Structural Models II

Viral V. Acharya and Stephen M Schaefer
NYU-Stern and London Business School (LBS), and LBS

Credit Risk Elective
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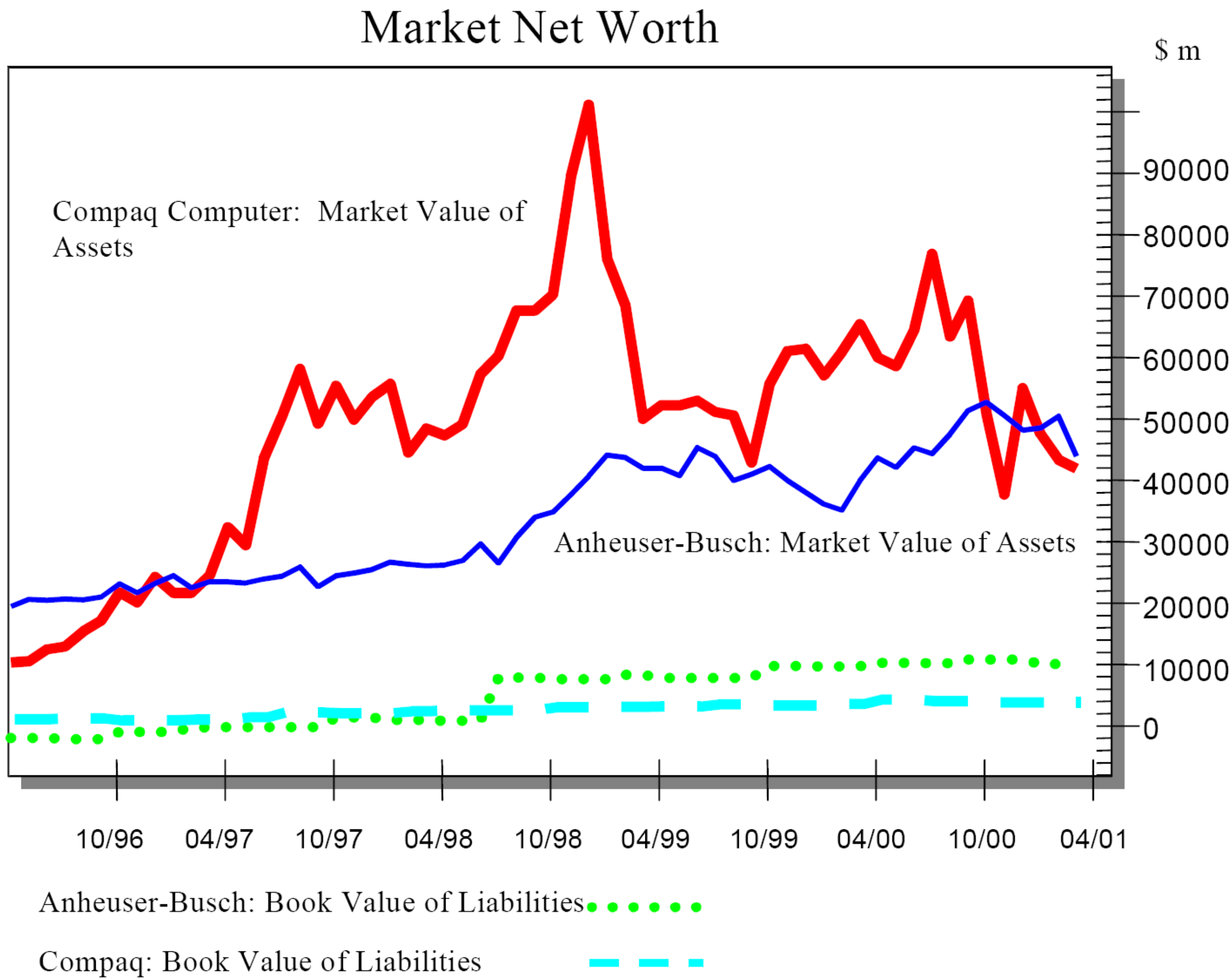

FIGURE 2 Evolution of asset values and default points for Compaq and Anheuser-Busch

