

Other Reduced-form Models

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Towards a default-risk adjusted discounting...

- Example: if risk neutral probability of default of a zero-coupon bond promising \$1 at maturity T is $p(T)$ and if recovery in default is zero, then risk-neutral expected payoff at T is $(1-p(T))$ and current price is:

$$e^{-r(T-t)} [(1 - p(T)) \times 1 + p(T) \times 0] = e^{-r(T-t)} (1 - p(T))$$

- If we substitute for $p(T)$ we obtain first “hint” of useful trick and a link between term structure and intensity based models:

$$e^{-r(T-t)} (1 - p(T)) = e^{-r(T-t)} e^{-\lambda(T-t)} = e^{-(r+\lambda)(T-t)}$$

- In other words price is just face value (\$1) discounted at the default risk adjusted rate of $(r+\lambda)$ and the **yield spread** is just λ . However, this result depends strongly on recovery assumptions.

Recovery of Market Value (RMV)

- Suppose that in default investors receive a constant fraction, R , of pre-default value of the defaultable bond
- Default time = τ , Value of bond instant before default = $V(\tau-)$
- Value of bond in default = $R V(\tau-)$, R = RMV fraction
- Under this assumption we can show that the current value is simply the promised value discounted at the default-adjusted rate $r + (1-R)\lambda$:

$$e^{-(r+(1-R)\lambda)(T-t)}$$

- Why is this valuation formula a neat analytical result?
 - ✓ Risk-less claims: Discount promised CFs at risk-free rate
 - ✓ Risky claims: Discount promised CFs at default risk-adjusted rate
- First, we provide an informal proof. Next, we apply it.

Informal Proof of RMV Result

- Suppose we are valuing a bond at time t , that λ is the risk neutral default intensity and R the recovery rate, then at time $t+1$ the investor receives:

$$\begin{array}{ll}
 V_{t+1} & \text{if no default with RN prob } (1 - \lambda\Delta t) \\
 RV_{t+1} & \text{if default with RN prob } \lambda\Delta t
 \end{array}$$

- The price at t is the risk neutral expected payoff discounted at r :

$$V_t = \frac{V_{t+1}(1 - \lambda\Delta t) + V_{t+1}R\lambda\Delta t}{1 + r\Delta t} = \frac{V_{t+1}}{1 + \hat{r}\Delta t}$$

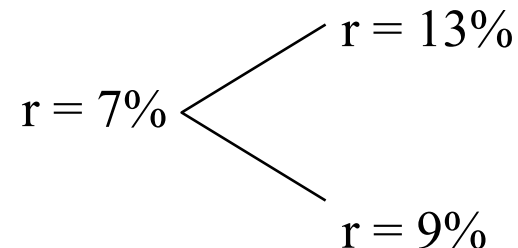
- Solving and then letting Δt tend to zero gives:

$$\hat{r} = \frac{r + (1 - R)\lambda}{1 - \lambda(1 - R)\Delta t} \quad \text{and as } \Delta t \rightarrow 0, \quad \hat{r} = r + (1 - R)\lambda$$

Implementing Intensity Models with Recovery of Market Value (RMV) as Default- Adjusted Short Rate Tree

Binomial Model with RMV Recovery: Duffie-Singleton Model – 2 Period Example – RMV Recovery

- Assume: risk-free short rate process (default-free yield curve):

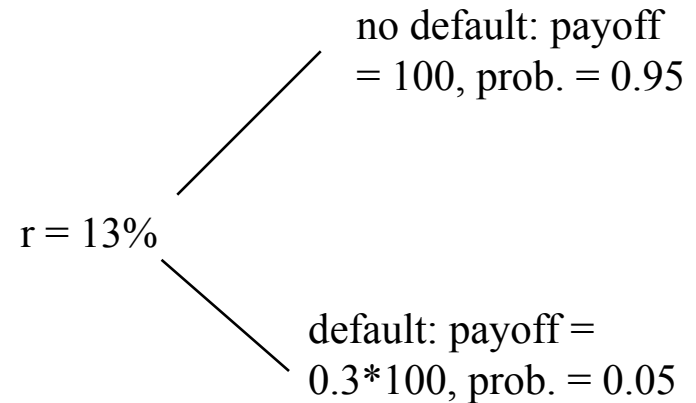


- assume :
 - ✓ recovery rate $R = 0.3$
 - ✓ annual (risk-neutral) default probability $\lambda = 0.05$
- In practice: use prices of credit risky bonds to fit default intensity

