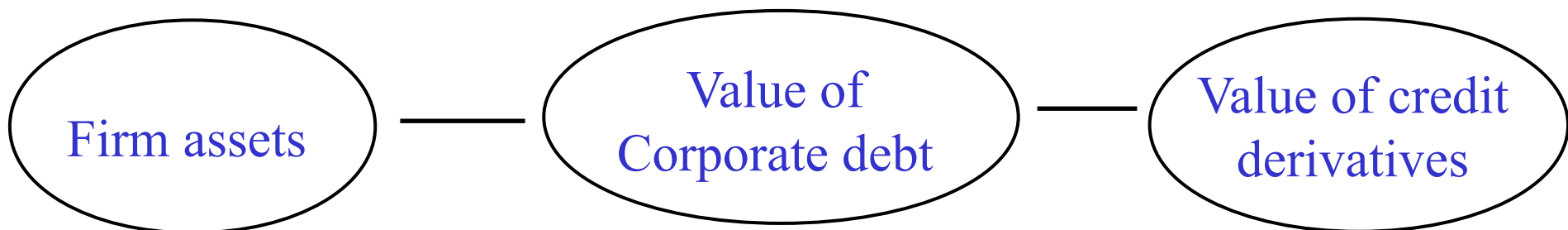


# Reduced-form Models: Preliminaries

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Credit Risk Elective  
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# Reduced-form Models – Motivation

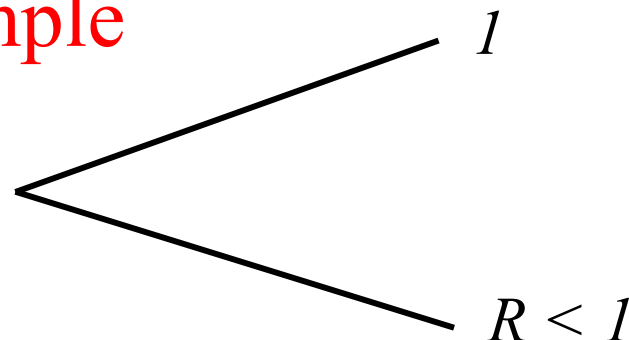


- For many credit derivatives (especially single-name credit default swaps - CDS) there is a “close-to-no-arbitrage” relation between the corporate bond / riskless bond yield spread and the CDS premium
- Most structural models – so far – do a relatively poor job of explaining the bond price (it is not just about default risk!) and so would do equally poor job of explaining CDS premia
- What is needed is a method of relative pricing that connects the CDS premium to the yield on the underlying bond. This is what reduced-form models do.
- Trade off: Lack economics of default, but price credit derivatives well (relative to corporate bonds)

# Reduced-form Model: Ingredients

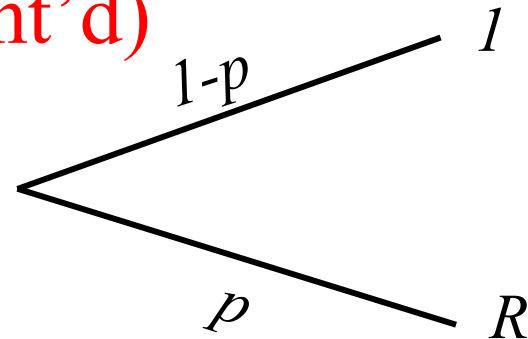
- Interest-rate process
  - ✓ This course: **Black-Derman-Toy** model-based interest rates
  - ✓ Others: Several possibilities
  - ✓ Details: Lecture notes will be available for background reading
- Default likelihood process
  - ✓ This course: **Litterman and Iben** model which assumes per-period probability of default
  - ✓ Others: Intensity models are natural extensions of Litterman-Iben
  - ✓ Details: Will be covered extensively in class
- Recovery or loss given default (LGD) assumption
  - ✓ This course: **Recovery of par, Recovery of treasury**
  - ✓ Others: Recovery of market value
  - ✓ Details: Focus of the next several slides

## Simple example



- Riskless bond price =  $B$
- Risky bond price =  $B^*$
- What is the no-arbitrage price of a credit insurance product that pays  $(1 - R)$  in case of default, and nothing otherwise?
  - ✓ Risky bond + Credit insurance = Riskless bond
- Can we use ideas from option-pricing and risk-neutral probabilities to this setup?

