# Structural Models I

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Credit Risk Elective
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# The Black-Scholes-Merton Option Pricing Model

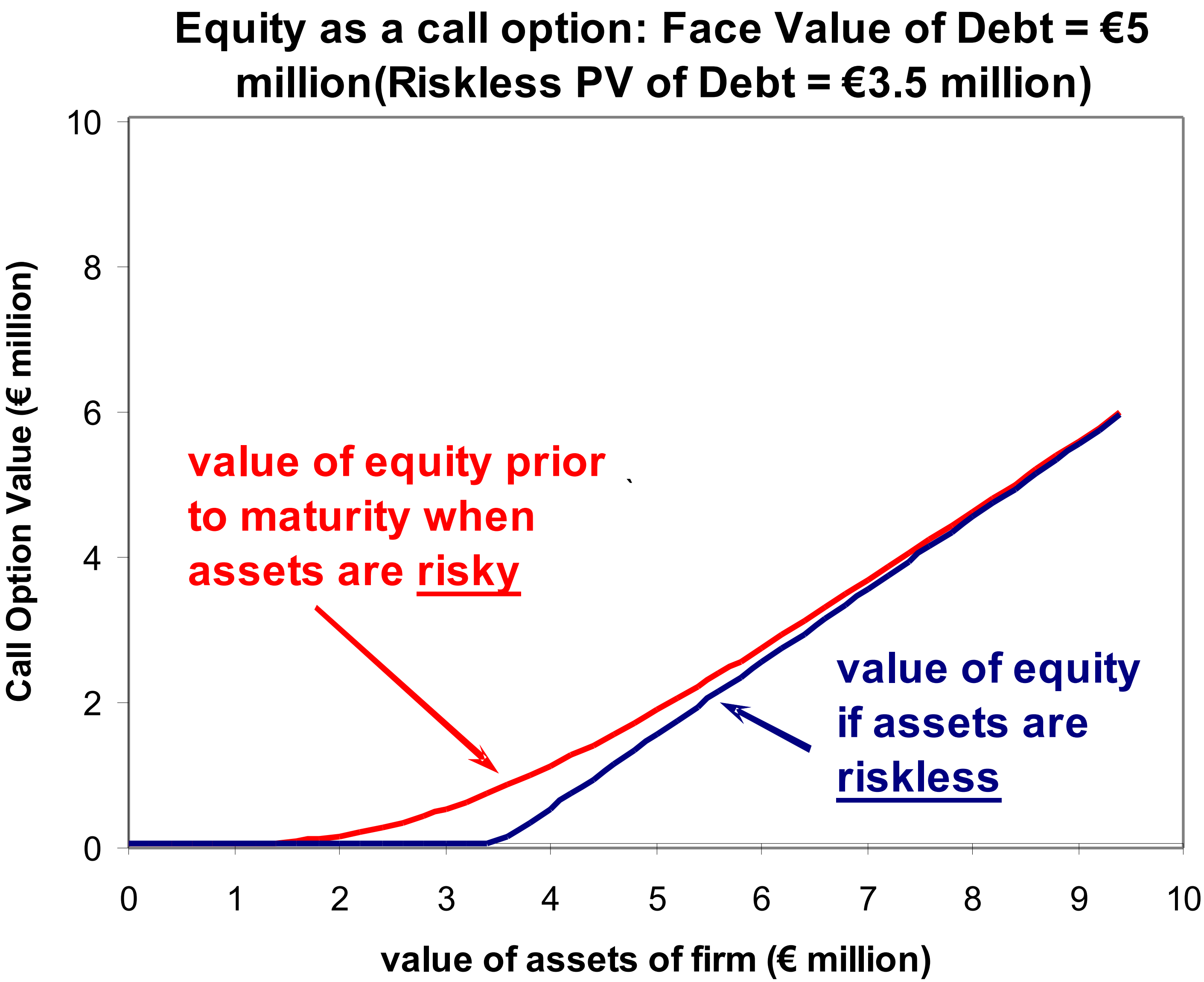
"… options are specialized and relatively unimportant financial securities …".

