Structural Models I

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Credit Risk Elective
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The Black-Scholes-Merton Option Pricing Model

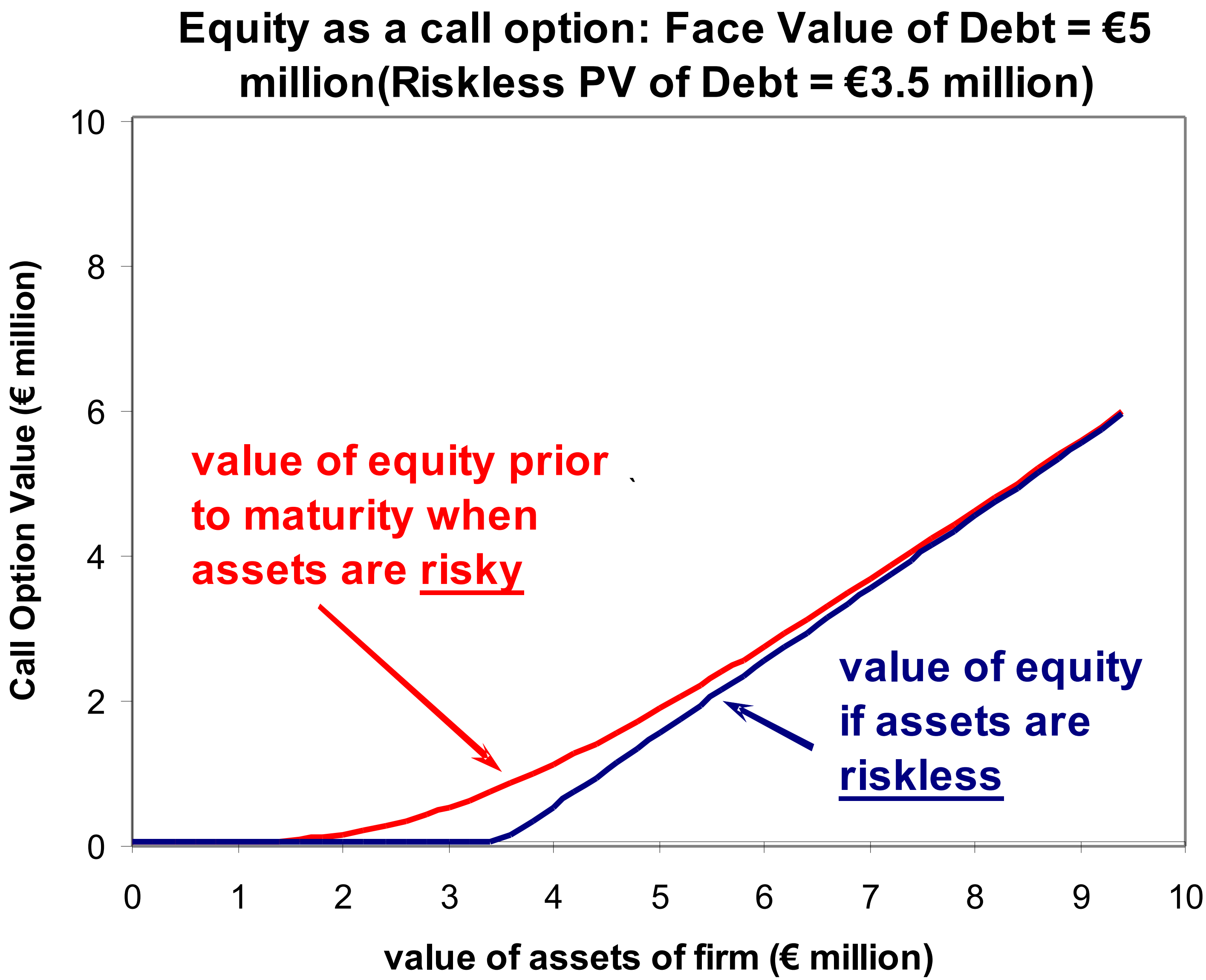
"… options are specialized and relatively unimportant financial securities …".