## Example (cont'd)



- Risky bond price:  
 $B^* = [ (1-p) \cdot 1 + p \cdot R ] B$   
(Risk-neutral) Expected loss =  $p \cdot (1-R) = (B - B^*)/B$
- Price of credit insurance product  
 $[ (1-p) \cdot 0 + p \cdot (1-R) ] B = B - B^*$
- Simple? At a fundamental level, this is all there is to reduced-form models
- This course:
  - ✓ Back out term-structure of risk-neutral probabilities of default
  - ✓ Use RNP's of default to price other instruments (in a relative sense)
  - ✓ However, we will be able to isolate RNP's of default only to a given assumption on the recovery rate
  - ✓ Recovery or loss given default (LGD): Focus of next few slides

# Recovery

# Recovery of par

- All instruments of a firm upon default recover a fraction of their face values
- That is, recoveries are *identical* within a class of instruments (by seniority, security)
- What is the rationale?
  - ✓ **Institutional:** Bond covenants
  - ✓ **Empirical:** Evidence from real-world examples
  - ✓ **Market:** Convergence of bond prices as default approaches is a popular strategy to bet on default

## Institutional: Enron covenant

*“If an Event of Default has occurred and is continuing with respect to Indenture Securities of any series, the Indenture provides that the Trustee or the holders of not less than 25% in principal amount of the outstanding Indenture Securities of that series may declare the principal amount of all of the Indenture Securities of that series to be due and payable immediately, and upon any such declaration such principal amount shall become immediately due and payable.”*

*- Prospectus supplement of the 6.875% Enron Corp bond, maturing 15 October 2007*



# Evidence: Worldcom's default

**Table I: Worldcom Bonds: Prices Leading up to Bankruptcy**

Shown below are the prices of 9 selected Worldcom bonds with final maturity shown. All bonds are senior unsecured debt. The official initial default event date defined by when Worldcom misses an interest payment is 15 July, 2002. The Chapter 11 bankruptcy filing occurs on 21 July, 2002. On 26 June, 2002 it became public information that the firm had concealed \$3.9 billion of losses for more than a year.

| <u>Bond Maturity</u> | 31-Jan-02 | 28-Feb-02 | 28-Mar-02 | 30-Apr-02 | 31-May-02 | 18-Jun-02 |
|----------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| 18-May-03            | 103.27    | 99.69     | 95.62     | 75        | 84.5      | 81        |
| 01-Apr-04            | 103.06    | 98.50     | 91.47     | 60        | 73.5      | 70        |
| 15-Aug-05            | 99.22     | 96.45     | 86.51     | 50        | 57        | 54.5      |
| 15-Jan-06            | 100.00    | 97.66     | 87.40     | 49        | 56        | 53        |
| 01-Apr-07            | 101.96    | 97.80     | 86.65     | 45.5      | 54        | 49.5      |
| 15-May-10            | 103.09    | 101.06    | 88.83     | 48        | 50        | 47.5      |
| 15-May-11            | 97.68     | 96.35     | 83.88     | 47        | 49        | 46.5      |
| 15-Aug-28            | 85.28     | 82.31     | 69.66     | 42        | 38.5      | 39        |
| 15-May-31            | 99.05     | 96.12     | 80.66     | 44        | 43        | 42        |
| <u>Bond Maturity</u> | 21-Jun-02 | 25-Jun-02 | 26-Jun-02 | 27-Jun-02 | 15-Jul-02 | 22-Jul-02 |
| 18-May-03            | 75        | 68.5      | 14        | 17        | 14.25     | 13.25     |
| 01-Apr-04            | 61.5      | 57.5      | 11.5      | 13.75     | 14.25     | 13.25     |
| 15-Aug-05            | 49        | 46.5      | 11.5      | 13.75     | 14.25     | 13.25     |
| 15-Jan-06            | 48.5      | 45        | 11.5      | 13        | 14        | 13.25     |
| 01-Apr-07            | 45.5      | 42        | 11.5      | 13        | 14        | 13.25     |
| 15-May-10            | 44        | 42        | 11.25     | 14        | 14        | 13.25     |
| 15-May-11            | 43.5      | 41        | 11.5      | 13.5      | 14        | 13.25     |
| 15-Aug-28            | 37.5      | 36.5      | 11        | 14        | 14        | 13.25     |
| 15-May-31            | 40        | 38        | 11.25     | 13        | 14        | 13.25     |

# Evidence: Enron's default

**Table II: Enron Bonds: Prices and Yields Leading up to Bankruptcy**

Shown below are the dealer-bid prices and yields of 9 selected Enron bonds whose contractual details are found in Table I. All bonds are senior unsecured debt. The initial default event date defined by when Enron files for Chapter 11 bankruptcy is 2 December, 2001. On 28 November, 2001 it became apparent that a potential merger bid by a rival company would not take place.

