT}\dfrac{1}{N}\displaystyle\sum_{j=1}^{N} TL_{m,j}}{(\beta_D - \beta_A)\,\dfrac{1}{r}\big(1 - \exp(-rT)\big)}$$

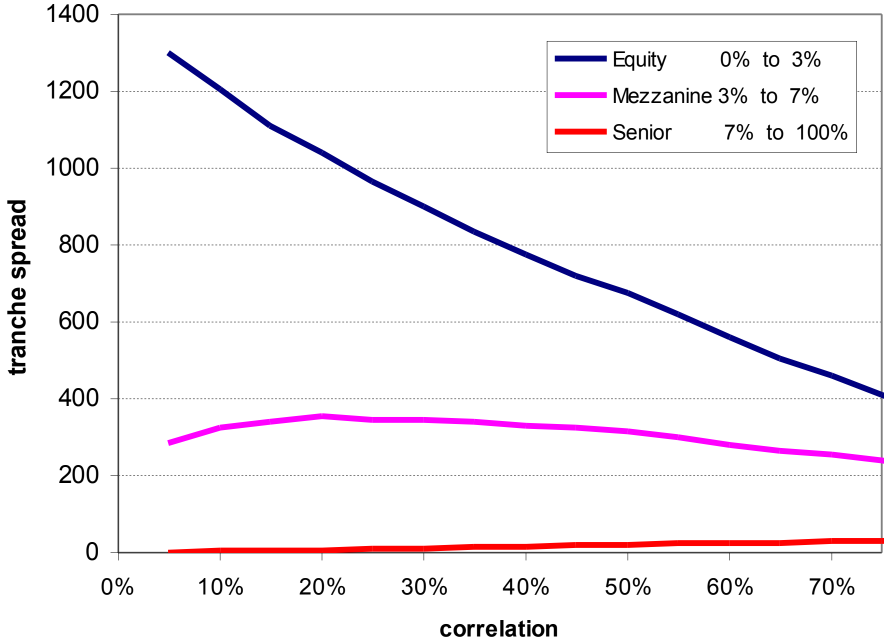
CDS Spreads on Indices and Tranches

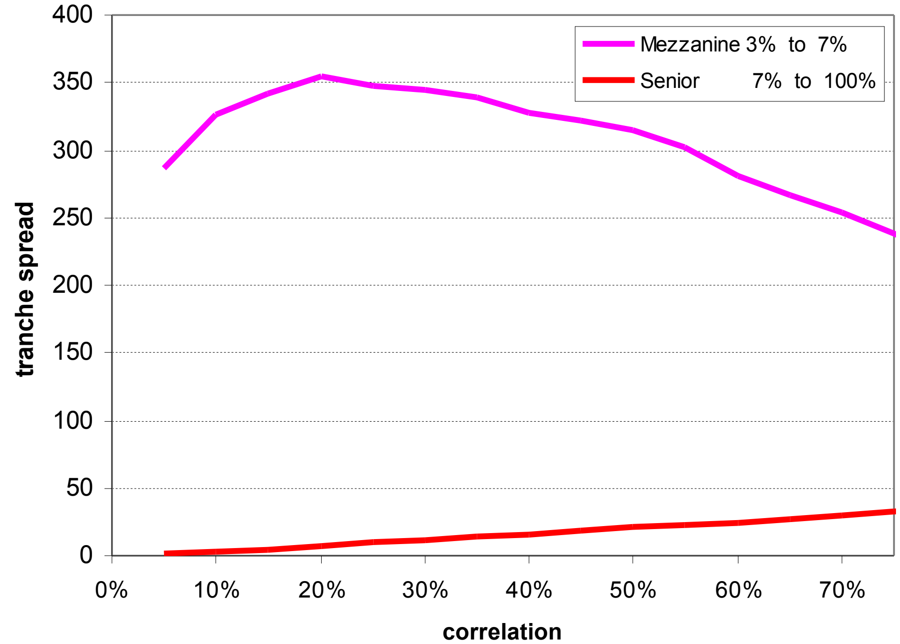


Source: Amato and Gyntelberg, "CDS Index Tranches and the Pricing of Credit Risk Correlations", BIS Quarterly Review, March 2005