***Robert Merton – Nobel prize winner for work on option pricing – in 1974 seminal paper on option pricing:***

- Great hope for the new theory was the valuation of corporate liabilities, in particular
  - ✓ equity
  - ✓ **corporate debt**

# Equity is a call option on the firm

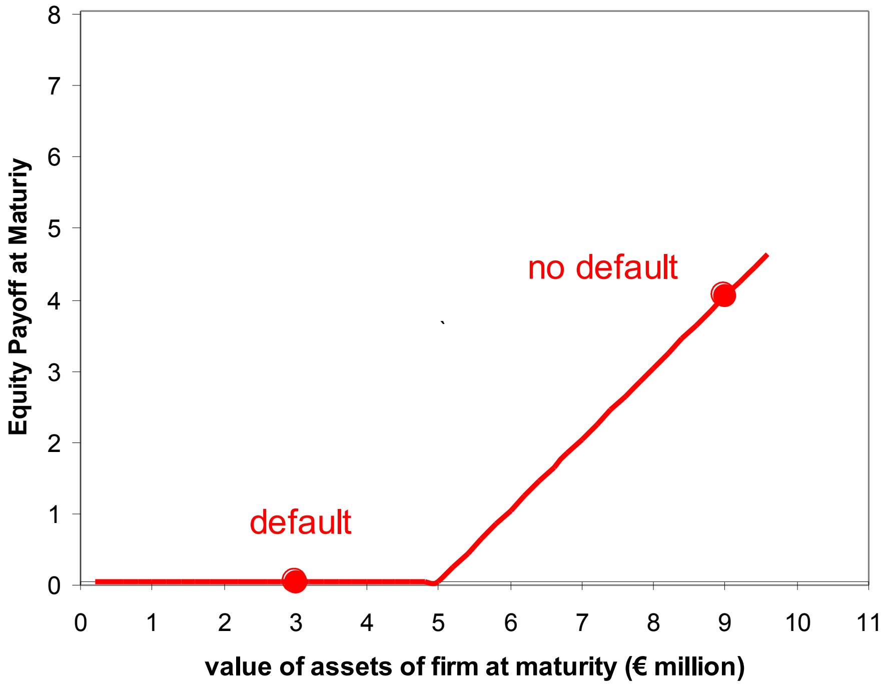
- Suppose a firm has borrowed ***€5 million*** (zero coupon) and that at the time the loan (5 years, say) is due
- ***Scenario I:*** the ***assets of the firm are worth €9 million***:
  - ✓ lenders get €5 million (paid in full)
  - ✓ equity holders get residual: €9 - €5 = ***€4 million***
- ***Scenario II:*** the ***assets are worth, say, €3 million***
  - ✓ firm defaults, lenders take over assets and get ***€3 million***
  - ✓ equity holders receive zero (Limited liability)
- Payments to equity holders are those of a call option written on the assets of the firm with a strike price of ***€5 million***, the face value of the debt


Equity as a call option: Face Value of Debt = €5 million(Riskless PV of Debt = €3.5 million)
Call Option Value (€ million)
value of assets of firm (€ million)
value of equity prior to maturity when assets are risky
value of equity if assets are riskless
0
2
4
6
8
10
0
1
2
3
4
5
6
7
8
9
10