Duffie-Singleton Valuation of 2-Period Bond without default- adjusted rates

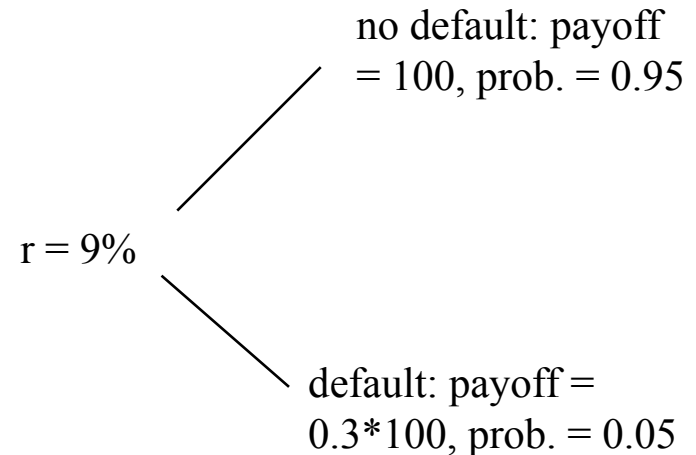
$$V_{13} = \frac{0.95 \cdot 100 + 0.05 \cdot 30}{1.13} = 85.398$$

no default: payoff = 85.398, prob. = 0.95
 default: payoff = $0.3 \cdot 85.398$, prob. = 0.05
 $E(\text{Payoff}) = 0.95 \cdot 85.398 + 0.05 \cdot 0.3 \cdot 85.398 = \mathbf{82.409}$



$r = 7\%$

no default: payoff = 88.532, prob. = 0.95
 default: payoff = $0.3 \cdot 88.532$, prob. = 0.05
 $E(\text{Payoff}) = 0.95 \cdot 88.532 + 0.05 \cdot 0.3 \cdot 88.532 = \mathbf{85.433}$