Source: Moody's-KMV

Default Probability

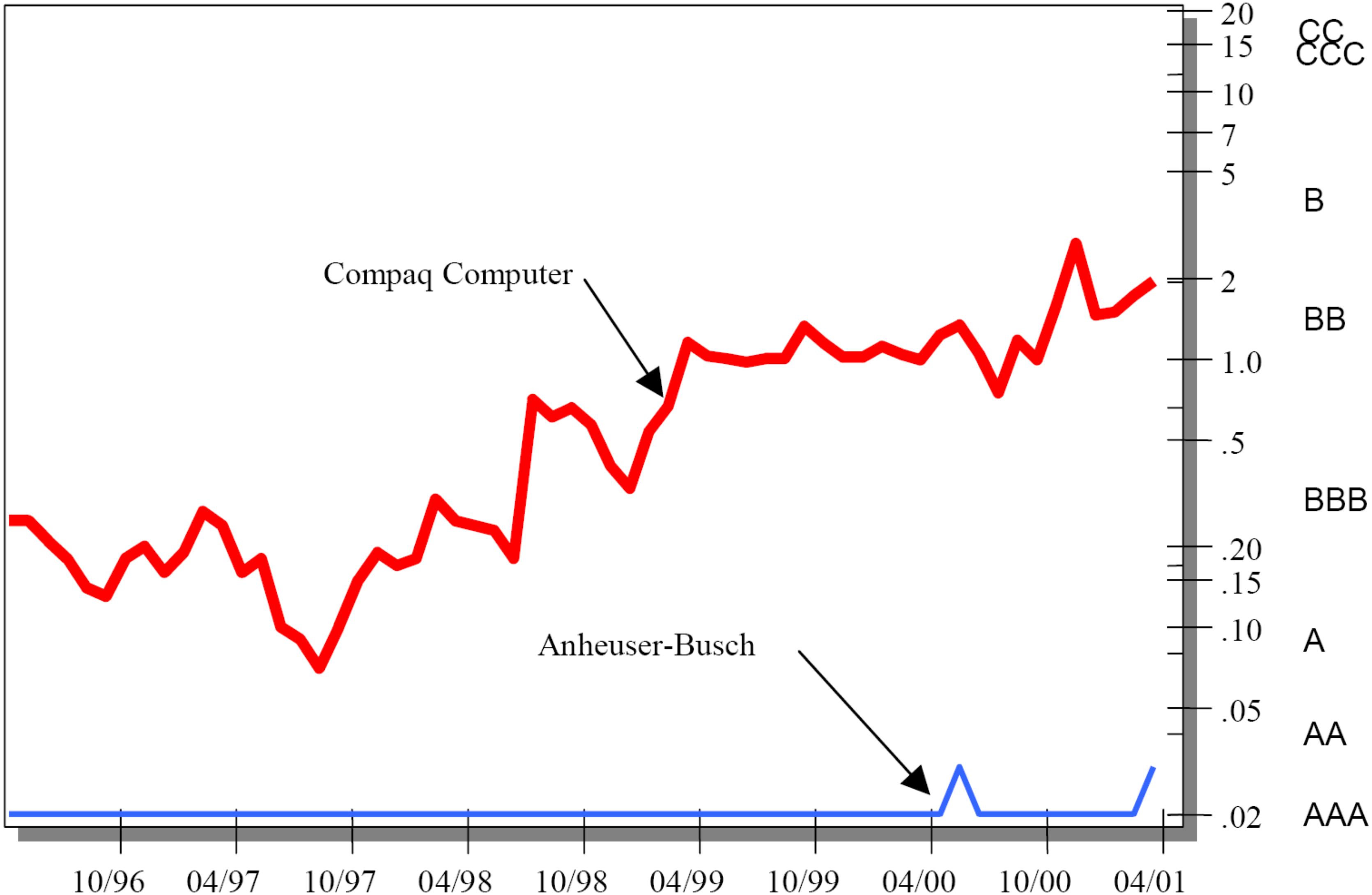

FIGURE 3 Corresponding evolution of the annual default probabilities

Source: Moody's-KMV

What do we learn from these plots?

- The volatility of a firm's assets is a major determinant of its risk of default
- But how do we estimate it?
- We need to use Merton model more powerfully than just use distance to default computations...

Implementing Structural Models: Estimating Value and Volatility of Firms Assets

Firm Value and Volatility

- Structural models depend on ***value*** and ***volatility*** of ***firm assets***
  - ✓ neither is directly observable

- Value of ***equity*** = stock price *x* number of shares

- Problem is the value of ***debt***
  - ✓ which ***debt to include***? (e.g., should some short-term debt be netted against short-term assets)
  - ✓ what is ***value of debt***? only part of total debt will be traded.
  - ✓ total borrowings observed only in periodic ***accounts***.