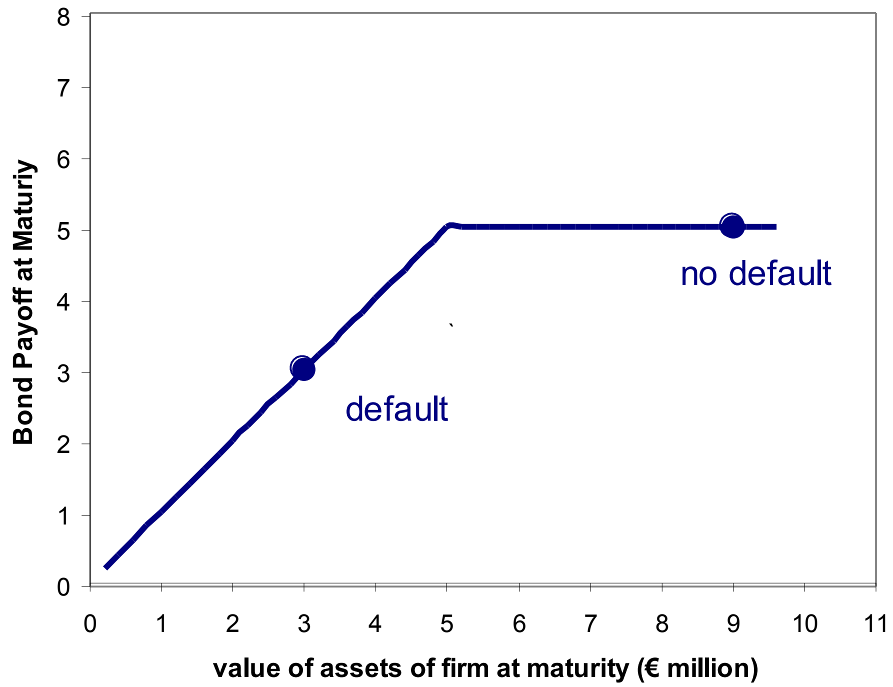
# Payoffs to Debt and Equity *at Maturity*

- Firm has single 5-year zero-coupon bond outstanding with face value *B=5 (million)*