$$V_7 = \frac{0.5 \cdot 82.409 + 0.5 \cdot 85.433}{1.07} = 78.431$$

$$V_9 = \frac{0.95 \cdot 100 + 0.05 \cdot 30}{1.09} = 88.532$$

DS Model: 2 Period Example

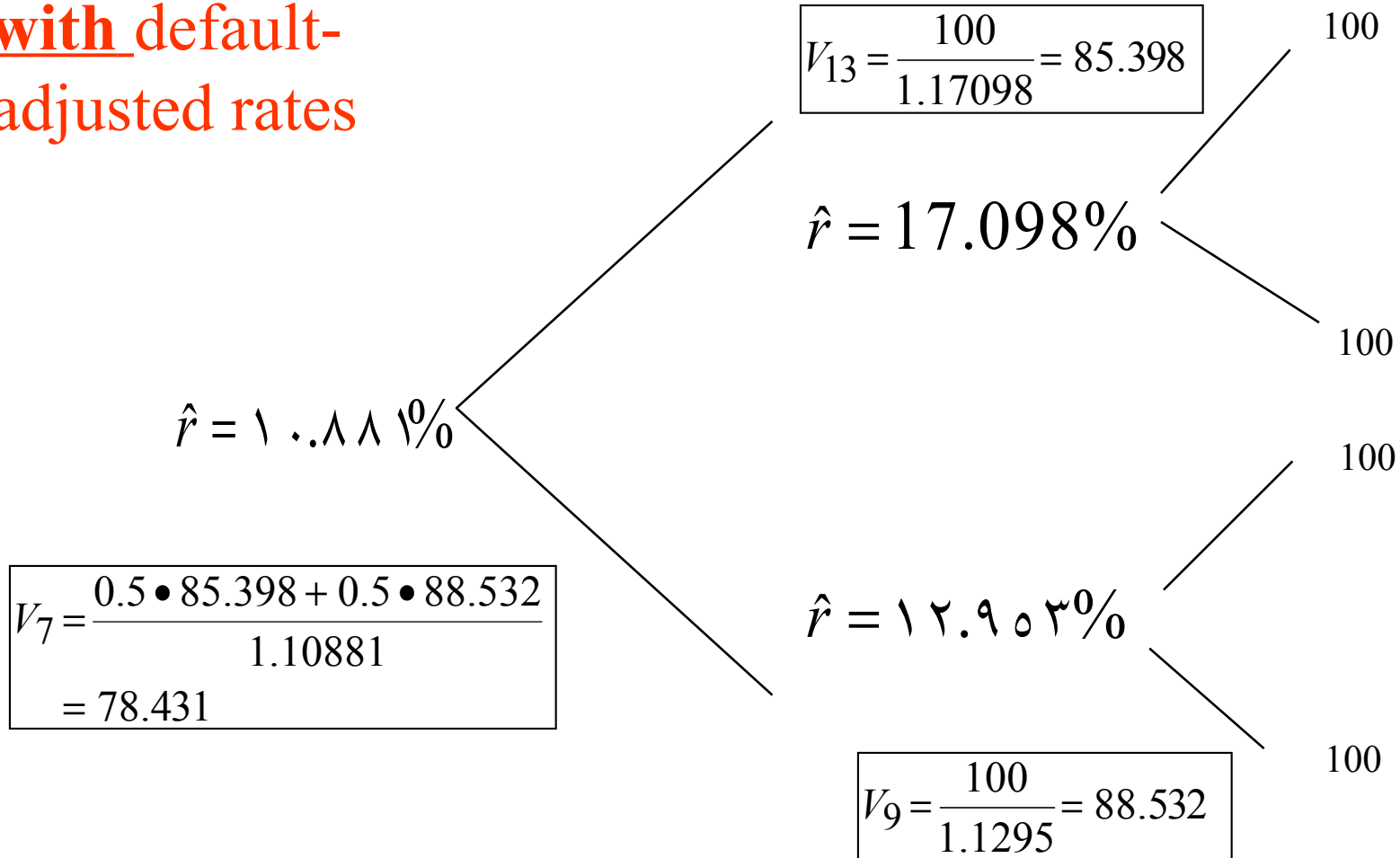
Using Default Adjusted Rates

- Assuming RMV we can rewrite the calculations in terms of a default-adjusted rate:

$$\hat{r} = \frac{r + \lambda(1 - R)}{1 - \lambda R} \approx r + \lambda \Delta \quad \text{..}\Delta \text{ (year)}$$

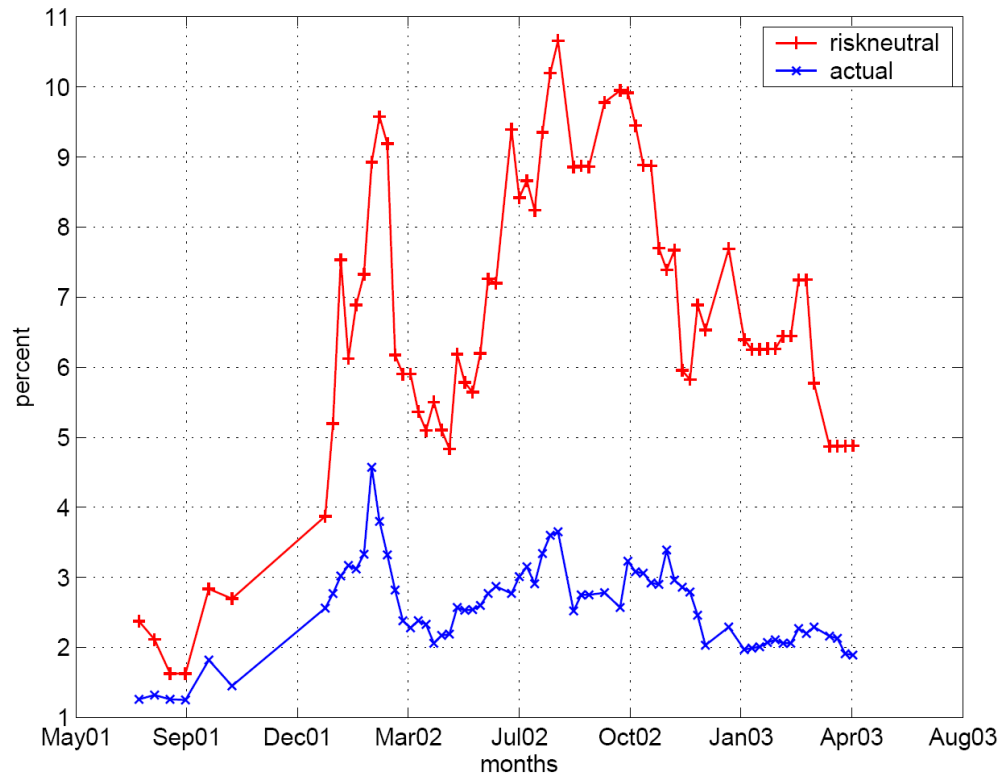
<i>riskless rate</i>	<i>default adjusted rate</i>
7%	10.881%
9%	12.953%
13%	17.098%