**Panel A: Prices**

| <u>ID</u> | 31-Jul-01 | 31-Aug-01 | 28-Sep-01 | 31-Oct-01 | 21-Nov-01 | 23-Nov-01 |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 1         | 106.23    | 106.28    | 106.89    | 87.09     | 69.89     | 64.95     |
| 2         | 104.87    | 104.91    | 105.47    | 82.19     | 62.88     | 63.92     |
| 3         | 102.61    | 102.74    | 103.75    | 78.19     | 57.76     | 61.84     |
| 4         | 101.13    | 101.78    | 103.13    | 77.7      | 57.74     | 60.82     |
| 5         | 104.3     | 104.59    | 105.74    | 77.9      | 56.72     | 59.82     |
| 6         | 102.53    | 103.25    | 104.33    | 76.89     | 65.4      | 59.81     |
| 7         | 100.49    | 100.81    | 100.68    | 77.73     | 61.66     | 58.76     |
| 8         | 101.04    | 101.93    | 97.69     | 74.66     | 56.63     | 57.81     |
| 9         | 94.5      | 96.15     | 91.37     | 73.93     | 54.86     | 56.81     |
| <u>ID</u> | 26-Nov-01 | 27-Nov-01 | 28-Nov-01 | 29-Nov-01 | 30-Nov-01 | 03-Dec-01 |
| 1         | 50.01     | 58.07     | 21        | 22.03     | 19        | 21        |
| 2         | 49.99     | 57.07     | 21        | 22.03     | 19        | 21        |
| 3         | 47.95     | 56.12     | 21        | 22.1      | 19.01     | 21        |
| 4         | 46.97     | 55.14     | 21        | 22.12     | 19.01     | 21        |
| 5         | 44.99     | 54.15     | 21        | 22.12     | 19.01     | 21        |
| 6         | 44.99     | 53.16     | 20.99     | 22.13     | 19.01     | 21        |
| 7         | 44.98     | 53.19     | 21.01     | 22.13     | 19        | 21        |
| 8         | 41.97     | 49.2      | 22        | 22.12     | 19        | 21        |
| 9         | 40.99     | 47.99     | 21.99     | 22.08     | 18.98     | 21        |

*- Rajiv Guha (London Business School MPhil Thesis)*

# Determinants of Recovery of Par

- Two notions:
  - ✓ Price at default (Pd)
  - ✓ Price at emergence (Pe)
    - Must be suitably discounted
  - ✓ Results are similar for Pd and Pe discounted at high yield returns
- Determinants:
  - ✓ Seniority
  - ✓ Security
  - ✓ Industry (?)
  - ✓ Business cycle
  - ✓ Business cycle \* Industry

## Across Seniority classes (1982-1999)

| Seniority Code | Seniority Description | Def  | Firm defaults | Avg   | Mdn   | St.Dev. |
|----------------|-----------------------|------|---------------|-------|-------|---------|
|                | Overall               | 1511 | 829           | 51.11 | 49.09 | 36.58   |
| 1              | Bank Loans            | 358  | 219           | 81.12 | 91.55 | 26.26   |
| 2              | Senior Secured        | 267  | 119           | 59.14 | 61.99 | 30.18   |
| 3              | Senior Unsecured      | 236  | 98            | 55.92 | 54.63 | 34.58   |
| 4              | Senior Subordinated   | 266  | 172           | 34.37 | 26.78 | 30.39   |
| 5              | Subordinated          | 346  | 186           | 27.07 | 16.66 | 30.37   |
| 6              | Junior Subordinated   | 38   | 35            | 18.28 | 6.25  | 27.11   |

*Source:* Acharya, Bharath and Srinivasan (2006), based on S&P data on defaulted bond and loan recoveries