Unobservability of firm values

- The value of equity, viewed as a call on the firm, depends on V and σ_V (as well as the ***observable*** variables (B, T, r))
- Letting f denote the call pricing function, and suppressing dependence on the observable variables, we write

$$E = f(V, \sigma_V)$$

- Under the Black-Scholes assumptions,

$$E = f(V, \sigma_V) = V\ N(d_1) - e^{-rT} B\ N(d_1 - \sigma\sqrt{T})$$

where

$$d_1 = \frac{\ln(V/B) + (r + \frac{1}{2}\sigma_V^2)T}{\sigma_V \sqrt{T}}$$

- But we have two unknowns, so one equation is not enough

Unobservability of firm volatility

- Since equity is an option on form value, the ***volatility*** of equity, denoted σ_E , is also a function of V and σ:

$$\sigma_E = g(V, \sigma)$$

- For example, in a Black-Scholes world, we have

$$\sigma_E = g(V, \sigma) = \sigma \; V \; \frac{f_V}{f} = \sigma \; V \; \frac{N(d_{=})}{E}$$

where f is the call pricing function and f_V is the partial derivative of f with respect to V. It is essentially the "delta" of equity with respect to firm value, $N(d_1)$

Two equations, two unknowns

- Expressions for E and σ_E give us two (***non-linear***) equations for two unknowns V and σ_V in terms of

  - Equity Value E

  - Equity volatility σ_E

  - Other observable variables (B, T, r).

- Since equity values are observable and equity volatility may be computed, we can use these three expressions to solve for the unknowns V and σ_V.

- The associated spreadsheet provides a numerical illustration of this procedure using spreadsheet.

The MKMV Model

- The MKMV model uses a four-step procedure to track changes in credit risk for publicly-traded firms:

1. Identify the ***default point*** B to be used in the computation.
   - Set to Short-term liabilities + 0.5 * Long-term liabilities
1. Use the default point in conjunction with the firm's equity value and equity volatility to identify the firm value V and the firm volatility $\sigma_{V.}$
2. Given these quantities, identify the number of standard deviation moves that would result in the firm value falling below B. This is the firm's ***distance-to-default*** .
3. * Use its database to identify the proportion of firms with distance-to-default who actually defaulted within a year. This is the ***expected default frequency*** or EDF.

Distance-to-Default and EDF

- In principle, we should be able to compute default frequencies using

  - the distance-to-default, and

  - the probability distribution governing the evolution of V.

- However, it turns out that under the usual assumptions, this method ***underpredicts*** defaults by a large margin.

  - It is typical to assume ***normality*** in returns and value distribution.

  - reality is ***fat-tailed*** (leptokurtic) and extreme events are far more likely.

- Using the empirical database improves default predictions enormously.

Distance-to-Default and EDF (Cont'd)

- For instance

  - For distance-to-default of 4.3, the EDF using KMV's database was 0.03%. Had we used normality, it would have been 0.00086%!

  - For distance-to-default of 3.2, the EDF using KMV's database was 0.25%. Using normality, it would have been 0.069%!

- In fact, the default probabilities predicted by the Merton Model-which is based on normality-would imply that well over 50% of all US companies are AAA or better!
- So MKMV uses Merton model to rank credit risk of firms in a relative sense but does not use its absolute probability

A direct alternative method

- Alternatively
  - ✓ use market value for equity (*E*) and equity volatility
  - ✓ book value for *total* debt (D^*) and market value of *traded* debt for debt volatility
  - ✓ calculate $V = E + D^*$ and calculate leverage ratio
  - ✓ Calculate asset volatility from debt and equity volatility, correlation and leverage:

$$R_V = \frac{E}{V} R_E + \frac{D^*}{V} R_D$$

$$\sigma_V^2 = {\frac{E}{V}}^{2} \sigma_E^2 + {\frac{D^*}{V}}^{2} \sigma_D^2 + 2 \frac{E}{V} \frac{D^*}{V} \operatorname{cov}(R_E, R_D)$$