Robert Merton – Nobel prize winner for work on option pricing – in 1974 seminal paper on option pricing:

- Great hope for the new theory was the valuation of corporate liabilities, in particular
  - ✓ equity
  - ✓ **corporate debt**

Equity is a call option on the firm

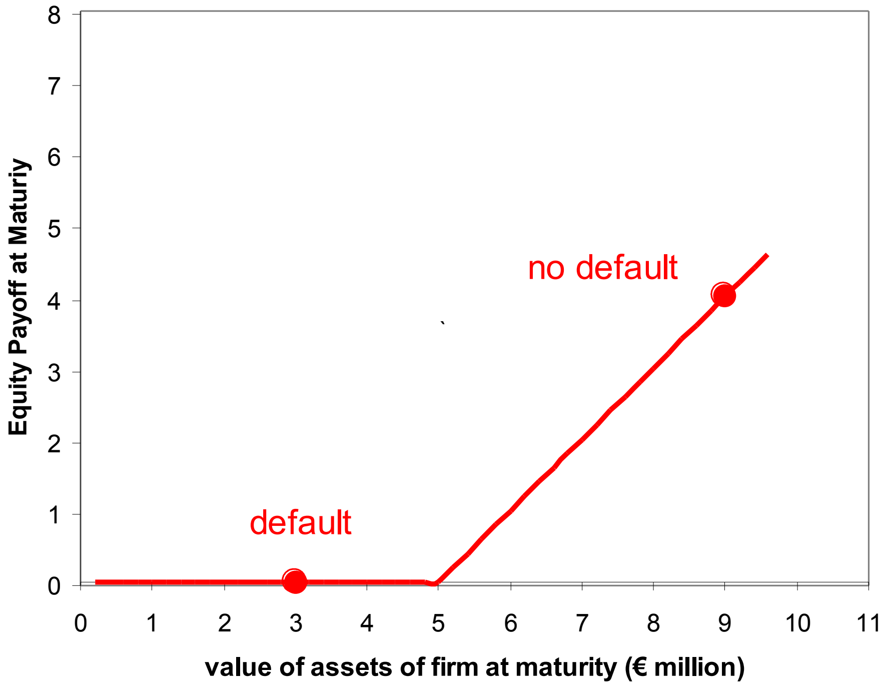
- Suppose a firm has borrowed ***€5 million*** (zero coupon) and that at the time the loan (5 years, say) is due
- ***Scenario I:*** the ***assets of the firm are worth €9 million***:
  - ✓ lenders get €5 million (paid in full)
  - ✓ equity holders get residual: €9 - €5 = ***€4 million***
- ***Scenario II:*** the ***assets are worth, say, €3 million***
  - ✓ firm defaults, lenders take over assets and get ***€3 million***
  - ✓ equity holders receive zero (Limited liability)
- Payments to equity holders are those of a <u>call option</u> written on the assets of the firm with a strike price of ***€5 million***, the face value of the debt


Equity as a call option: Face Value of Debt = €5 million(Riskless PV of Debt = €3.5 million)
Call Option Value (€ million)
10
8
6
4
2
0
0
1
2
3
4
5
6
7
8
9
10
value of assets of firm (€ million)
value of equity prior to maturity when assets are risky
value of equity if assets are riskless