Correlation

Tranche Spreads and Correlation

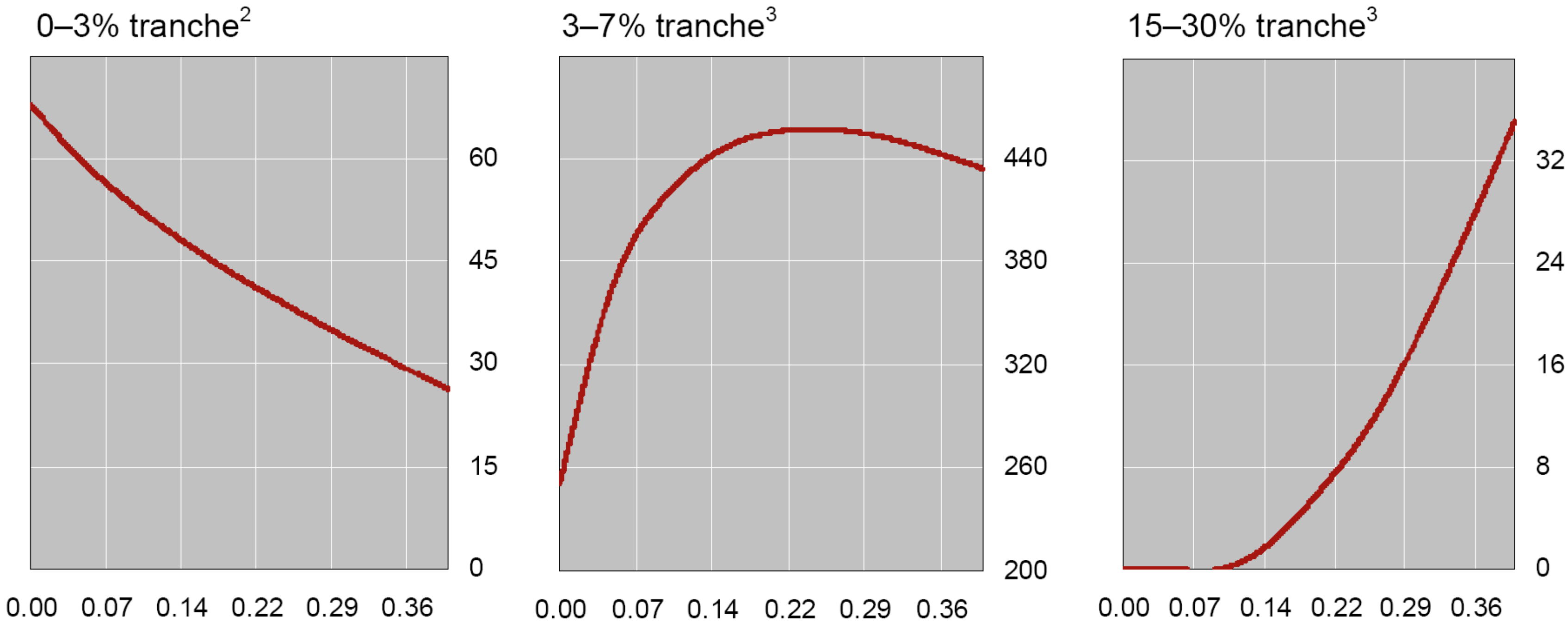




Note: number of names = 100; CDS spread =100 bps; LGD = 0.6;

Dependence of Value of CDX Tranches on Correlation (Gaussian Copula)

Price sensitivity of CDX tranches to default time correlation[1]



[1] Based on correlation sensitivities reported in Hull and White (2004). [2] Upfront payment, in per cent. [3] Spread, in basis points.

Sources: Hull and White (2004); BIS calculations. Graph 4

Source: Amato and Gyntelberg, "CDS Index Tranches and the Pricing of Credit Risk Correlations", BIS Quarterly Review, March 2005

