# Other Reduced-form Models

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# Towards a default-risk adjusted discounting…

- Example: if risk neutral probability of default of a zero-coupon bond promising \$1 at maturity *T* is *p(T)* and if recovery in default is zero, then risk-neutral expected payoff at *T* is *(1-p(T))* and current price is:

$$e^{-r(T-t)}[(1-p(T))\times 1+p(T)\times 0]=e^{-r(T-t)}(1-p(T))$$

- If we substitute for p*(T)* we obtain first "hint" of useful trick and a link between term structure and intensity based models:

$$e^{-r(T-t)}(1-p(T))=e^{-r(T-t)}e^{-\lambda(T-t)}=e^{-(r+\lambda)(T-t)}$$

- In other words price is just face value (\$1) discounted at the default risk adjusted rate of $(r+\lambda)$ and the ***yield spread*** is just $\lambda$ However, this result depends strongly on recovery assumptions.

# Recovery of Market Value (RMV)

- Suppose that in default investors receive a constant fraction, *R,* of pre-default value of the defaultable bond
- Default time = $\tau$, Value of bond instant before default = $V(\tau-)$
- Value of bond in default = $R\ V(\tau-)$, $R$ = RMV fraction
- Under this assumption we can show that the current value is simply the promised value discounted at the default-adjusted rate $r + (1-R)\lambda$ :

$$e^{-(r+(1-R)\lambda)(T-t)}$$

- Why is this valuation formula a neat analytical result?
  - ✓ Risk-less claims: Discount promised CFs at risk-free rate
  - ✓ Risky claims: Discount promised CFs at default risk-adjusted rate
- First, we provide an informal proof. Next, we apply it.

# Informal Proof of RMV Result

- Suppose we are valuing a bond at time *t*, that $\lambda$ is the risk neutral default intensity and *R* the recovery rate, then at time *t+1* the investor receives:

$$V_{t+1} \quad \text{if no default with RN prob } (1-\lambda\Delta t)$$

$$RV_{t+1} \quad \text{if default with RN prob } \lambda\Delta t$$

- The price at t is the risk neutral expected payoff discounted at *r*:

$$V_t = \frac{V_{t+1}(1-\lambda\Delta t) + V_{t+1}R\lambda\Delta t}{1+r\Delta t} \qquad \frac{V_{t+1}}{1+\hat{r}\Delta t}$$

- Solving and then letting $\Delta t$ tend to zero gives:

$$\hat{r} = \frac{r+(1-R)\lambda}{1-\lambda(1-R)\Delta t} \quad \text{and as } \Delta t \quad 0, \; \hat{r} \quad r + (1-R)\lambda$$

# Implementing Intensity Models with Recovery of Market Value (RMV) as Default-Adjusted Short Rate Tree

# Binomial Model with RMV Recovery: Duffie-Singleton Model – 2 Period Example – RMV Recovery

- Assume: risk-free short rate process (default-free yield curve):

$$
\begin{tikzcd}[row sep=small]
 & r = 13\% \\
r = 7\% \arrow[ur, dash] \arrow[dr, dash] & \\
 & r = 9\%
\end{tikzcd}
$$

- assume :
  - ✓ recovery rate $R = 0.3$
  - ✓ annual (risk-neutral) default probability $\lambda = 0.05$
- In practice: use prices of credit risky bonds to fit default intensity

# Duffie-Singleton Valuation of 2-Period Bond <u>without default-</u>adjusted rates

$$V_{13} = \frac{0.95 \bullet 100 + 0.05 \bullet 30}{1.13} = 85.398$$

r = 7%

no default: payoff = 85.398, prob. = 0.95

default: payoff = 0.3*85.398, prob. = 0.05

E(Payoff) = 0.95*85.398+0.05*0.3*85.398= **82.409**

r = 13%

no default: payoff = 100, prob. = 0.95

default: payoff = 0.3*100, prob. = 0.05

no default: payoff = 88.532, prob. = 0.95

default: payoff = 0.3*88.532, prob. = 0.05

E(Payoff) = 0.95*88.532 +0.05*0.3*88.532 = **85.433**

r = 9%

no default: payoff = 100, prob. = 0.95

default: payoff = 0.3*100, prob. = 0.05

$$V_7 = \frac{0.5 \; +82.409 \quad 0.5 \quad 85.433}{1.07}$$

$= 78.431$

$$V_9 = \frac{0.95 \bullet 100 + 0.05 \bullet 30}{1.09} = 88.532$$

# DS Model: 2 Period Example Using Default Adjusted Rates

- Assuming RMV we can rewrite the calculations in terms of a default-adjusted rate:

$$\hat{r} = \frac{r + \lambda(1-R)}{1-\lambda(1-R)\Delta t} \qquad \lambda = [illegible] \qquad R = 0.30 \qquad \Delta t = 0.5\,(\text{year})$$

| *riskless* **rate** | *default adjusted* **rate** |
|---|---|
| 7% | 10.881% |
| 9% | 12.953% |
| 13% | 17.098% |

# Duffie-Singleton Valuation of 2-Period Bond **with** default-adjusted rates

$$V_{13} = \frac{100}{1.17098} = 85.398$$

100

$\hat{r} = 17.098\%$

100

$\hat{r} =$ ۱۰.۸۸۱%

100

$\hat{r} =$ ۱۲.۹۵۳%

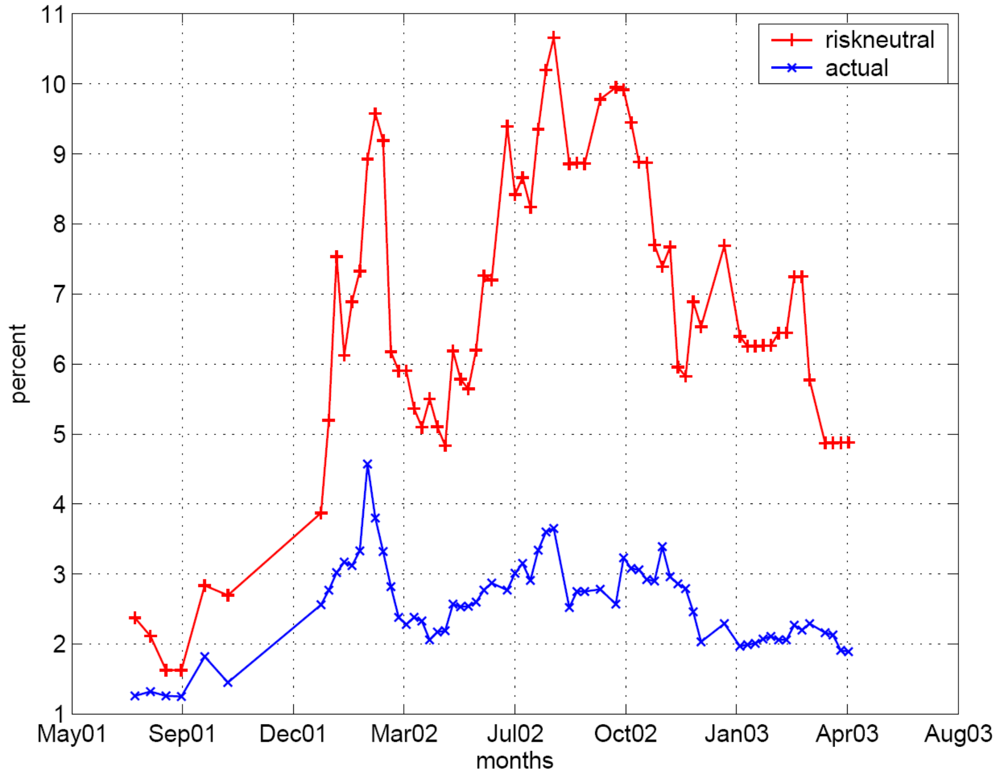
100

$$V_7 = \frac{0.5 \bullet 85.398 + 0.5 \bullet 88.532}{1.10881} = 78.431$$