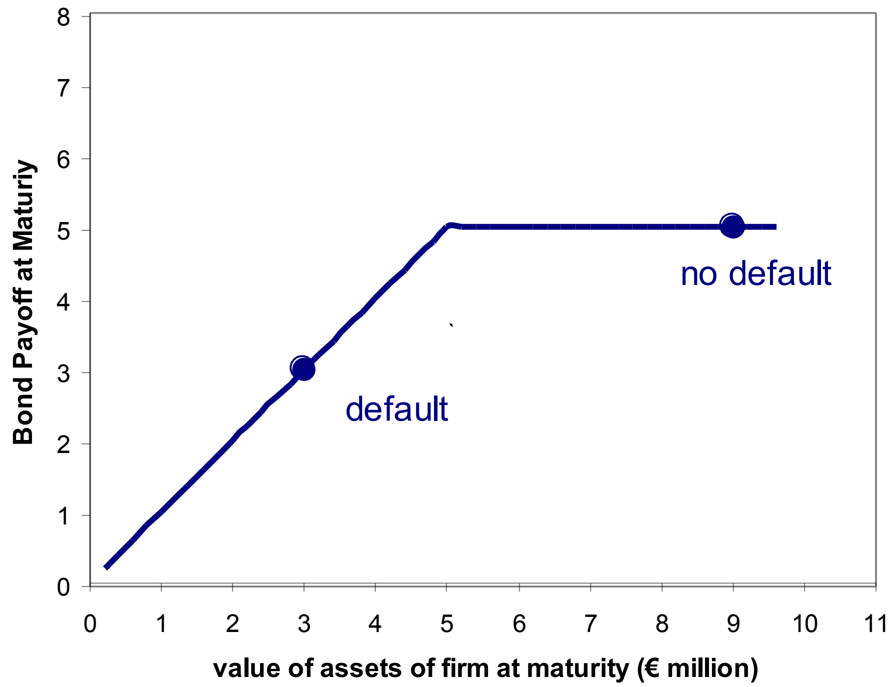
Payoffs to Debt and Equity *at Maturity*

- Firm has single 5-year zero-coupon bond outstanding with face value *B=5 (million)*