Implied Correlation 1/2

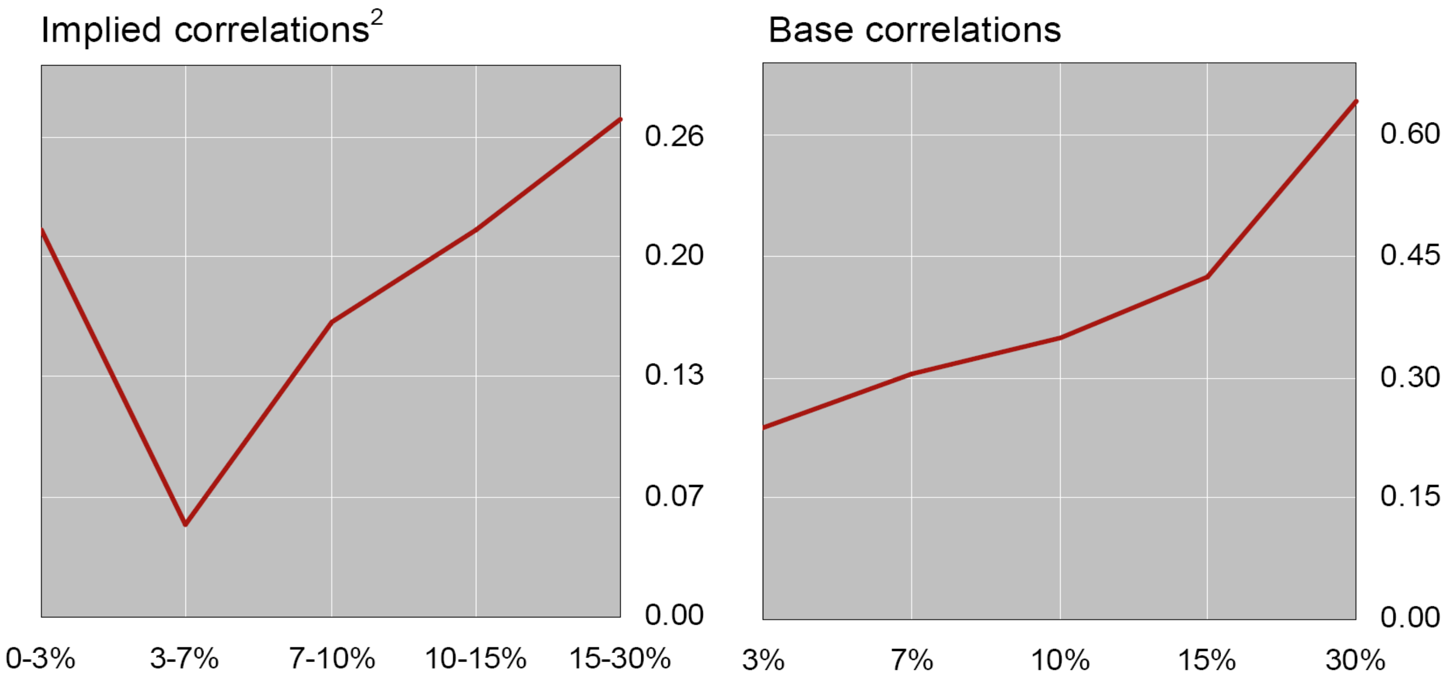
- In applications of the Black-Scholes model, there is most disagreement/uncertainty about the ***volatility parameter***
  - ✓ Traders use ***implied volatility*** (volatility calculated from the option price) as a measure of the relative value of the option
- In the same way, for CDO tranches there is most disagreement/uncertainty about the pattern of ***correlation***
- ***Implied correlation*** for CDO tranches is calculated in two main ways.

Implied Correlation 2/2

- ***Implied*** correlation
  - ✓ tranche specific correlation is the value that is consistent with the market spread of each tranche (***one at a time***)
- ***Base*** correlation
  - ✓ Consider a structure with detachment points of 3%, 10%, 30% etc.
  - ✓ The base correlation for the 10% detachment point is the correlation that is consistent with the ***combined market values*** of the 0%-3% and 3%-10% tranches.

Implied and Base Correlation

Implied and base default time correlations[1]



Note: the x-axis is tranche (left-hand panel) and tranche size (right-hand panel).

[1] Based on on-the-run five-year contracts. [2] Averages based on correlation sensitivities reported in Hull and White (2004) and market quotes from JPMorgan Chase spanning the period 13 November 2003 to 28 September 2004.

Sources: Hull and White (2004); JPMorgan Chase; BIS calculations. Graph 5

Source: Amato and Gyntelberg, "CDS Index Tranches and the Pricing of Credit Risk Correlations", BIS Quarterly Review, March 2005

Some Perspective on Correlation

- Need to remember that implied correlation is a single statistic that is supposed to the summarise ***7750 different correlation parameters*** in a basket of 125 names (125x124/2)

- CDO value (and therefore correlation) coefficient also depends strongly on the ***type of copula*** being used
  - ✓ we have used Gaussian copula – market standard for "communication" between market practitioners – but no empirical evidence that this is a good model.

- Pattern of default correlation major area for ***future work***

Mark-to-Market (MTM) Impact of Spread Change

- Suppose protection ***seller*** via a particular tranche ***receives*** a spread of s_0
- Now suppose that market spread increases to s_1:
  - ✓ if seller wishes to close out position s/he will have to purchase protection at s_1, leading to a net ***outflow*** of $(s_1 - s_0)$
  - ✓ the ***MTM impact*** of this change is the ***PV*** of the change in the spread, i.e., the value of an ***annuity*** paying $(s_1 - s_0)$
  - ✓ ***value*** to seller ***increases*** for ***reduction in spread***

Delta and MTM Hedging

- Suppose we wish to ***hedge a CDO*** against changes in the CDS spreads of the underlying names
- The ***delta*** of a CDO measures the sensitivity of the value of a tranche to a change in the value of a particular given underlying CDS

$$\text{Delta of Credit} = \frac{\text{Change in Mark-to-Market of Tranche}}{\text{Change in Mark-to-Market of Credit}}$$