***Equity*** is a ***call option*** on the ***assets of the firm***

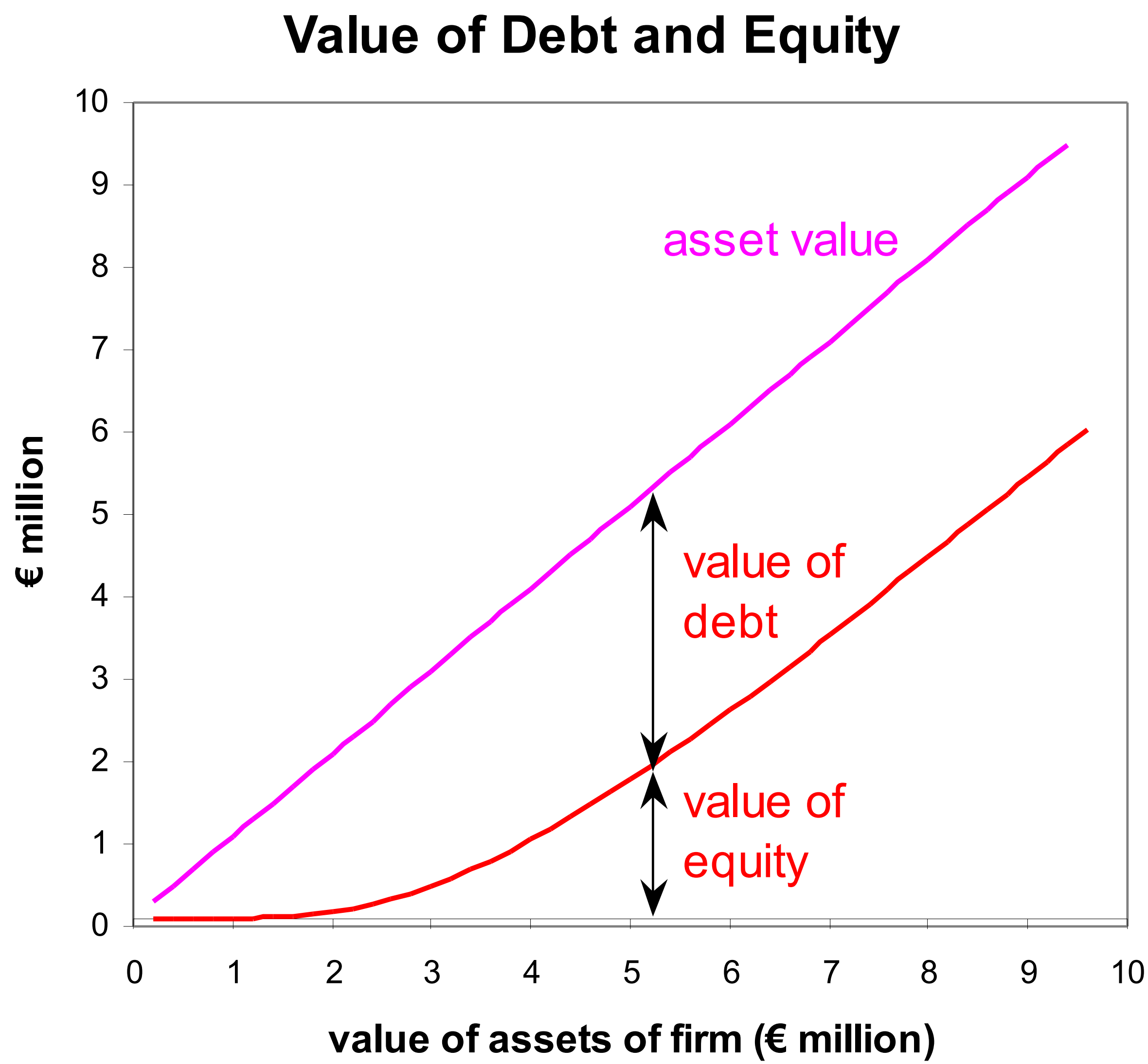



Payoff on ***risky debt*** looks like this

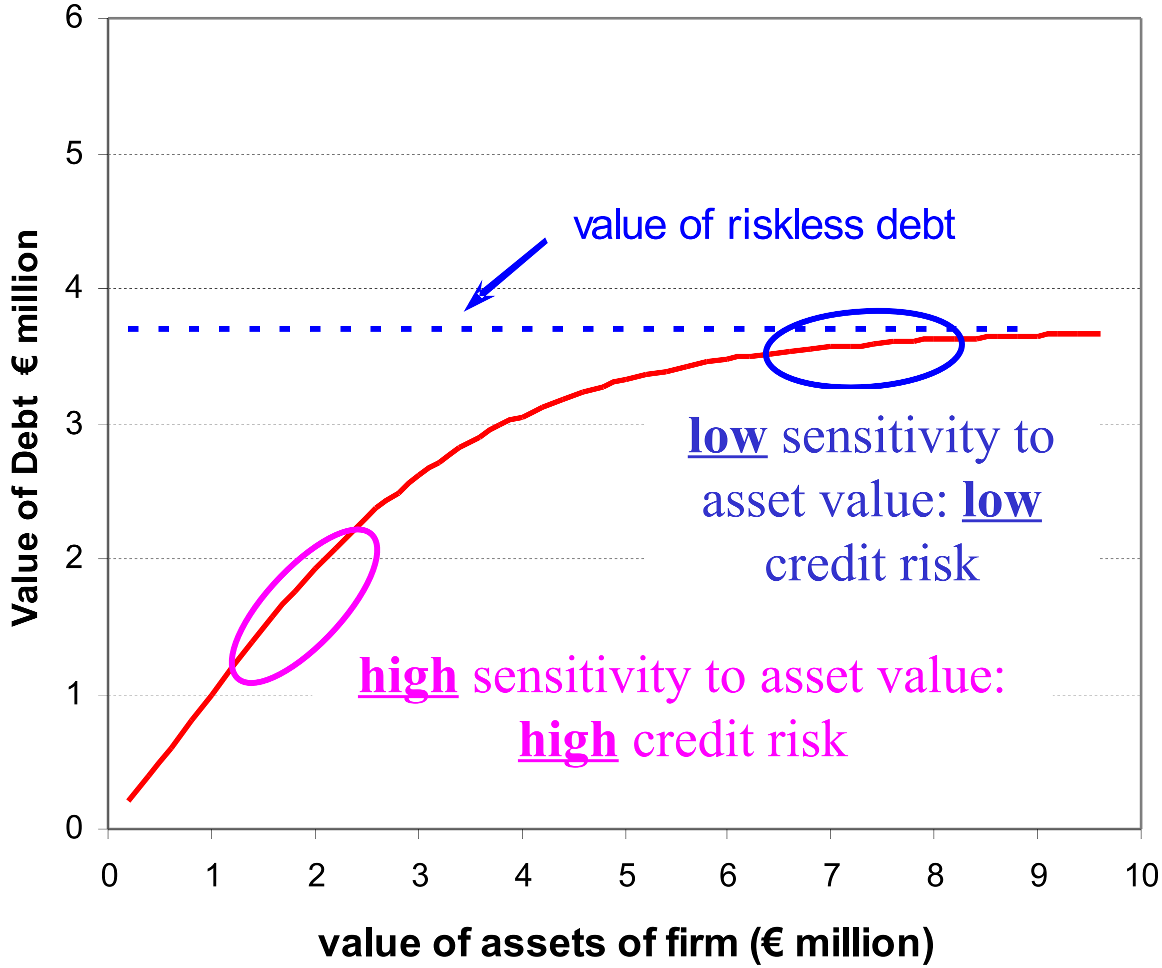


# *Prior* to maturity …

- value of the debt is value of firm's assets less the value of the equity (a call)



# Value of Debt and Credit Exposure



# What is the price discount on credit risky debt?

put-call parity

$$\text{underlying asset} = \boxed{\text{riskless bond} - \text{put option}} + \boxed{\text{call option}}$$

Modigliani-Miller

$$\text{value of firm assets} = \boxed{\text{bond value}} + \boxed{\text{equity value}}$$

- Since equity is a ***call option***

$$\text{value of risky debt} = \text{riskless bond} - \text{value of put option on assets}$$

- ***Merton model*** uses Black-Scholes to value the (default) put.

# Limited Liability and the "Default Put"

- ***Limited liability*** of equity means that no matter how bad things get, equity holders can walk away from firm's debt in exchange for payoff of zero

- ***Limited liability*** equivalent to equity holders:
  - ✓ issuing ***riskless debt***

  **BUT**

  - ✓ lenders giving equity holders a ***put*** on the ***firm's assets*** with a ***strike*** price equal to the ***face amount*** of the debt (***"default put")***

# Understanding Credit

- This insight by Merton provides the basis for one of the two most useful ways of thinking about and analysing credit risky instruments

# Outline of Session

- Merton model (direct application of Black-Scholes) to valuing zero-coupon risky debt
  - ✓ Default only at Maturity