## Across Security classes (1982-1999)

| Collateral Code | Collateral Description    | Pehyld |               |         |       |         |
|-----------------|---------------------------|--------|---------------|---------|-------|---------|
|                 |                           | Def    | Firm defaults | Avg     | Mdn   | St.Dev. |
|                 | Overall                   | 1511   | 644           | 51.11   | 49.09 | 36.58   |
| 1               | Current Assets            | 52     | 46            | 94.19** | 98.81 | 15.96   |
| 2               | PP and E                  | 83     | 44            | 71.36   | 77.74 | 27.51   |
| 3               | Real Estate               | 38     | 23            | 71.83   | 77.77 | 31.07   |
| 4               | All or Most assets        | 228    | 126           | 80.05   | 89.16 | 26.35   |
| 5               | Other                     | 33     | 20            | 60.94   | 53.67 | 31.21   |
| 6               | Unsecured                 | 32     | 25            | 63.71   | 63.79 | 33.48   |
| 7               | Secured                   | 40     | 17            | 63.59   | 67.42 | 36.43   |
| 8               | Information Not available | 1005   | 343           | 38.64** | 30.91 | 33.48   |

*Source:* Acharya, Bharath and Srinivasan (2006), based on S&P data on defaulted bond and loan recoveries

# Across Industries (1982-1999)

| S&P Code | Industry Description                 | Pehyld |               |         |       |         |
|----------|--------------------------------------|--------|---------------|---------|-------|---------|
|          |                                      | Def    | Firm defaults | Avg     | Mdn   | St.Dev. |
|          | Overall                              | 1511   | 424           | 51.11   | 49.09 | 36.58   |
| 1        | Utility                              | 82     | 9             | 74.49** | 76.94 | 18.79   |
| 2        | Insurance and Real Estate            | 77     | 23            | 37.13   | 27.92 | 30.96   |
| 3        | Telecommunications                   | 26     | 6             | 53.01   | 49.49 | 44.29   |
| 4        | Transportation                       | 99     | 20            | 38.92   | 18.69 | 40.76   |
| 5        | Financial Institutions               | 76     | 24            | 58.79   | 51.94 | 42.13   |
| 6        | Healthcare / Chemicals               | 111    | 35            | 55.67   | 49.41 | 38.13   |
| 7        | High Technology/ Office Equipment    | 63     | 22            | 47.05   | 40.11 | 38.07   |
| 8        | Aerospace / Auto / Capital Goods     | 138    | 46            | 52.08   | 48.43 | 37.18   |
| 9        | Forest, Building Prod / Homebuilders | 114    | 30            | 53.50   | 53.33 | 32.35   |
| 10       | Consumer / Service Sector            | 472    | 126           | 47.22   | 41.09 | 35.57   |
| 11       | Leisure Time / Media                 | 167    | 54            | 51.82   | 48.50 | 36.05   |
| 12       | Energy and Natural Resources         | 86     | 29            | 60.41   | 58.80 | 35.41   |

*Source:* Acharya, Bharath and Srinivasan (2006), based on S&P data on defaulted bond and loan recoveries

# Over time (1982-1999)

| Year    | Defaults | Firm defaults | Average | Median | St.Dev. |
|---------|----------|---------------|---------|--------|---------|
| Overall | 1511     | 465           | 51.11   | 49.09  | 36.58   |
| 1982    | 12       | 5             | 44.86   | 51.66  | 16.57   |
| 1983    | 5        | 4             | 46.17   | 35.94  | 34.95   |
| 1984    | 6        | 3             | 50.70   | 48.57  | 26.91   |
| 1985    | 12       | 8             | 21.71   | 10.82  | 30.13   |
| 1986    | 37       | 16            | 21.53   | 15.48  | 23.49   |
| 1987    | 56       | 11            | 55.59   | 58.80  | 36.11   |
| 1988    | 101      | 24            | 56.59   | 64.64  | 33.73   |
| 1989    | 110      | 29            | 43.76   | 36.02  | 37.49   |
| 1990    | 245      | 69            | 41.24   | 34.14  | 35.78   |
| 1991    | 326      | 81            | 48.97   | 47.62  | 35.06   |
| 1992    | 137      | 53            | 58.80   | 62.58  | 33.89   |
| 1993    | 103      | 36            | 55.84   | 49.09  | 38.18   |
| 1994    | 60       | 25            | 66.02   | 82.54  | 38.23   |
| 1995    | 97       | 35            | 63.22   | 68.30  | 36.96   |
| 1996    | 75       | 27            | 60.64   | 62.40  | 36.55   |
| 1997    | 38       | 11            | 61.18   | 73.71  | 40.27   |
| 1998    | 49       | 16            | 36.69   | 38.76  | 29.47   |
| 1999    | 42       | 12            | 67.18   | 80.00  | 37.19   |