Duffie-Singleton Valuation of 2-Period Bond with default- adjusted rates



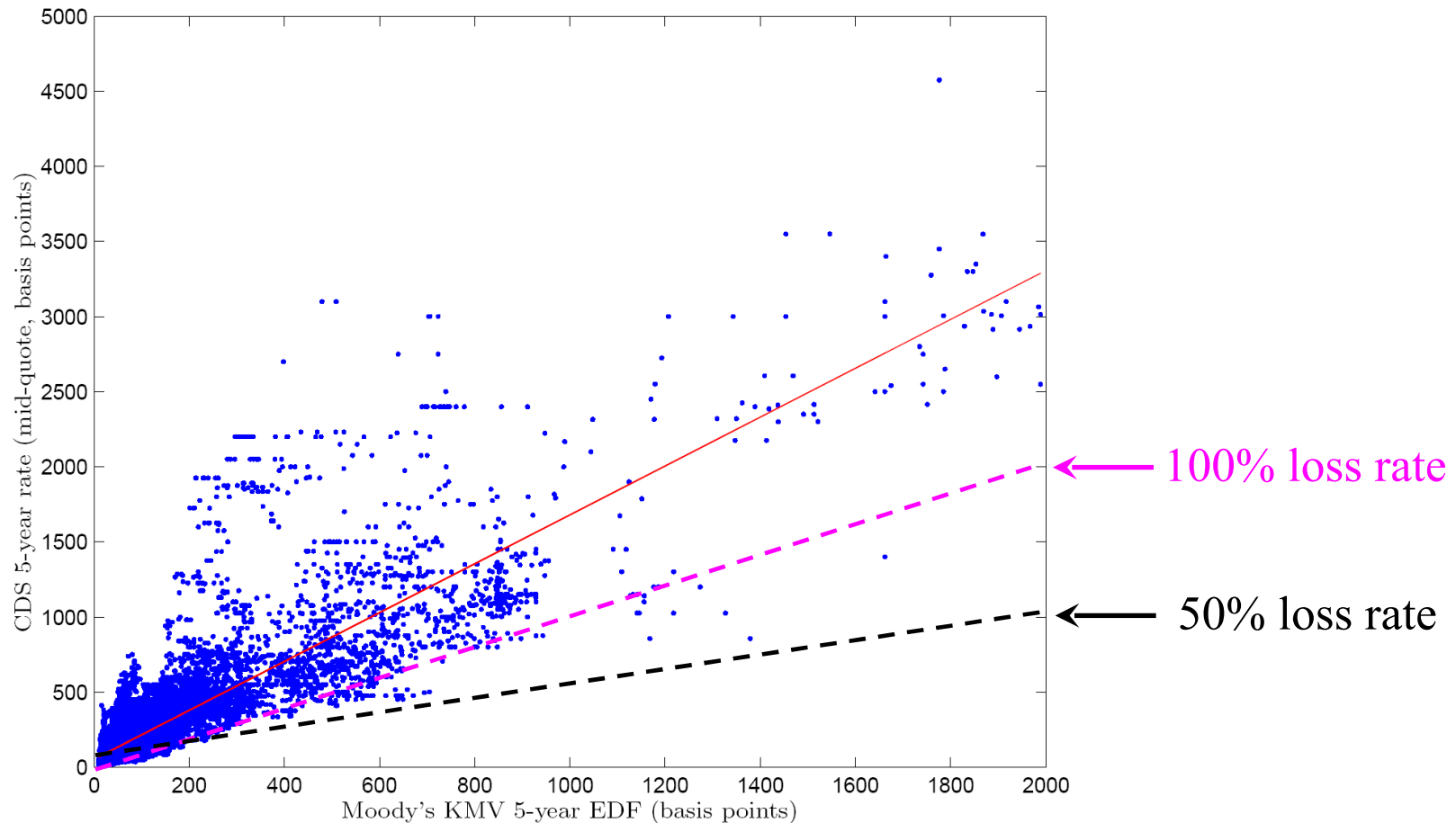
Risk-neutral versus Actual Default Probabilities

Estimated 1-year default probabilities for Vintage Petroleum.



Source: Berndt, Douglas, Duffie, Ferguson and Schranz, "Measuring Default Risk Premia from Default Swap Rates and EDFs", 2004

CDS Rates (approx. equal to spread) and Natural Default Probabilities



Source: Berndt, Douglas, Duffie, Ferguson and Schranz, "Measuring Default Risk Premia from Default Swap Rates and EDFs", 2004

What is going on?