- MKMV Approach
  - ✓ A sketch of how the approach works
  - ✓ Exact details to follow

# The Merton Model

# Valuation Theory: Merton Model

- ***Merton*** (1974) the first to use option pricing theory to value credit risky debt
- Assumptions follow ***Black-Scholes*** model
  - ✓ lognormal distribution for value of assets of firm
  - ✓ no uncertainty in interest rates
- Value of ***risky bond*** is simply value of equivalent ***riskless bond minus Black-Scholes value of put*** on assets
- Merton model:
  - ✓ ***basis for all structural models***
  - ✓ has been generalised and extended by Longstaff/Schwartz, Leland, Leland-Toft, and others

# The Merton Model: Assumptions

- ***Parameters***
  - ✓ constant interest rate: *defines risk-free rate process*
  - ✓ constant volatility of firm value
- ***Structure of debt***
  - ✓ zero coupon bond is only liability
- ***Nature of bankruptcy***
  - ✓ costless bankruptcy: *nothing to the lawyers*
  - ✓ strict priority of claims preserved: *defines recovery rate (1-L)*
  - ✓ bankruptcy triggered at maturity when value of assets falls below face value of debt: *defines default event*

# Natural Distribution of Firm Value in the Merton Model

- If the firm value at time $T$ is $V_T$, then total continuously compounded return from time zero to $T$ is:

$$\ln \frac{V_T}{V} \quad , \qquad \text{where } V \text{ is the current firm value}$$

- In Black-Scholes-Merton, the ***total continuously compounded return*** has a ***normal distribution***:

$$\ln \frac{V_T}{V} \sim N\left(\left(\mu - \frac{1}{2}\sigma_V^2\right) T, \sigma_V^2 T\right)$$