- Delta usually expressed as percentage of CDS position in underlying portfolio

Calculating Delta 1/2

- In principle calculating delta is straightforward: e.g., for ***credit number i*** (of the 125 underlying credits)
  - ✓ calculate the tranche spread using the existing CDS spreads (base case)
  - ✓ change the CDS spread of credit *i* by (say) 10 bps.
  - ✓ calculate the MTM change in the value of
    - ➢ CDO tranche
    - ➢ the underlying CDS

    and take the ratio

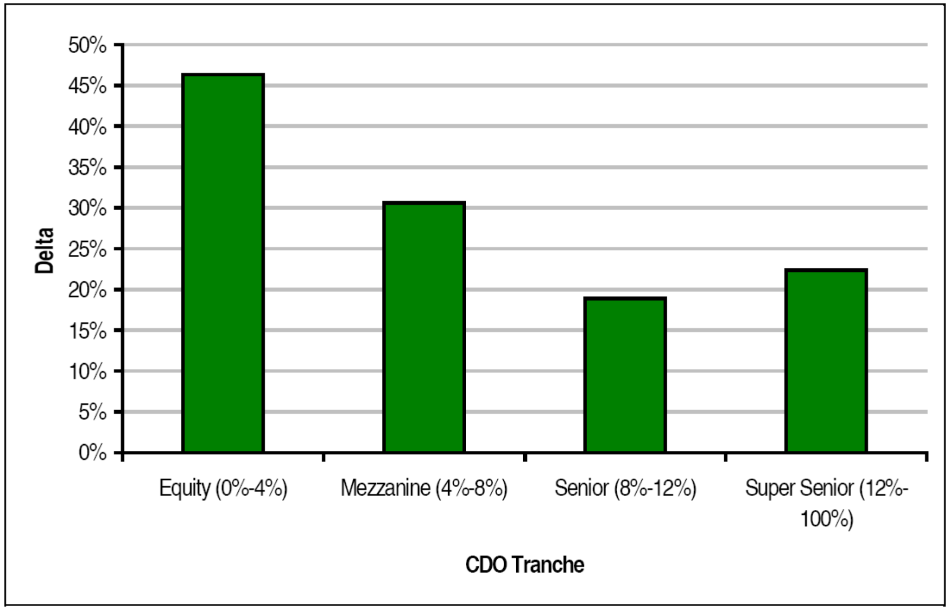
Calculating Delta 2/2

- ***Problem***: Monte Carlo method produces values that are ***subject to error***
  - ✓ Problematic to calculate change in value using ***separate*** MC runs – change will contain two (independent) errors and will tend to be inaccurate

- ***Solution***:
  - ✓ compute both base case value and revised value using ***same set of random numbers*** (e.g., in same MC run)
  - ✓ MC errors in valuation will cancel out and estimate delta will be quite accurate (known as control variate technique)

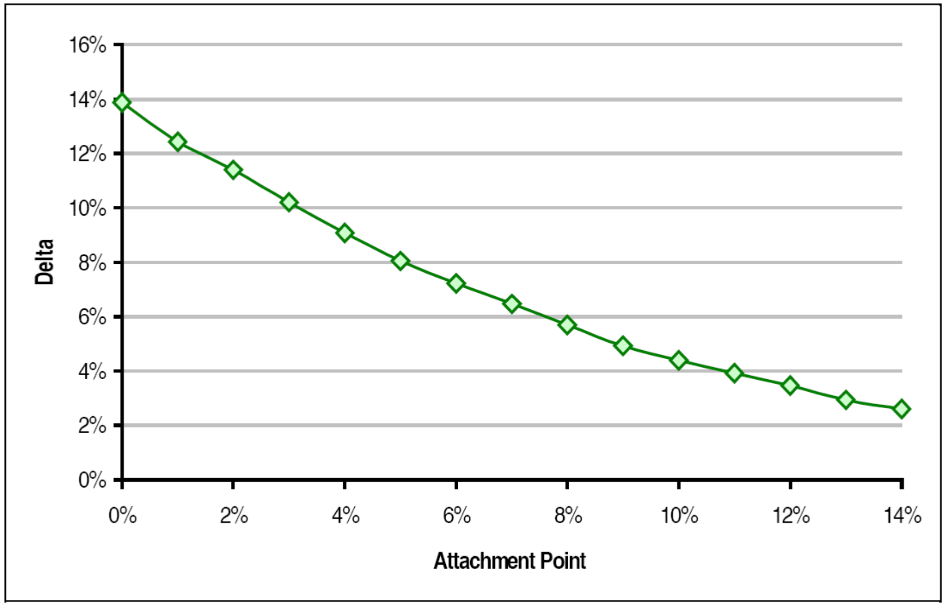
Tranche Deltas

Delta as a Function of Subordination (Discrete CDO)



Source: Merrill Lynch

Delta as a Function of Subordination (Continuous CDO)



Source: Merrill Lynch

Summary

- Important new market for credit risk transfer
  - ✓ new indices high liquidity
  - ✓ high rate of innovation in contract design
- Correlation a major area for future research