- It is possible that there are large risk premia associated with default.
- But is also possible that credit spreads are influenced by other factors such as
 - ✓ Limited liquidity of corporate debt
 - ✓ Institutional limitations on arbitrage between debt and equity
- It turns out that for some derivatives this will make little difference but for others it will be important
 - ✓ GM and Ford downgrades of 2004-05

Do Recovery Rate Assumptions Matter?

How much difference do the recovery assumptions make?

- Recovery of market value leads to very convenient valuation formulae but may (or may not) be empirically realistic.
- How much difference does this assumption make?
- The jury is still out, but in many cases, the recovery assumption choice is second order to getting the likelihood of default right

Effect of recovery Assumptions

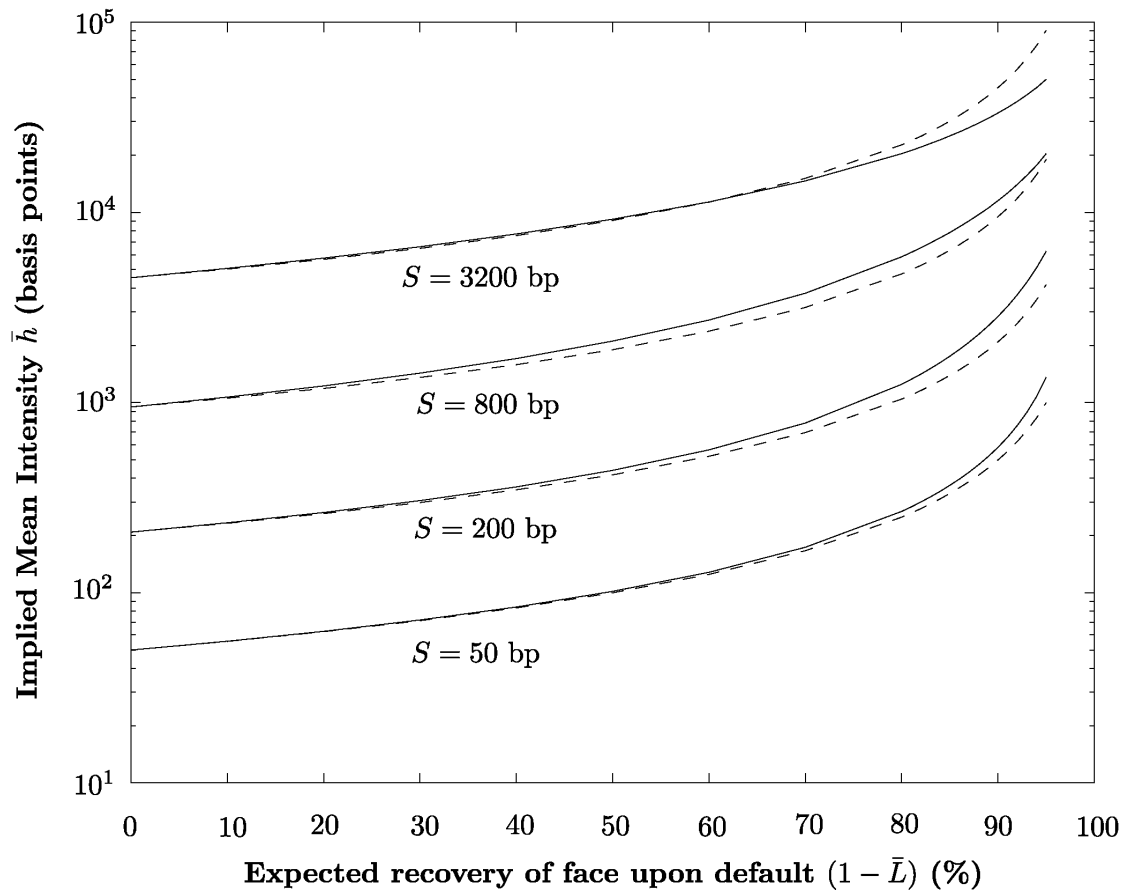


Figure 2: For fixed ten-year par-coupon spreads, S , this figure shows the dependence of the mean hazard rate \bar{h} on the assumed fractional recovery $1 - \bar{L}$. The solid (dashed) lines correspond to the model RFV (RMV).

Source: Duffie & Singleton