Equity is a ***call option*** on the ***assets of the firm***

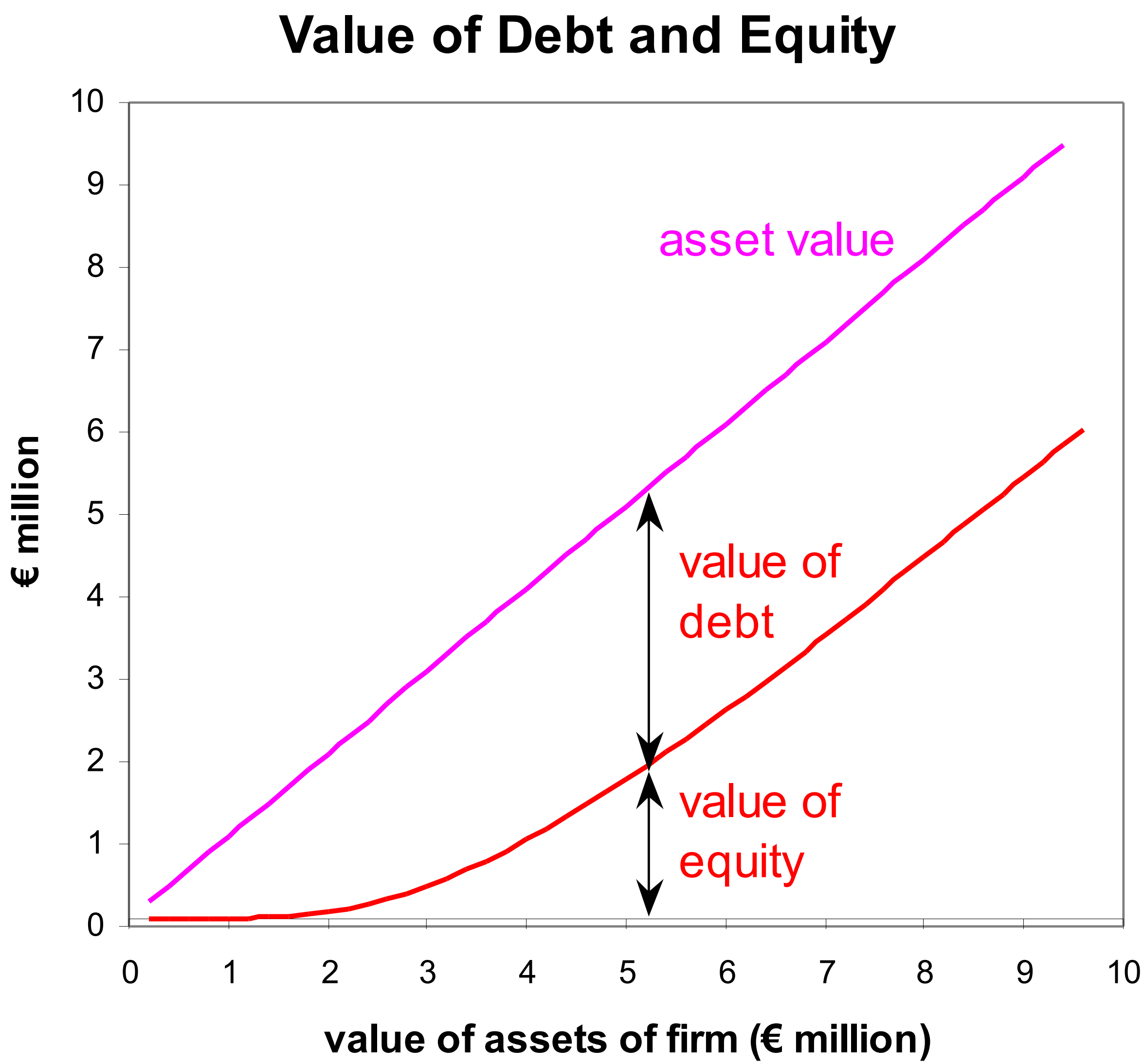


Payoff on ***risky debt*** looks like this

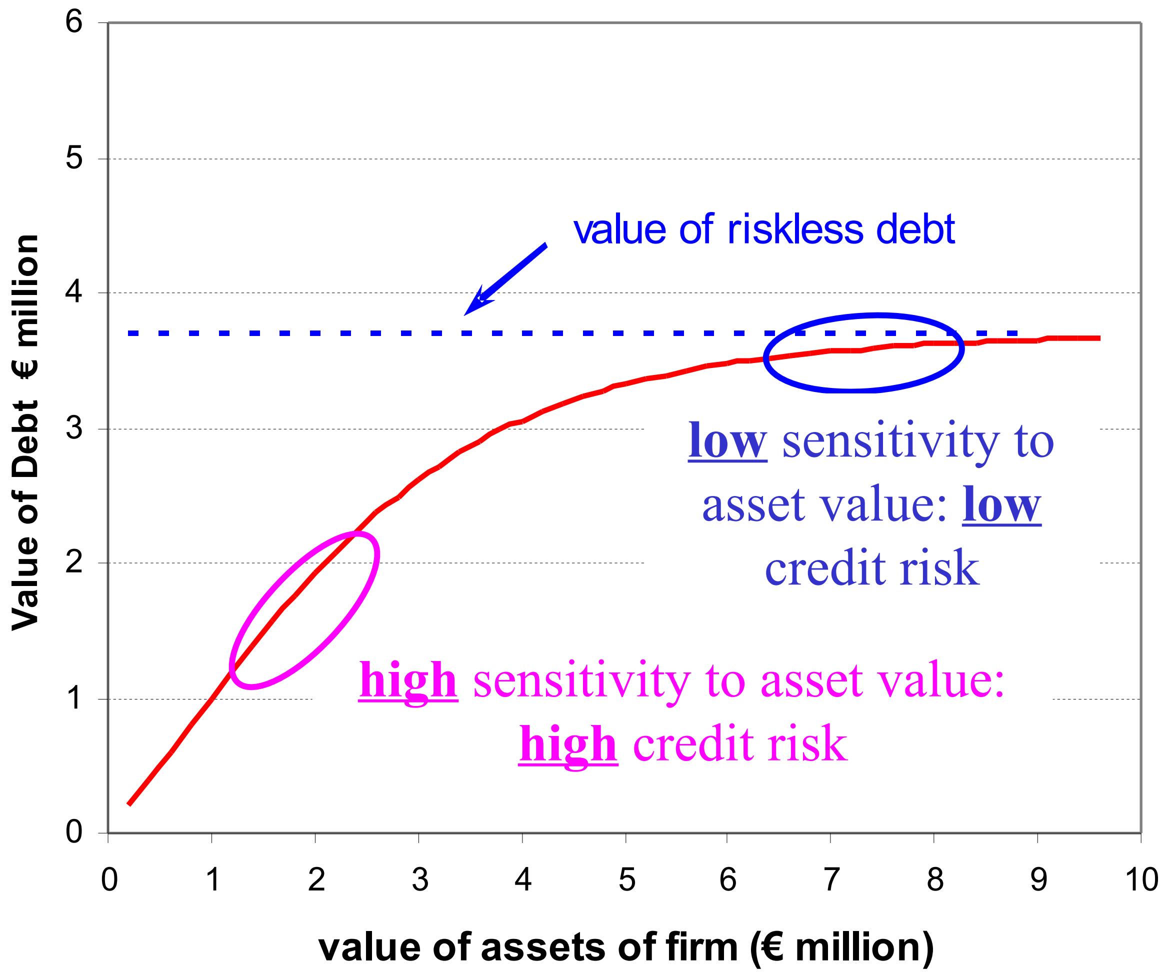


Prior to maturity …

- value of the debt is value of firm's assets less the value of the equity (a call)



Value of Debt and Credit Exposure



What is the price discount on credit risky debt?

put-call parity	underlying asset	=	riskless bond - put option	+	call option
Modigliani-Miller	value of firm assets	=	bond value	+	equity value

- Since equity is a ***call option***

$$\text{value of risky debt} = \text{riskless bond} - \text{value of put option on assets}$$

- ***Merton model*** uses Black-Scholes to value the (default) put.

Limited Liability and the "Default Put"

- ***Limited liability*** of equity means that no matter how bad things get, equity holders can walk away from firm's debt in exchange for payoff of zero

- ***Limited liability*** equivalent to equity holders:
  - ✓ issuing ***riskless debt***

  BUT

  - ✓ lenders giving equity holders a ***put*** on the ***firm's assets*** with a ***strike*** price equal to the ***face amount*** of the debt (***"default put"***)

Understanding Credit

- This insight by Merton provides the basis for one of the two most useful ways of thinking about and analysing credit risky instruments

Outline of Session

- Merton model (direct application of Black-Scholes) to valuing zero-coupon risky debt
  - ✓ Default only at Maturity

- MKMV Approach
  - ✓ A sketch of how the approach works