$$V_9 = \frac{100}{1.1295} = 88.532$$

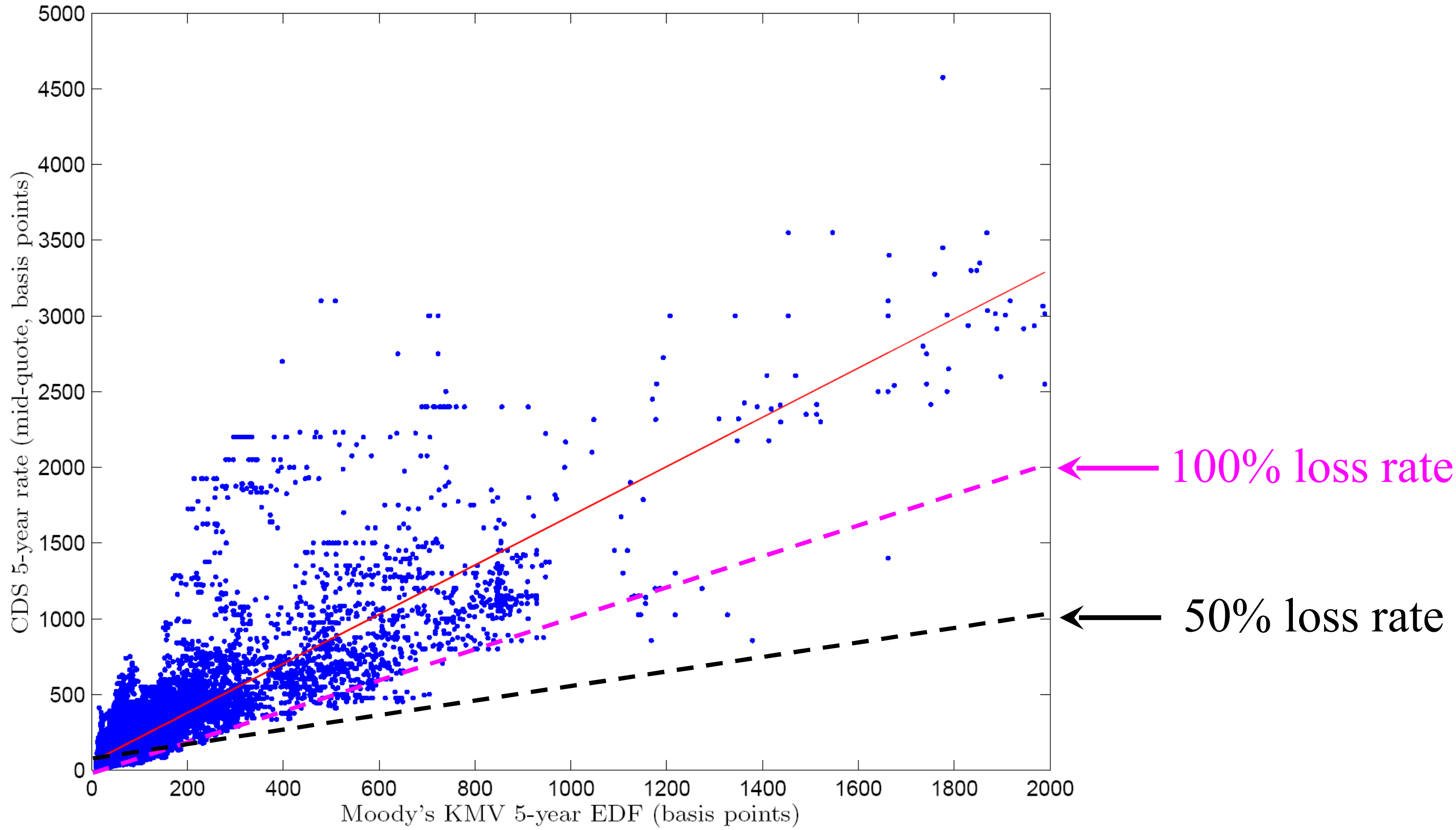
# Risk-neutral versus Actual Default Probabilities

# Estimated 1-year default probabilities for Vintage Petroleum.




*Source*: Berndt, Douglas, Duffie, Ferguson and Schranz, “Measuring Default Risk Premia from Default Swap Rates and EDFs”, 2004

# CDS Rates (approx. equal to spread) and Natural Default Probabilities




*Source*: Berndt, Douglas, Duffie, Ferguson and Schranz, "Measuring Default Risk Premia from Default Swap Rates and EDFs", 2004