*Source:* Acharya, Bharath and Srinivasan (2006)

# Industry-level Distress (1982-1999)

| S and P Code | Description                             | Year |
|--------------|---|------|
| 4            | Transportation                          | 1984 |
| 12           | Energy and Natural Resources            | 1986 |
| 5            | Financial Institutions                  | 1987 |
| 6            | Healthcare/Chemicals                    | 1987 |
| 2            | Insurance and Real Estate               | 1990 |
| 4            | Transportation                          | 1990 |
| 5            | Financial Institutions                  | 1990 |
| 6            | Healthcare/Chemicals                    | 1990 |
| 7            | High Technology/Office Equipment        | 1990 |
| 8            | Aerospace/Auto /Capital goods           | 1990 |
| 9            | Forest, Building Products/Home Builders | 1990 |
| 10           | Consumer/Service Sector                 | 1990 |
| 11           | Leisure Time/Media                      | 1990 |
| 5            | Financial Institutions                  | 1991 |
| 10           | Consumer/Service Sector                 | 1993 |
| 2            | Insurance and Real Estate               | 1994 |
| 6            | Healthcare/Chemicals                    | 1994 |
| 11           | Leisure Time/Media                      | 1994 |
| 6            | Healthcare/Chemicals                    | 1995 |
| 10           | Consumer/Service Sector                 | 1995 |
| 11           | Leisure Time/Media                      | 1995 |
| 10           | Consumer/Service Sector                 | 1996 |
| 6            | Healthcare/Chemicals                    | 1998 |

*Source:* Acharya, Bharath and Srinivasan (2006)