  - ✓ Exact details to follow

The Merton Model

Valuation Theory: Merton Model

- ***Merton*** (1974) the first to use option pricing theory to value credit risky debt
- Assumptions follow ***Black-Scholes*** model
  - ✓ lognormal distribution for value of assets of firm
  - ✓ no uncertainty in interest rates
- Value of ***risky bond*** is simply value of equivalent ***riskless bond minus Black-Scholes value of put*** on assets
- Merton model:
  - ✓ ***basis for all structural models***
  - ✓ has been generalised and extended by Longstaff/Schwartz, Leland, Leland-Toft, and others

The Merton Model: Assumptions

- ***Parameters***
  - ✓ constant interest rate: *defines risk-free rate process*
  - ✓ constant volatility of firm value
- ***Structure of debt***
  - ✓ zero coupon bond is only liability
- ***Nature of bankruptcy***
  - ✓ costless bankruptcy: *nothing to the lawyers*
  - ✓ strict priority of claims preserved: *defines recovery rate (1-L)*
  - ✓ bankruptcy triggered at maturity when value of assets falls below face value of debt: *defines default event*

Natural Distribution of Firm Value in the Merton Model

- If the firm value at time T is V_T, then total continuously compounded return from time zero to T is:

$$\ln \frac{V_T}{V} \,, \quad \text{where } V \text{ is the current firm value}$$

- In Black-Scholes-Merton, the ***total continuously compounded return*** has a ***normal distribution***:

$$\ln \frac{V_T}{V} \sim N\left(\left(\mu - \frac{1}{2}\sigma_V^2\right)T, \sigma_V^2 T\right)$$