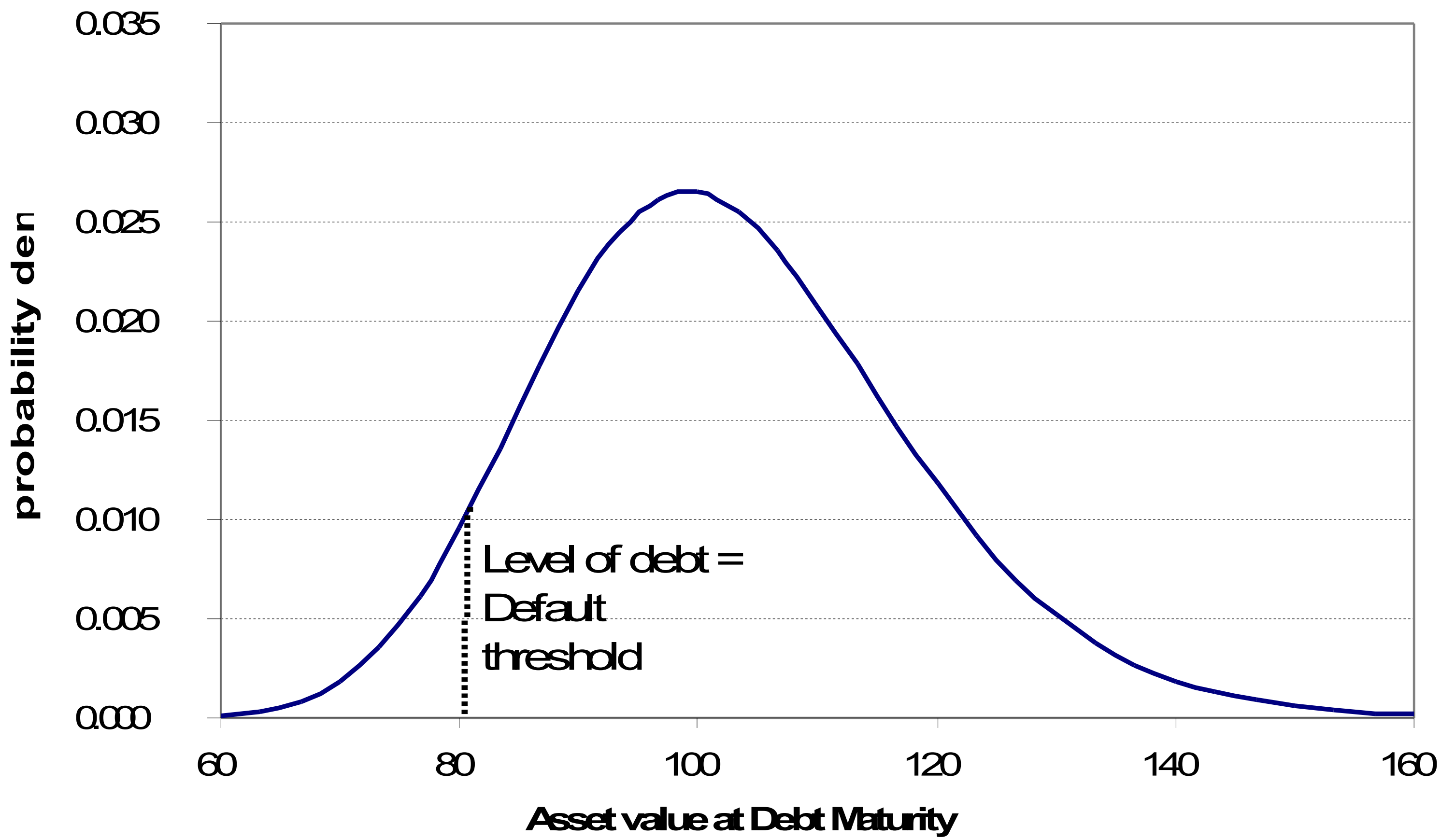
where $\mu$ and $\sigma$ are the (natural) expected return and standard deviation of continuously compounded returns on the firm's assets

- We can therefore write $V_T$ as:

$$V_T = V \exp\left(\left(\mu - \frac{1}{2}\sigma_V^2\right)T + \sigma_V \sqrt{T}\varepsilon\right) \qquad \text{where } \varepsilon \sim N(0,1)$$