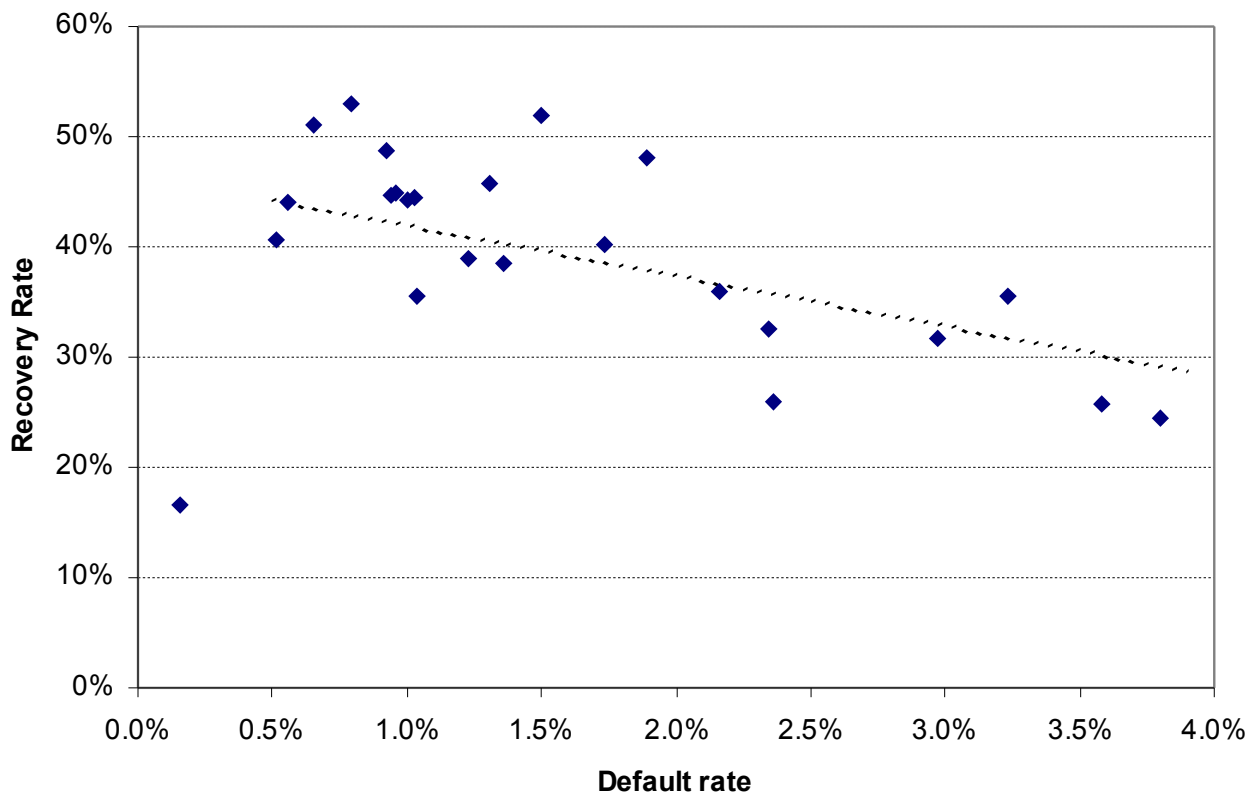


# Industry-level Distress (cont'd)

| Recovery rates                              | Full sample    | Obs  | No Industry Distress<br>(B) | Obs  | Distress<br>(C) | Obs | t-statistic<br>(z- statistic) |
|---|----------------|------|-----------------------------|------|-----------------|-----|-------------------------------|
| Pehyld                                      | 50.8<br>(48.4) | 1443 | 52.4<br>(50.3)              | 1285 | 37.8<br>(24.9)  | 158 | 4.77***<br>(4.92)***          |
| Pehyld (excl. 1990)                         | 52.8<br>(50.5) | 1209 | 53.2<br>(51.2)              | 1167 | 40.2<br>(27.5)  | 42  | 2.30**<br>(2.33)**            |
| Pehyld (excl. utilities<br>and finl instns) | 49.0<br>(43.4) | 1293 | 51.1<br>(47.6)              | 1154 | 31.9<br>(18.5)  | 139 | 5.92***<br>(6.07)***          |

*Source:* Acharya, Bharath and Srinivasan (2006)

# Evidence that Recovery Rate is Negatively Related to Default Frequency (Speculative Grade, 1981-2004)



*\*Source: Moody's*

# Altman, Brady, Resti and Sironi

- **BDA:** aggregate face value of defaulted bonds measured at mid-year
  - ✓ 10 bln \$ increase -> 5% decrease in recoveries
- **BDR:** aggregate weighted average default rate of high yield bonds
  - ✓ 1% increase -> 4% decrease in recoveries
- Total Depressed prices of defaulted securities in 2001-2002 period

*“As the huge volume of defaulted debt floods the market, trading prices for distressed debt have become depressed, a response to increased supply meeting a generally shallow, illiquid market.”*

- Standard and Poors, January 2002

# Alternate view: United Airlines case (2003)

- A large number of aircraft leaseholders
  - ✓ GE, Disney, Whirlpool, Boeing, Morgan Stanley, Bank of New York, Philip Morris, Pitney Bowes, Fort Motor Credit
  - ✓ Total exposure to airlines of 20 bln \$
- The problem:
  - ✓ 400 Parked Planes at the Mojave Boneyard!
  - ✓ Prices of used jets down by 40% since 2000.
  - ✓ Gives tremendous leverage to bankrupt airlines during bankruptcy negotiations