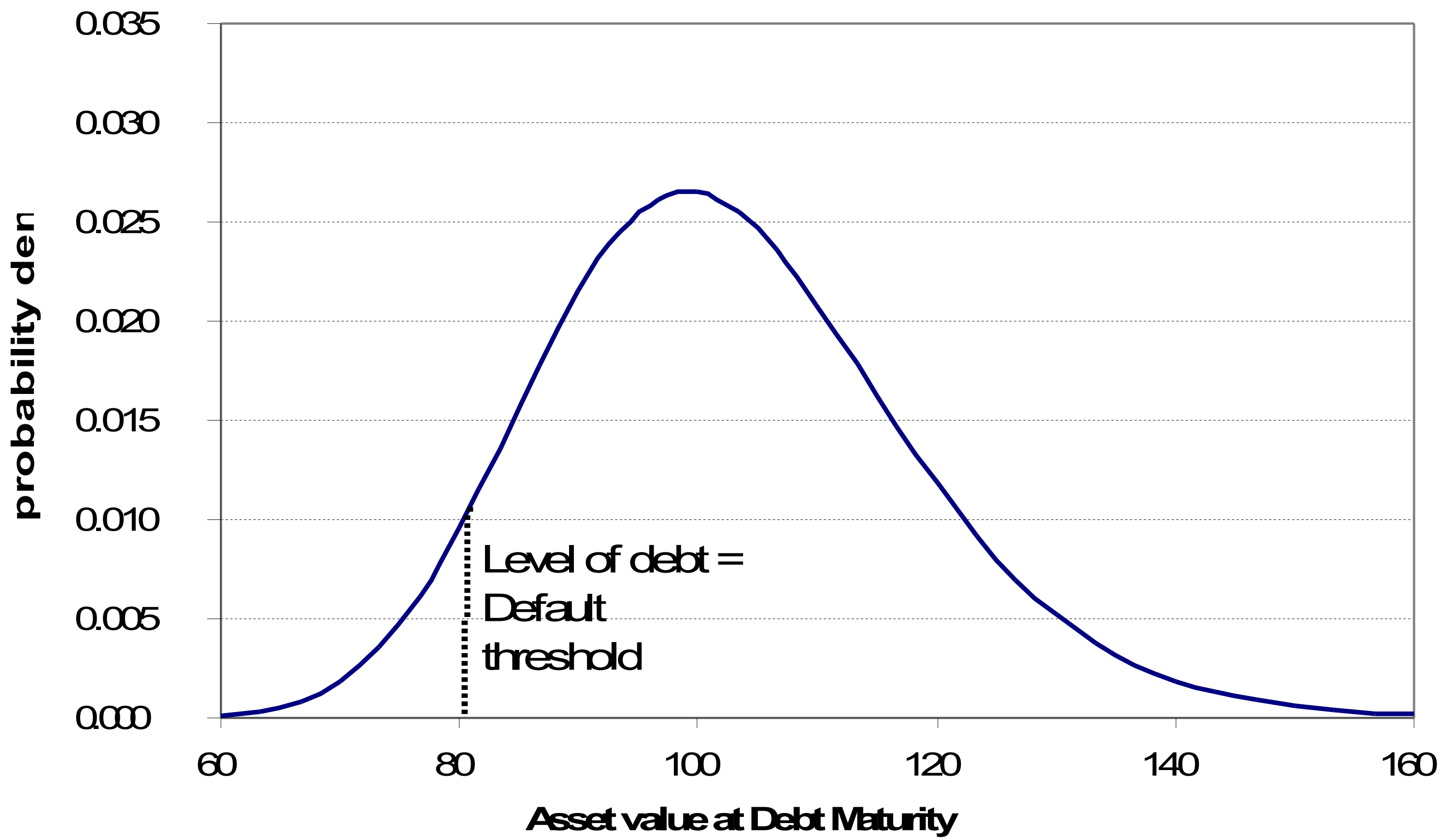
where μ and σ are the (natural) expected return and standard deviation of continuously compounded returns on the firm's assets