The Volatility of Corporate Assets

	All	AAA	AA	A	BBB	BB	B
			Quasi-Market Leverage				
Mean	0.34	0.10	0.21	0.32	0.37	0.50	0.66
Std.Dev.	0.21	0.08	0.19	0.20	0.17	0.23	0.22
			Equity Volatility				
Mean	0.32	0.25	0.29	0.31	0.33	0.42	0.61
Std.Dev.	0.13	0.06	0.10	0.11	0.13	0.19	0.19
			Estimated Asset Volatility				
Mean	0.22	0.22	0.22	0.21	0.22	0.23	0.28
Std.Dev.	0.08	0.05	0.07	0.08	0.08	0.08	0.08

$$L_{jt} = \frac{\text{Book Value of Debt (Compustat items 9 and 34)}}{\text{Book Value of Debt + Market Value of Equity}}$$

L_{jt}: Quasi-market leverage ratio of firm j, time t

Estimated asset volatility

$$\sigma_{Ajt}^2 = (1-L_{jt})^2\sigma_{Ejt}^2 + L_{jt}^2\sigma_{Djt}^2 + 2L_{jt}(1-L_{jt})\sigma_{ED,jt}$$

Bond Prices in the Merton Model

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- Recall: The Black-Scholes value for a call on the firm assets with exercise price *B*, i.e., the value of equity, *E*, is:

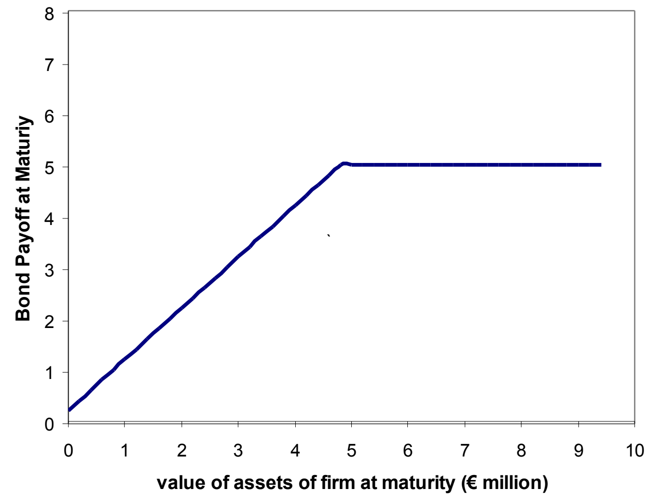
$$E = VN(d_1) - PV(B)N(d_2);\ d_1 = \frac{\ln(V/B) + (r + \frac{1}{2}\sigma_V^2)T}{\sigma_V \sqrt{T}} \quad \text{and } d_2 = d_1 - \sigma\sqrt{T}$$

- Since the bond value, *D*, is the firm value minus the equity value:

$$\begin{aligned} D &= V - E \\ &= V - \ VN(d_1) - PV(B)N(d_2) \\ &= V(1 - N(d_1)) + PV(B)N(d_2) \\ &= VN(-d_1) + PV(B)N(d_2) \quad \text{since } 1 - N(x) = N(-x) \end{aligned}$$

Understanding the Pricing Formula

$$D = VN(-d_1) + PV(B)N(d_2)$$



- $N(d_2)$: is the risk-neutral probability of survival. (To see this, look at the expression for the default probability derived earlier and substitute the riskless rate for the expected return μ).
- The payoff in the case of survival (no-default) is B, and its value is therefore the risk-neutral expected payoff ($BN(d_2)$) discounted at the riskless rate, i.e., $PV(B)N(d_2)$.
- The first term, $VN(-d_1)$, is the value of the payment in the case of default.

$$D = \underbrace{VN(-d_1)}_{\substack{\text{value of payment} \\ \text{(V) in default}}} + \underbrace{PV(B)\overbrace{N(d_2)}^{\substack{\text{risk-neutral prob} \\ \text{of survival}}}}_{\substack{\text{value of payment of} \\ \text{B if no default}}}$$