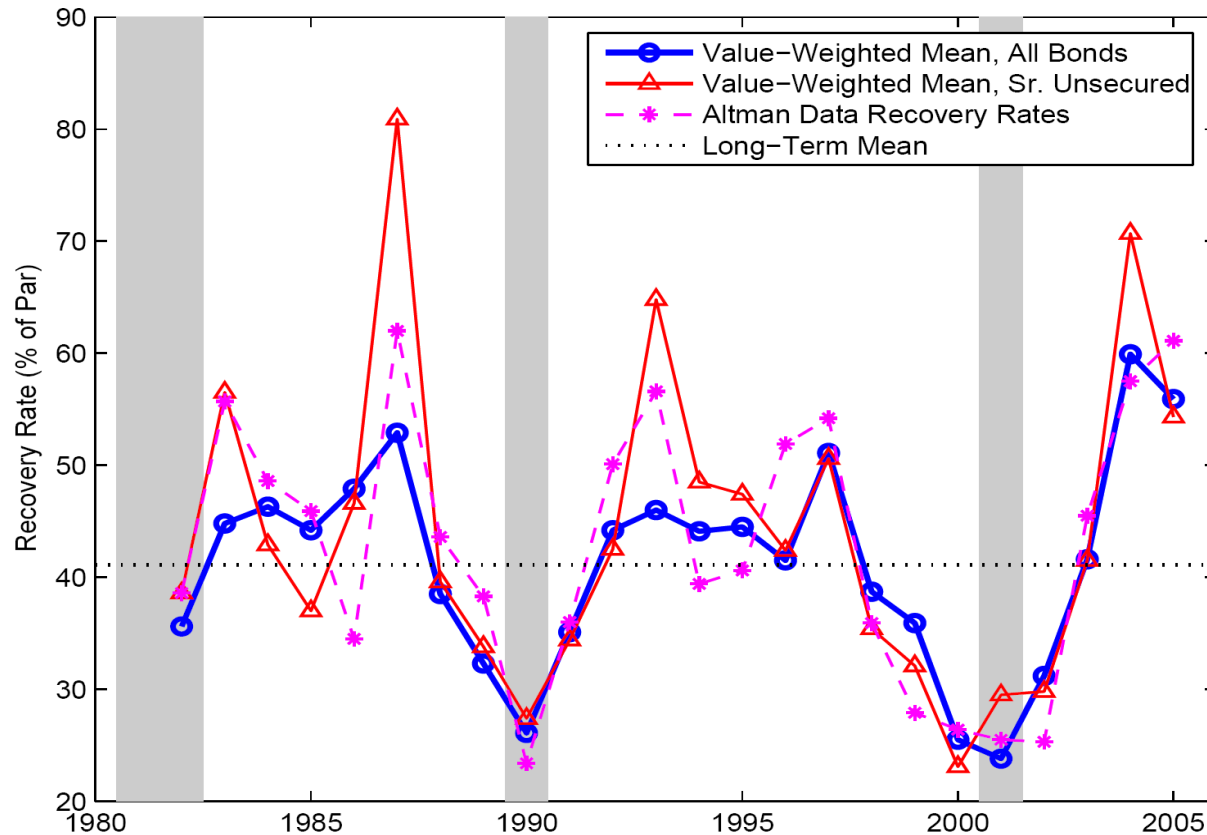
*“... Ford and Philip Morris are facing billions of dollars of losses on United Airlines leases... The US airline believes it can slash its costs by renegotiating its \$8bln of aircraft leases ... It plans to send revised terms to leaseholders over the next three days... United’s advisers argue it is in a strong negotiating position because of the weak market for used aircrafts.”*

- Financial Times, December 13 2002

# Exact terms

- United has asked its airplane owners to accept a reduction in monthly payments
  - ✓ Inspired by US Airways bargains
  - ✓ 50% for less desirable models: Boeing 757-200s and older 737s
    - Aircraft price discount: 60% (Morgan Stanley)
  - ✓ Hard bargains even for newer Boeing 737s
    - Aircraft price discount: 16%
- So, is it bond-market supply effects, or is it liquidity in the market for sale of bankrupt firms?
- Interesting, but for this course, what matters is that there is cyclical recovery rates regardless of the exact cause