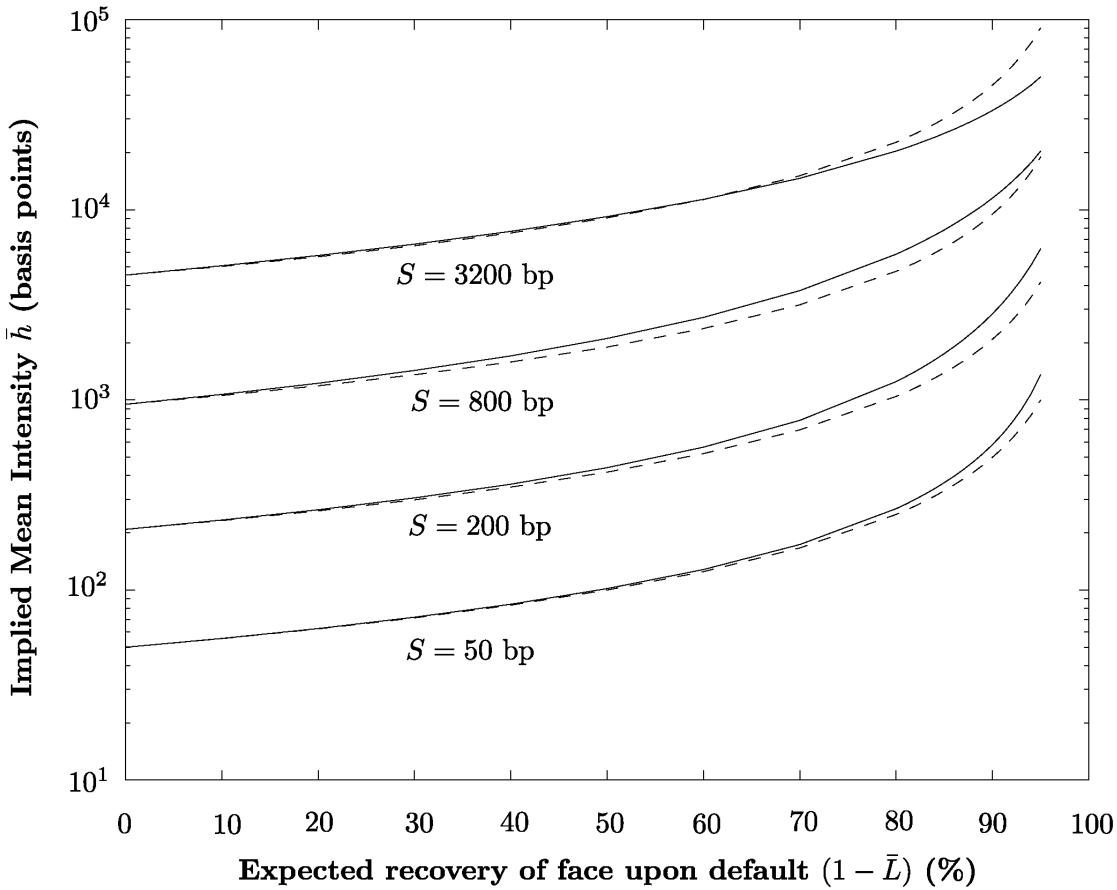
# What is going on?

- It is possible that there are large risk premia associated with default.
- But is also possible that credit spreads are influenced by other factors such as
  - ✓ Limited liquidity of corporate debt
  - ✓ Institutional limitations on arbitrage between debt and equity
- It turns out that for some derivatives this will make little difference but for others it will be important
  - ✓ GM and Ford downgrades of 2004-05

# Do Recovery Rate Assumptions Matter?

# How much difference do the recovery assumptions make?

- Recovery of market value leads to very convenient valuation formulae but may (or may not) be empirically realistic.
- How much difference does this assumption make?
- The jury is still out, but in many cases, the recovery assumption choice is second order to getting the likelihood of default right




# Effect of recovery Assumptions

Figure 2: For fixed ten-year par-coupon spreads, $S$, this figure shows the dependence of the mean hazard rate $\bar{h}$ on the assumed fractional recovery $1 - \overline{L}$. The solid (dashed) lines correspond to the model $RFV$ ($RMV$).

Source: Duffie & Singleton