Credit Spreads in the Merton Model

- The promised yield on the bond in the Merton model is

$$y = r + s \text{ where } s \text{ is the "credit spread" and } y \text{ is defined by:}$$

$$D = e^{-yT}B = e^{-(r+s)T}B = e^{-sT}PV(B)$$

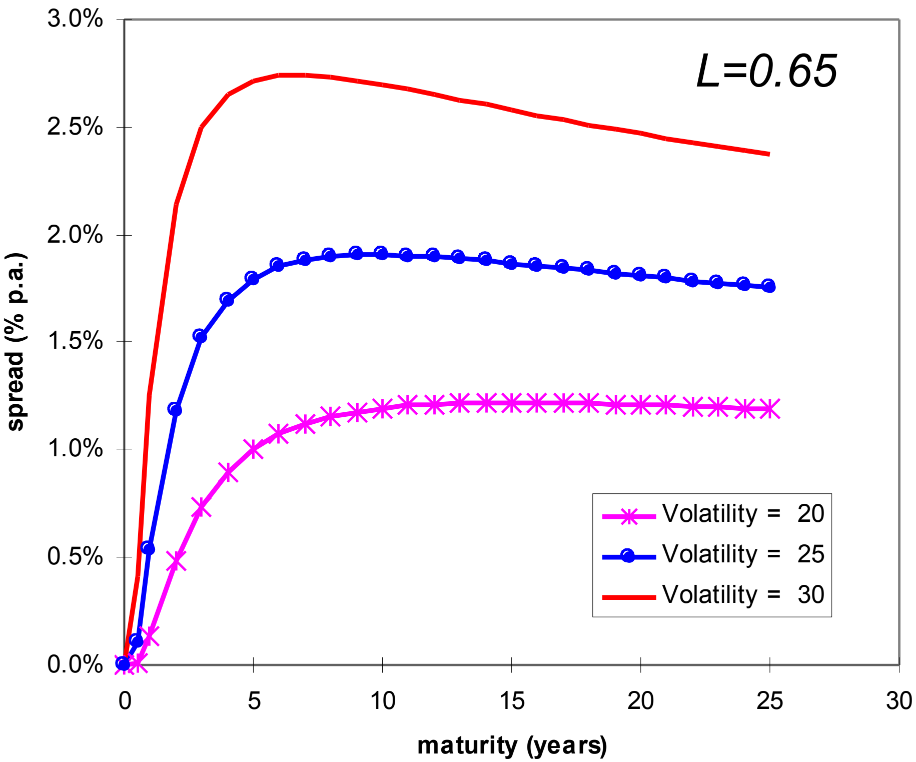
$$\text{and} \quad D = VN(-d_1) + PV(B)N(d_2)$$

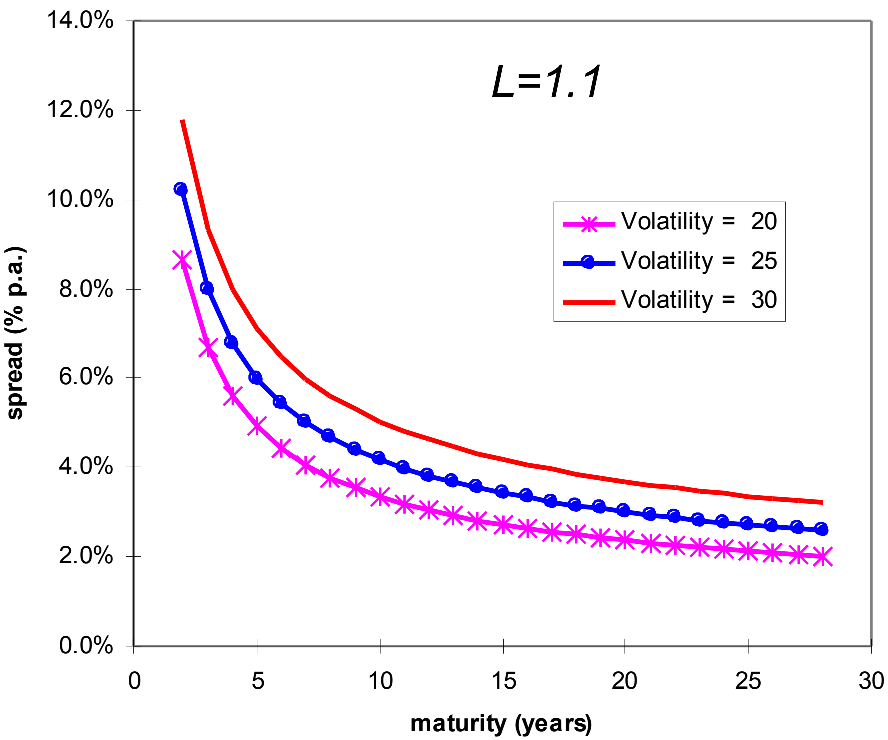
- If we now define the "quasi leverage ratio", L, as $PV(B)/V$, i.e., the debt-to-firm value ratio, but valuing the debt using the riskless rate , then we can express the spread simply as a function of L and $\sigma\sqrt{T}$

$$s = -\frac{1}{T}\ln\ \ \frac{1}{L}N(-d_1) + N(d_2)\ \ , \text{ where } d_1 = \frac{\ln(1/L)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}, \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

The Merton Model, contd.