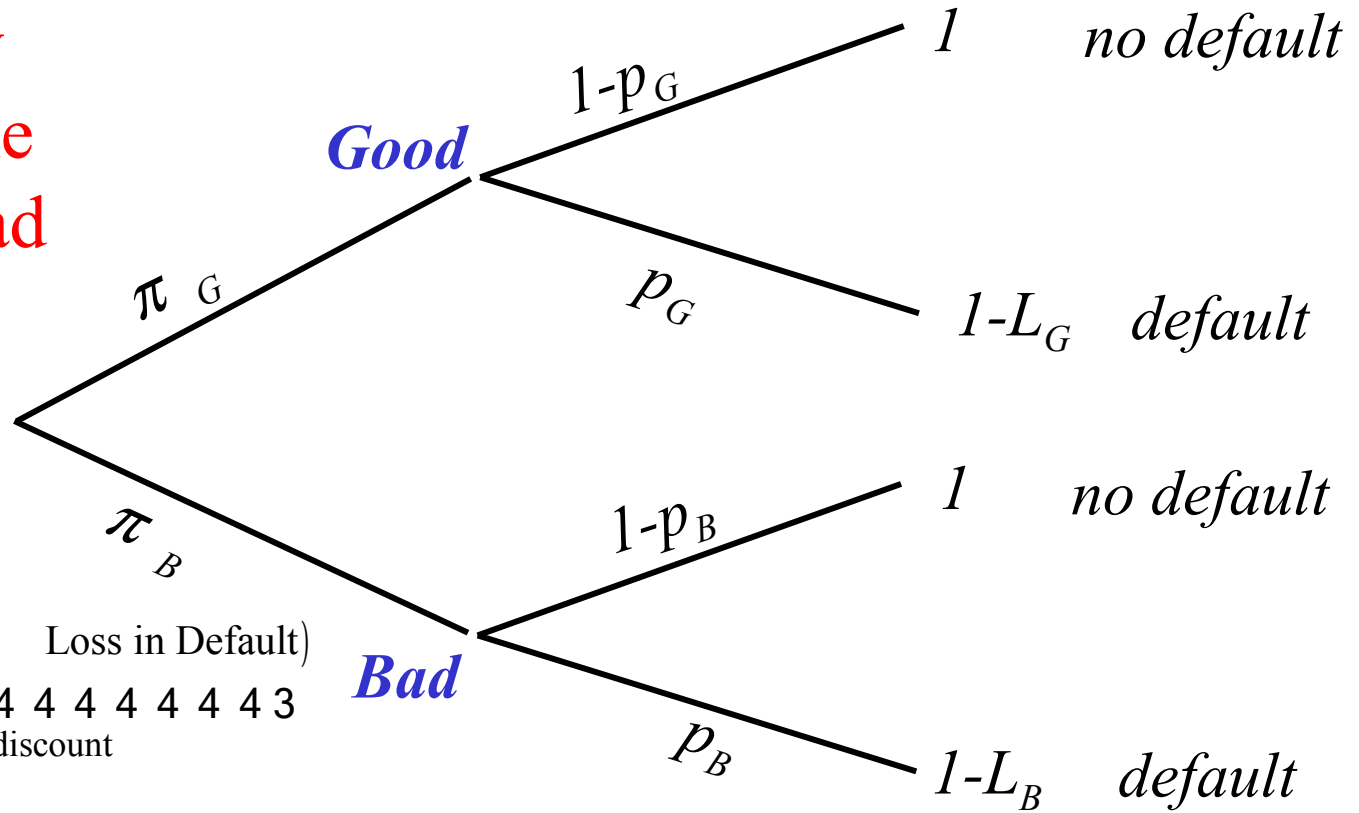
# Recovery Rates are *Cyclical*



Data source: Value-weighted average recovery rates for “All Bonds” and “Sr. Unsecured” are from Moody's. “Altman Data Recovery Rates” are from Altman and Pasternack (2006). Shaded areas are NBER-dated recessions.

*Source:* “Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure”, Hui Chen, Working Paper, Graduate School of Business, University of Chicago, Jan. 2007.

# Cyclical Increases the Credit Spread



$$\frac{1}{1+R_f} \left( \text{RN-Prob}(\text{default}) \quad \text{Loss in Default} \right)$$

Credit risk price discount

$$= \frac{1}{1+R_f} (\pi_G p_G L_G + \pi_B p_B L_B)$$

- Cyclical increases credit spread since, in “bad” state:
  - ✓ higher market value of losses ( $\pi$  high relative to natural probability)
  - ✓ higher default probabilities ( $p$ ) and loss-given-default ( $L$ )

# Other Recovery Assumptions

- In default intensity models three forms for recovery have been assumed:
  - ✓ **Recovery of Par (RP)** – a fraction of the face value of the bond, that we have examined so far and justified as being realistic
  - ✓ **Recovery of Treasury (RT)** – recovery of a fraction of an otherwise identical riskless (“Treasury”) bond
    - Employed in Litterman-Iben model, but not necessary
  - ✓ **Recovery of Market Value (RMV)** – recovery of fraction of value bond would have at that time if default had not occurred
    - Duffie and Singleton model
    - Unnatural, but leads to a convenient analytical result
  - ✓ Evidence mixed on the impact of different assumptions