- We can therefore write V_T as:

$$V_T = V \exp((\mu - \frac{1}{2}\sigma_V^2)T + \sigma_V \sqrt{T}\varepsilon) \qquad \text{where } \varepsilon \sim N(0,1)$$

- The ***distribution of V_T is lognormal***

Default scenario on normal distribution



Default when $B > V_T = V \exp((\mu - \frac{1}{2}\sigma_V^2)T + \sigma_V \sqrt{T}\varepsilon)$ where $\varepsilon \sim N(0,1)$

Natural Default Probability in the Merton Model

- ***Default*** occurs in the Merton model when, at maturity, V_T is lower than the face amount of the debt, *B*. In other words when:

$$V_T = V\exp((\mu - \frac{1}{2}\sigma_V^2)T + \sigma_V\sqrt{T}\varepsilon) \quad B \ \text{ i.e. when }, \varepsilon \quad \frac{\ln(B/V) - (\mu - \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}$$

- And so the ***probability of default*** is simply

$$\text{prob} \ \ \varepsilon \ = \frac{\ln(B/V) - (\mu - \frac{١}{٢}\sigma_V^{٢})T}{\sigma_V\sqrt{T}} \quad N \ \frac{\ln(B/V) - (\mu - \frac{١}{٢}\sigma_V^{٢})T}{\sigma_V\sqrt{T}}$$

- Where *N(.)* is the cumulative normal distribution

The Distance-to-Default

$$\text{default value for } \varepsilon = \frac{\ln(B/V) - (\mu - \frac{1}{2}\sigma_V^2)T}{\sigma_V \sqrt{T}}$$

- In the Merton model, default occurs when the "surprise" term, ε, is large enough (typically a large ***negative*** number). ***What does this number mean?***
- In the numerator, *ln(B/V)* is the ***actual*** continuously compounded return on the assets that is necessary to lead to default.
  - ✓ if $V > B$, this return is negative (i.e., the asset value must fall to lead to default).
- The term $(\mu - \sigma^2 / 2)\ T$ is the ***expected value*** of the continuously compounded return (usually positive)
- Thus the numerator is the difference between the actual continuously compounded rate of return required for default and the expected value of the return, i.e., it is the "***surprise***", or unexpected component of the rate of return necessary for default.
- The ***denominator*** is the standard deviation of the rate of return

The Distance-to-Default, contd.

$$\text{default value for } \varepsilon = \frac{\ln(B/V) - (\mu - \frac{1}{2}\sigma_V^2)T}{\sigma_V \sqrt{T}}$$

- Therefore, the ***ratio*** (again typically negative) measures the number of standard deviations of return necessary to lead to default at time T
- ***The negative of this ratio (a positive number) is called the distance-to-default***

$$\text{Distance-to-Default} = \frac{\ln(V/B) + (\mu - \frac{1}{2}\sigma_V^2)T}{\sigma_V \sqrt{T}}$$

* Note: Sometimes, the term "distance to default" is applied to other, closely related, quantities

Merton Model Distance to Default and Default Probabilities

- Distance to default is smaller (and default probability higher) when volatility is higher and maturity is longer

Distance-to-Default*

		V			
Vol	***T***	***150***	***100***	***80***	***60***
20%	1	5.89	3.87	2.75	1.31
20%	20	3.02	2.56	2.31	1.99
40%	1	2.80	1.78	1.23	0.51
40%	20	0.84	0.61	0.49	0.33

Default Probabilities*

		V			
Vol	***T***	***150***	***100***	***80***	***60***
20%	1	0.00%	0.01%	0.30%	9.48%
20%	20	0.13%	0.52%	1.03%	2.31%
40%	1	0.26%	3.73%	11.03%	30.65%
40%	20	20.11%	27.06%	31.34%	37.24%

*Note: Assumptions - expected return on assets = 10%; face value of debt = 50