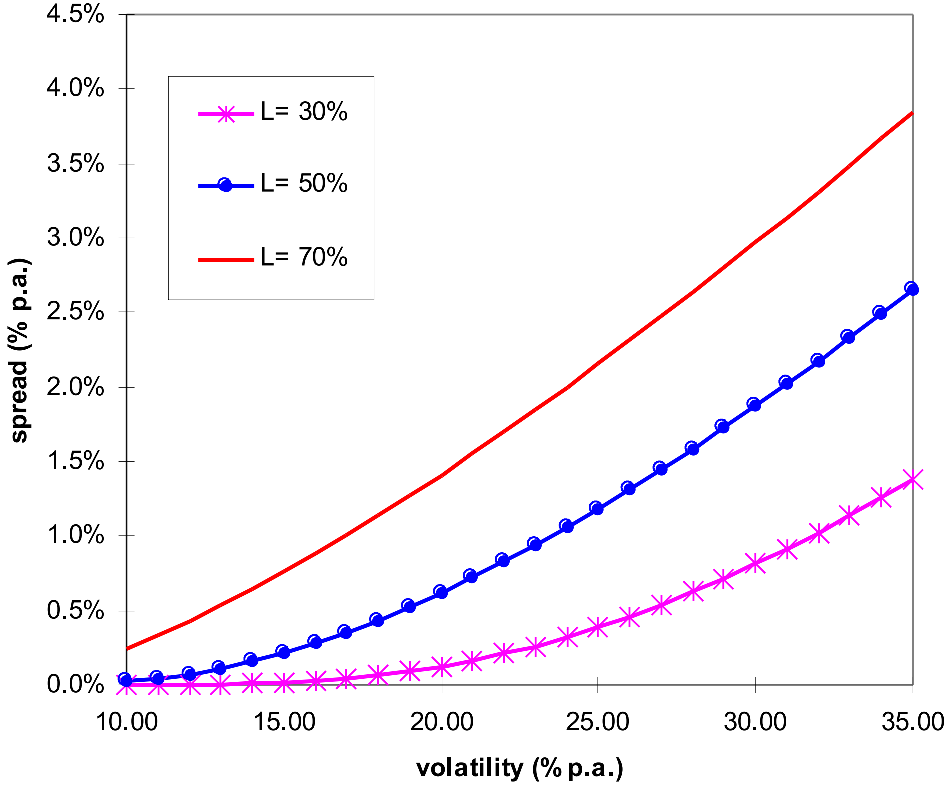
- The credit spread depends on two variables: ***asset volatility*** and the ***quasi-leverage ratio:*** *L=PV(B)/V*
- As expected the credit spread is increasing with respect to both variables
- Even in this simple model the term structure of credit spreads can be upward sloping, downward sloping or "hump" shaped





Merton model: spreads vs. leverage and volatility

- In the Merton model the spread over riskless rate increases with volatility and leverage (L)



The Merton Model: Limitations

- Merton model provides important insight into the problem of pricing default.
- But some limitations...
  - ✓ the ***value of the firm***, V, and its ***volatility*** hard to estimate (more on this later)
  - ✓ constant ***risk-free rates*** … cannot model relation between interest rate risk, asset risk and default risk.
  - ✓ specification of ***default time restrictive***
  - ✓ ***default is never a surprise*****:** so long as $V>B$ default can never occur in next instant (Enron, Parmalat etc.)

Merton - Summary

- Important first step in modelling default
- Predictions of default are too low
- Predictions of credit spreads appear too low (more later)
- Occurrence of default only at maturity is major limitation
  - ✓ ***BUT***, inclusion of early default does not necessarily increase spreads.

Summary

- Merton model
  - ✓ basis for all structural models of credit risk
  - ✓ default probability and prices in Merton model
  - ✓ concept of distance-to-default (used in MKMV is estimating default probabilities)

- Next Session: Models with early default
  - ✓ default probabilities
  - ✓ empirical evidence