- The ***distribution of $V_T$ is lognormal***

# Default scenario on normal distribution




Default when $B > V_T = V \exp((\mu - \frac{1}{2}\sigma_V^2)T + \sigma_V \sqrt{T}\varepsilon)$ where $\varepsilon \sim N(0,1)$

# Natural Default Probability in the Merton Model

- ***Default*** occurs in the Merton model when, at maturity, $V_T$ is lower than the face amount of the debt, $B$. In other words when:

$$V_T = V\exp((\mu - \frac{1}{2}\sigma_V^2)T + \sigma_V\sqrt{T}\varepsilon) \quad B \ \text{ i.e. when }, \varepsilon \quad \frac{\ln(B/V) - (\mu - \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}$$

- And so the ***probability of default*** is simply

$$\text{prob} \ \ \varepsilon \ = \frac{\ln(B/V) - (\mu - \frac{\text{١}}{\text{٢}}\sigma_V^{\text{٢}})T}{\sigma_V\sqrt{T}} \quad N \ \frac{\ln(B/V) - (\mu - \frac{\text{١}}{\text{٢}}\sigma_V^{\text{٢}})T}{\sigma_V\sqrt{T}}$$

- Where *N(.)* is the cumulative normal distribution

# The Distance-to-Default

$$\text{default value for } \varepsilon = \frac{\ln(B/V) - (\mu - \frac{1}{2}\sigma_V^2)T}{\sigma_V \sqrt{T}}$$

- In the Merton model, default occurs when the "surprise" term, $\varepsilon$, is large enough (typically a large ***negative*** number). ***What does this number mean?***
- In the numerator, *ln(B/V)* is the ***actual*** continuously compounded return on the assets that is necessary to lead to default.
  - ✓ if $V > B$, this return is negative (i.e., the asset value must fall to lead to default).
- The term $(\mu - \sigma^2/2)T$ is the ***expected value*** of the continuously compounded return (usually positive)
- Thus the numerator is the difference between the actual continuously compounded rate of return required for default and the expected value of the return, i.e., it is the "***surprise***", or unexpected component of the rate of return necessary for default.
- The ***denominator*** is the standard deviation of the rate of return

# The Distance-to-Default, contd.

$$\text{default value for } \varepsilon = \frac{\ln(B/V) - (\mu - \frac{1}{2}\sigma_V^2)T}{\sigma_V \sqrt{T}}$$

- Therefore, the ***ratio*** (again typically negative) measures the number of standard deviations of return necessary to lead to default at time $T$
- ***The negative of this ratio (a positive number) is called the distance-to-default***

$$\text{Distance-to-Default} = \frac{\ln(V/B) + (\mu - \frac{1}{2}\sigma_V^2)T}{\sigma_V \sqrt{T}}$$

* Note: Sometimes, the term "distance to default" is applied to other, closely related, quantities

# Merton Model Distance to Default and Default Probabilities

- Distance to default is smaller (and default probability higher) when volatility is higher  and maturity is longer

## Distance-to-Default*

| | | V | | | |
|---|---|---|---|---|---|
| ***Vol*** | ***T*** | ***150*** | ***100*** | ***80*** | ***60*** |
| 20% | 1 | 5.89 | 3.87 | 2.75 | 1.31 |
| 20% | 20 | 3.02 | 2.56 | 2.31 | 1.99 |
| 40% | 1 | 2.80 | 1.78 | 1.23 | 0.51 |
| 40% | 20 | 0.84 | 0.61 | 0.49 | 0.33 |

## Default Probabilities*

| | | V | | | |
|---|---|---|---|---|---|
| ***Vol*** | ***T*** | ***150*** | ***100*** | ***80*** | ***60*** |
| 20% | 1 | 0.00% | 0.01% | 0.30% | 9.48% |
| 20% | 20 | 0.13% | 0.52% | 1.03% | 2.31% |
| 40% | 1 | 0.26% | 3.73% | 11.03% | 30.65% |
| 40% | 20 | 20.11% | 27.06% | 31.34% | 37.24% |

*Note: Assumptions - expected return on assets = 10%; face value of debt = 50